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Novel of statistical quality control development and econometric applications

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Abstract

Purpose – The purpose of this paper is to investigate and review the impact of the use of statistical quality control (SQC) development and analytical and numerical methods on average run length for econometric applications.

Design/methodology/approach – This study used several academic databases to survey and analyze the literature on SQC tools, their characteristics and applications. The surveys covered both parametric and nonparametric SQC.

Findings – This survey paper reviews the literature both control charts and methodology to evaluate an average run length (ARL) which the SQC charts can be applied to any data. Because of the nonparametric control chart is an alternative effective to standard control charts. The mixed nonparametric control chart can overcome the assumption of normality and independence. In addition, there are several analytical and numerical methods for determining the ARL, those of methods; Markov Chain, Martingales, Numerical Integral Equation and Explicit formulas which use less time consuming but accuracy. New ideas of mixed parametric and nonparametric control charts are effective alternatives for econometric applications.

Originality/value – In terms of mixed nonparametric control charts, this can be applied to all data which no limitation in using of the proposed control chart. In particular, the data consist of volatility and fluctuation usually occurred in econometric solutions. Furthermore, to find the ARL as a performance measure, an explicit formula for the ARL of time series data can be derived using the integral equation and its accuracy can be verified using the numerical integral equation.

Keywords Average run length, Detection, Analytical results, Nonparametric control chart, Time series

Paper type Research paper

1. Introduction

Due to intense market competition and economic growth, quality control of production processes and services in industrial factories and operational facilities is now critical. In order to identify and manage any modifications to these procedures, these operational facilities must utilize statistical quality control (SQC) tools. Many kinds of quality control instruments are used in statistical control of production processes. The check sheet, histogram, Pareto diagram, graph, cause and effect diagram, scatter diagram, DMAIC and control chart are the seven tools that Julan (1954) developed and compiled for use in quality control. The control chart is the most effective tool for identifying process changes because it provides clear results, which makes it popular and widely used.

Statistical process control (SPC) charts are widely used in many fields of application to measure, monitor, control and improve quality. These charts are used to monitor the

JEL Classification — C10, C14, C15

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performance of output processes (manufacturing, etc.) in order to identify any potential changes and swiftly return the particular process to its target value. Shewhart proposed the use of basic SPC charts for quality control in 1931 (Montgomery, 2009). The traditional control chart, x-bar and R control charts, S control chart and other types of control charts are frequently employed in the measurement of variables. The charts that regulate a production process by measuring the characteristics of the products to classify them as being of good or bad quality, defective or not, having flaws, etc. are known as attribute control charts. Control charts for defect proportion (*p* chart), number of defects per unit (*c* chart) and other popular control charts are used to measure attributes.

In addition to industry, SPC charts are now widely used in many other fields with practical uses, including health care (Frisén, 1992; Hawkins and Olwell, 1998; Hillson *et al.*, 1998), Epidemiology according to Sitter *et al.* (1990), Westgard *et al.* (1977) published Clinical Chemistry, Bioterrorism (refer to Hutwagner *et al.*, 2003), Economics and Finance (Ergashev, 2003; Anderson, 2002), detection of computer intrusions (see Ye *et al.*, 2002, 2003), Environmental Sciences (Basseville and Nikiforov, 1993). One of the main goals of SPC charts is to identify process changes as soon as possible, but it is also ideal to have a low false alarm rate. An in-control process's parameter is supposed to be maintained at a given target value, but the target value may fluctuate at an unidentified time point, at which point the process is said to be out of control. Before concluding that the process is out of control, a controller watches it until an alarm time.

Let us assume that the consecutive observations $\{\xi_1, \xi_2, \ldots\}$ are random variables that are independent and have a distribution function $F(x, \alpha)$ the parameter α_0 being present. In addition, let us assume that the process is "in-control" prior to the change-point (θ) ("out-of-control"), and that the distribution function of t is $F(x, \alpha)$ where $\alpha \neq \alpha_0$, following the change-point

$$\xi_t \sim \begin{cases} F(x, \alpha_0), & t = 1, 2, .., \theta - 1 \\ F(x, \alpha), & t = \theta, \theta + 1, \ldots \end{cases}$$

The Shewhart charts only consider the most recent observation when determining an alarm time; previous observations are entirely ignored. Because of this, these charts are ineffective for tracking minute changes. The control charts that were previously discussed make use of the Shewhart control limit construction principles, also referred to as the control chart or standard control chart. Both the process of major shifts and changes in the mean of production processes can be effectively detected by these charts. Control charts that consider historical data were created because the standard control chart does not. One such example is the cumulative sum control chart (CUSUM), which was put forth by Page in 1954. Eventually, Roberts (1959) developed the exponentially weighted moving average control chart (EWMA), which is useful for identifying even the smallest process variations (Montgomery, 2009). Next came the proposal for the double exponentially weighted moving average control chart (DEWMA) by Butler and Stefani (1994). The moving average control chart (MA), which is a control chart that calculates the average by locating the moving average (w), was later developed by Khoo (2004). It works well with both discrete and continuous distributions and is capable of detecting small changes. Subsequently, Khoo and Wong (2008) collaborated to create the Double Moving Average Control Chart (DMA), a chart that regulates the MA chart's statistical value in order to determine the moving value once more. Using the Monte Carlo method to compare the effectiveness of the DMA chart with the MA, CUSUM and EWMA charts, it was discovered that the EWMA and CUSUM charts will most effectively detect changes when a small change in the process ($\delta \leq 0.10$) occurs. The DMA control chart performs best when there is a moderate change in a process ($0.20 \leq \delta \leq 1.50$). These statistical control charts are typically employed in computer science, communications, finance,

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economics, environmental research and the construction of industrial factories. The ability to quickly identify changes in the process parameters while minimizing the rate of false alarms when the production process is under control is a key goal when utilizing these control charts. When a process is out of control, both the true alarm rate and the in-control process are at their highest.

Mostly, the attributes or quality characteristics often have normal distribution, such as the quantity of soft drink containers, average length of wire, average quantity of containers of rice, etc. However, in practice, the interesting characteristics most likely have the attributes of non-normal distributions, such as when measuring the quality and attributes, for example, the time spent processing a telephone payment or the time spent queuing for services in a department store. The proper distribution for these measurements is the exponential distribution, as well as other types. The most widely used control chart nowadays for determining a process's change in mean or standard deviation is the parametric control chart, which is a control chart that makes use of a parameter. The assumption of normality, homogeneity of variances and independence affects the data and, consequently, the characteristics of the parametric control chart, which is its drawback. In practice, the distribution of the information collected during the production process is either unknown or merely an identified distribution; the parameters of the distribution are unknown. The non-parametric control chart was consequently developed as a solution for these issues.

2. Control charts and properties

Statistical quality controls are one of the most useful and versatile tools in production quality control, along with control charts and acceptance sampling. A control chart is a statistical tool that allows a manufacturer to monitor the quality of production processes in the form of a graph and identify any issues that need to be fixed or the process adjusted back to normal. Although, there are two categories for control charts; first the variable control charts, which use a quantitative measurement – for example, the weight of wheat or the thickness of steel to regulate the production process. Second, the attribute control charts use a qualitative measurement to regulate the production process by classifying the products based on factors like the quantity of NGs or part defects, among others and whether or not they are qualified or disqualified or have defects. There are numerous varieties of attribute control charts, including *p, np, c* and *u* charts. Although, there are usually categorized into two types of control chart but in the point of view of this survey article will review a perspective of control chart in two dimension as parametric and nonparametric control charts as follows.

2.1 Parametric control charts

Depending on the kind of data being tracked, parametric control charts come in a variety of forms. The X-bar chart, which tracks the process mean and the R (Range) chart, which tracks the process variability, are typical examples. It is crucial to remember that parametric control charts have a limitation in that they require the assumption of a particular distribution. A large deviation of the data from the assumed distribution could potentially impair the control chart's performance. The use of non-parametric or distribution-free control charts may be appropriate in these circumstances.

Although parametric control charts have shown to be useful instruments for process monitoring and control, they have certain drawbacks. The following are some of the main drawbacks of parametric control charts:

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(1) Assumption of Normality: The underlying process data are assumed to follow a normal distribution by parametric control charts, especially those that are based on the normal distribution (like X-bar and R charts). In the event that this supposition is broken, the control chart might draw the wrong conclusions and the control limits might not be trustworthy.

(2) Sensitivity to Outliers: Outliers and extreme values in the data may cause parametric control charts to become sensitive. Inaccurate control limits and possibly false alarms can result from outlier distortion of central tendency and variability estimates.

(3) Difficulty in Handling Non-Stationary Processes: The mean and variance of a process are assumed to be constant over time in parametric control charts, based on the idea that the process is stationary. Parametric control charts might not work as well in real-world scenarios especially in econometric applications where processes change or evolve over time.

(4) Inaccuracy in the Presence of Autocorrelation: The effectiveness of parametric control charts can be impacted by autocorrelation, or the correlation between subsequent observations. The independence assumption that underpins control chart computations may be broken if observations lack independence.

(5) Limited Adaptability to Non-Normal distributions: Normally distributed data sets are the most common use case for parametric control charts. Using non-parametric control charts or other alternative techniques may be more appropriate when working with non-normal data distributions.

In order to tackle some of these constraints, professionals might think about employing strong statistical techniques, adjusting data to attain normalcy or investigating non-parametric control charts when necessary. It is critical to evaluate parametric control chart assumptions and constraints in light of the particular process that needs to be considered. In this part, some importance time weighted parametric control charts are presented as follows.

(1) Cumulative Sum Control Chart: CUSUM Chart

In 1954, Page introduced the Cumulative Sum (CUSUM) control chart (Page, 1954). The results of calculating the cumulative sum of observations from the start of the current period to the present are obtained by a CUSUM control chart. A control chart is then created using the total. Compared to Shewhart's control chart, this one can identify even more subtle changes in process averages. Given a normal distribution with mean and variance and a random sample taken from an independent population. The statistics of CUSUM consider two factors: when the production process shifts in the direction of an increase in the mean. Additionally, the average direction decreases as the production process changes. In this study, we examine the scenario where the upward average (also known as one-sided CUSUM) changes. The cumulative sum C_t, which can be computed as follows, is the CUSUM statistic at time t

$$C_t = \max(0, X_t - K + C_{t-1}) \tag{1}$$

where C_t is the cumulative sum of CUSUM statistics at time period t, where $t = 1, 2, \ldots, K$ is a constant value called the reference value of the CUSUM chart and set the C_t statistic to the default value $C_0 = 0$.

The control limits of the CUSUM chart are shown in Figure 1. The CUSUM control chart warns that the process is out of control. When the C_t statistic is greater than the upper control limit (H_1) where $H_1 = UCL_{CUSUM} > 0$.

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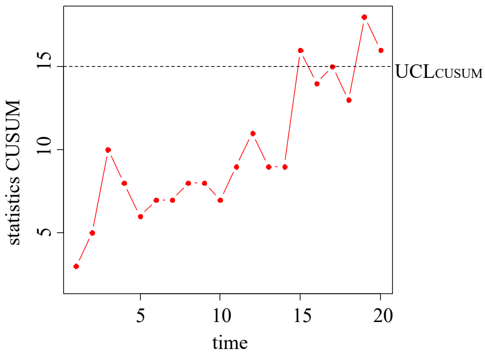

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Figure 1. Control limits of the CUSUM control chart

(2) Exponentially Weighted Moving Average Control Chart: EWMA Chart

Roberts (1959) introduced the EWMA control chart (see also Lucas and Saccucci, 1990). It is a useful tool for identifying even minute changes in process parameters. Based on the statistic, an EWMA control chart is created to track a process mean.

$$Z_t = \lambda X_t + (1-\lambda)Z_{t-1}, t = 1,2,\ldots \tag{2}$$

where Z_t is the process observation at time t. At the first time point, an initial value is given as $Z_0 = \mu_{0,}$, where X_t are the independent and normally distributed observations, and λ is the weighing parameter of the historical data with values ranging from 0 to 1, the mean and variance of are:

$$E(Z_t) = \mu_0$$

$$\text{and } Var(Z_t) = \sigma^2\left(\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2t}\right)\right), t = 1,2,\ldots \tag{3}$$

From Equation (3), when $t \to \infty$, the asymptotic variance of EWMA statistics is

$$Var(Z) = \sigma^2\left(\frac{\lambda}{2-\lambda}\right). \tag{4}$$

Therefore, the control limits of the EWMA control chart are as the following

$$UCL/LCL = \mu_0 \pm H_2\sqrt{\sigma^2\left(\frac{\lambda}{2-\lambda}\right)} \tag{5}$$

where H_2 is a coefficient of the control limit of the EWMA control chart, μ_0 is the mean of the process and the variance is σ^2.

(3) Double Exponentially Weighted Moving Average Control Chart: DEWMA Chart

The DEWMA control chart was developed by Butler and Stefani (1994) from the EWMA control chart by double exponentially weighting where Y_t is the statistics of DEWMA control chart obtained from the second weighting of EWMA and the weighted parameter of the previous observation $\lambda_1 = \lambda_2 = \lambda$ is between 0 and 1. The statistic of the DEWMA Chart is as follows.

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$$Y_t = \lambda_2 Z_t + (1-\lambda_2)Y_{t-1}, t = 1, 2, \ldots \tag{6}$$

where Z_t is the EWMA statistics: $Z_t = \lambda_1 X_t + (1-\lambda_1)Z_{t-1}$. Then,

$$E(Y_t) = E(Z_t) = E(X_t) = \mu_0$$

$$V(Y_t) = \left(\lambda^4 \frac{1+(1-\lambda)^2 - \left(t^2+2t-1\right)(1-\lambda)^{2t} + \left(2t^2+2t-1\right)(1-\lambda)^{2t+2} - t^2(1-\lambda)^{2t+4}}{\left(1-(1-\lambda)^2\right)^3}\right)\sigma^2. \tag{7}$$

From Equation (7) where $t \to \infty$, the asymptotic variance is

$$V(Y) = \left(\frac{\lambda\left(2-2\lambda+\lambda^2\right)}{(2-\lambda)^3}\right)\sigma^2.$$

Therefore, the control limit of the DEWMA Chart is

$$UCL/LCL = \mu_0 \pm H_3 \sigma \sqrt{\frac{\lambda\left(2-2\lambda+\lambda^2\right)}{(2-\lambda)^3}} \tag{8}$$

where H_3 is the coefficient of the control limit of the DEWMA Chart.

(4) Moving Average Control Chart: MA Chart

A Moving Average (MA Chart) is a control chart that uses historical data to find an equally weighted moving average. If the data are highly dispersed, using a moving average will help smooth out the data. And the MA control chart can detect changes in the mean and changes in the production process variance. Both are changes in the direction of increasing and decreasing (Khoo, 2004).

Given $X_1, X_2, \ldots$ a random sample from an independent population and follows the normal distribution with mean μ and variance σ^2. A moving average with averaging width w at period t is denoted by the symbol MA_t can be calculated as follows

$$MA_t = \begin{cases} \dfrac{X_t + X_{t-1} + X_{t-2} + \ldots}{t}, & t < w \\[2ex] \dfrac{X_t + X_{t-1} + \ldots + X_{t-w+1}}{w}, & t \geq w \end{cases} \tag{9}$$

where w is the width of the MA control chart, and the mean and variance of MA statistics are $E(MA_t) = \mu$ and

$$Var(MA_t) = \begin{cases} \dfrac{\sigma^2}{t}, & t < w \\[2ex] \dfrac{\sigma^2}{w}, & t \geq w. \end{cases} \tag{10}$$

Then, the control limits of the MA chart are as the following,

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$$UCL/LCL = \begin{cases} \mu \pm \frac{H_4 \sigma}{\sqrt{t}}, & t < w \\ \mu \pm \frac{H_4 \sigma}{\sqrt{w}}, & t \geq w \end{cases} \tag{11}$$

where H_4 is a coefficient of the control limit of an MA Chart.

(5) Double Moving Average Control Chart: DMA Chart

A Double Moving Average (DMA Chart) is a control chart that uses historical data to find an unweighted moving average, similar to an MA Chart, by using the moving average *(MA_t)* to find the moving average. The second time DMA Charts are able to detect changes in process averages when the changes are small (Khoo and Wong, 2008) and DMA Chart can be written as

$$DMA_t = \begin{cases} \frac{1}{t} \sum_{j=1}^{t} MA_j & ; t \leq w \\ \frac{1}{w} \sum_{j=t-w+1}^{t} MA_j & ; w < t < 2w-1 \\ \frac{1}{w} \sum_{j=t-w+1}^{t} MA_j & ; t \geq 2w-1 \end{cases} \tag{12}$$

where MA_t is the MA statistics calculated from Equation (9) and the DMA_t statistics can be divided into 3 periods t, namely $t \leq w$, $w < t < 2w-1$ and $t \geq 2w-1$ the mean and variance of DMA_t statistics can therefore be considered in three cases as follows.

Case I: $t \leq w, E(DMA_t) = \mu$ and $Var(DMA_t) = \frac{\sigma^2}{t^2} \sum_{j=1}^{t} \frac{1}{j}$

Case II: $w < t < 2w-1, E(DMA_t) = \mu$ and $Var(DMA_t) = \frac{\sigma^2}{w^2} \left(\sum_{j=t-w+1}^{w-1} \frac{1}{j} + \frac{(t-w+1)}{w} \right)$

Case III: $t \geq 2w-1, E(DMA_t) = \mu$ and $Var(DMA_t) = \frac{\sigma^2}{w^2}$

Therefore, the control limits of DMA Chart when $t \leq w$,

$$UCL_1/LCL_1 = \mu_0 \pm H_5 \sqrt{\frac{\sigma^2}{t^2} \sum_{j=1}^{t} \frac{1}{j}}.$$

Next, the control limits of DMA Chart when $w < t < 2w-1$,

$$UCL_2/LCL_2 = \mu_0 \pm H_5 \sqrt{\frac{\sigma^2}{w^2} \left(\sum_{j=t-w+1}^{w-1} \frac{1}{j} + \frac{(t-w+1)}{w} \right)}.$$

Next, the control limits of DMA Chart when $t \geq 2w-1$,

$$UCL_3/LCL_3 = \mu_0 \pm H_5 \sqrt{\frac{\sigma^2}{w^2}}$$

where H_5 is a coefficient of the control limit of an DMA Chart.

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(6) Mixed Moving Average – Exponentially Weighted Moving Average Control Chart: MME Chart

The MA and EWMA charts from Sukparungsee *et al.* (2022) are combined to create this chart. The EWMA Chart plot statistic is fed into the MA Chart in the mathematical model created for the MME chart design. Consequently, the following is the MME chart's statistic:

$$MA_t = \begin{cases} \dfrac{Z_t + Z_{t-1} + Z_{t-2} + \ldots}{t}, & t < w \\ \dfrac{Z_t + Z_{t-1} + \ldots + Z_{t-w+1}}{w}, & t \geq w. \end{cases} \tag{13}$$

Thus, the asymptotical control limit of the MME Chart is as follows:

$$UCL/LCL = \begin{cases} \mu_Z \pm H_6 \sqrt{\left(\dfrac{\sigma_Z^2}{t}\right)\left(\dfrac{\lambda}{2-\lambda}\right)}, & t < w \\ \mu_Z \pm H_6 \sqrt{\left(\dfrac{\sigma_Z^2}{w}\right)\left(\dfrac{\lambda}{2-\lambda}\right)}, & t \geq w \end{cases} \tag{14}$$

where H_6 is the coefficient of the control limits for the MME Chart, μ_Z and σ_Z^2 are the mean and variance of the EWMA statistics.

(7) Mixed Exponentially Weighted Moving Average – Moving Average Control Chart: MEM Chart

The EWMA and the MA charts were combined to create the MEM chart proposed by Taboran *et al.* (2019). The EWMA and MA charts can be effectively substituted with these charts. The statistics remain part of the EWMA chart, as Equation (15) illustrates.

$$Z_t = \lambda MA_t + (1-\lambda)Z_{t-1}, t = 1, 2, \ldots \tag{15}$$

where Z_0 is the initial value and is set to equal the target mean for the weighing parameter λ of the data in the past. The expected values for the data are represented by the UCL and LCL of the MEM Chart, and they will coincide with the value of the MA Chart. The following control limits will be used for variance between the EWMA and MA Charts:

$$UCL/LCL = \mu_{MA} \pm H_7 \sqrt{\left(\frac{\sigma_{MA}^2}{w}\right)\left(\frac{\lambda}{2-\lambda}\right)} \tag{16}$$

where H_7 is a coefficient of the control limits of the MEM Chart, μ_{MA} and σ_{MA}^2 are the mean and variance of the MA statistics.

(8) Modified Exponentially Weighted Moving Average Control Chart: MEWMA Chart

Assuming that the observations (X_t) at the time t follow a normal distribution, Khan *et al.* (2017) developed the MEWMA Chart, which is very effective at detecting small and abrupt shifts. The MEWMA Chart's statistic is defined as follows

$$M_t = \lambda X_t + (1-\lambda)M_{t-1} + k(X_t - X_{t-1}), t = 1, 2, \ldots \tag{17}$$

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where M_t is the MEWMA statistic at time tth, λ is a smoothing parameter between 0 and 1 and k is an additional parameter ($k \neq 0$), see also Patel and Divecha (2011). Then the mean and variance of M_t are $E(M_t) = \mu$, and

$$V(M_t) = \sigma^2 \left(\frac{(\lambda + 2\lambda k + 2k^2) - \lambda(1-\lambda-k)^2(1-\lambda)^{2(t-1)}}{(2-\lambda)} \right). \tag{18}$$

When $t \to \infty$, the variance from Equation (18) become to be asymptotic variance is

$$V(M_t) = \sigma^2 \left(\frac{\lambda + 2\lambda k + 2k^2}{2-\lambda} \right).$$

The upper and lower control limits of MEWMA Chart are:

$$UCL/LCL = \mu \pm H_8 \sigma \sqrt{\frac{\lambda + 2\lambda k + 2k^2}{2-\lambda}}$$

where H_8 is a coefficient of the control limits of the MEWMA Chart. μ is the mean of the process and σ^2 is variance of the process.

(9) Mixed Moving Average – Modified Exponentially Weighted Moving Average Control Chart: MMME Chart

This chart is a combination of the MA and MEWMA control charts. The statistic M_t of the MEWMA chart is used input for the MA chart. The statistic of the MMME control chart is defined as:

$$MMME_t = \begin{cases} \dfrac{M_t + M_{t-1} + M_{t-2} + \ldots}{t}, & t < w \\[2ex] \dfrac{M_t + M_{t-1} + \ldots M_{t-w+1}}{w}, & t \geq w. \end{cases} \tag{19}$$

Therefore, the asymptotical upper and lower control limits of the MMME chart are given as follow:

$$UCL/LCL = \begin{cases} \mu_M \pm H_9 \sqrt{\left(\dfrac{\sigma_M^2}{t}\right)\left(\dfrac{\lambda + 2\lambda k + 2k^2}{2-\lambda}\right)}, & t < w \\[3ex] \mu_M \pm H_9 \sqrt{\left(\dfrac{\sigma_M^2}{w}\right)\left(\dfrac{\lambda + 2\lambda k + 2k^2}{2-\lambda}\right)}, & t \geq w \end{cases}$$

where H_9 is a coefficient of the control limits of the MMME Chart. μ_M is the mean of MEWMA and σ_M^2 is variance of the MEWMA statistics.

(10) Mixed Modified Exponentially Weighted Moving Average – Moving Average Control Chart: MMEM Chart

The MMEM chart is a combination of MEWMA and MA Charts proposed by Talordphop *et al.* (2022). The statistic of MMEM control chart is defined as:

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$$MMEM_t = \lambda MA_t + (1-\lambda)MMEM_{t-1} + k(MA_t - MA_{t-1}), t = 1, 2, \ldots$$

where λ is the weighing parameter of the data in the past and k is constant ($k \neq 0$). The upper and lower control limits of the MMEM Chart are given as follow:

$$UCL/LCL = \mu_{MA} \pm H_{10}\sqrt{\left(\frac{\sigma_{MA}^2}{w}\right)\left(\frac{\lambda + 2\lambda k + 2k^2}{2-\lambda}\right)}$$

where H_{10} is a coefficient of the control limits of the MMEM Chart. μ_{MA} is the mean of MA and σ_{MA}^2 is variance of MA statistics.

In an effort to increase the precision of the monitoring techniques, mixed control charts were developed. Several articles in the literature discuss combining control charts to enhance the control chart's capacity to identify abrupt changes in data. Wong *et al.* (2004) proposed an MA-Shewhart Charts combined structure to achieve a straightforward construction in the finance field. Abbas *et al.* (2012) investigated the tracking process combined EWMA-CUSUM Charts. The mixed CUSUM-EWMA chart (MCE) was proposed by Zaman *et al.* (2015) as a tool for monitoring the status of a procedure. Mixed EWMA-MA Charts were proposed by Sukparungsee *et al.* (2022), while MA-EWMA Charts were designed by Taboran *et al.* (2019). A combined MA-CUSUM Chart was presented by Saengsura *et al.* (2022) to track changes in parameters. All of the previously mixed control charts performed admirably in standard operating environments. The efficacy of these displays is questioned when the process is unable to meet the assumption of normalcy. Nowadays, there are several mixed control charts to achieve quick detection both in process mean and dispersion. The proposed mixed control charts are example to present in this survey paper.

2.2 Non-parametric control charts

Nonparametric charts are becoming more and more accepted and are being used for real-world problems in circumstances where the population's distribution is unknown and parameter estimation is not feasible. However, the nonparametric chart's interface is easy to use even with these shortcomings. The most widely used control chart nowadays for determining a process change in mean or dispersion is the parametric control chart, which is a control chart that makes use of a parameter. The assumption of normality, homogeneity of variances and independence affects the data and, consequently, the characteristics of the parametric control chart, which is its drawback. In actuality, the distribution of the information gathered during the production process is either unknown or only the identified distribution; the parameters of the distribution are not known. The nonparametric control chart was consequently developed as a solution for these issues.

Ryan (2000) introduced the Arcsine control chart and it is an extremely effective tool for determining the process mean. The Tukey's Control Chart (TCC), a control chart intended for examining the data with a single observation or subgroup size, was later proposed by Alemi (2004). Additionally, TCC can be applied to the analysis of a process in cases where the process's quality features are non-normally distributed or its quality attributes fall within a normal distribution. Later, Yang *et al.* (2011) proposed the Arcsine Exponentially Weighted Moving Average Sign Chart (Arcsine EWMA Sign), which is an Arcsine control chart that uses exponential weighting to detect small changes without the need for a parameter, which was modified from the EWMA Sign Chart to detect process mean error by transforming data to the standard normal distribution using the Arcsine method. Tukey's Control Chart may not be able to fully control symmetry and asymmetry in cases where the process has a positively or negatively skewed distribution, according to Sukparungsee (2012). The study's

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findings showed that the control chart with restrictions on asymmetry control has a stronger effect on distribution skewness than the control chart with restrictions on symmetry control. The Tukey-Cumulative Control Chart (TCC-CUSUM), then, was proposed by Khaliq and Riaz in 2014 to identify average changes in processes with symmetric and asymmetric distributions. Compared to TCC and CUSUM, the suggested control chart was more effective at identifying the change. The Exponentially Weighted Moving Average-Tukey's Control Chart (EWMA-Tukey), which was created from the TCC and combines EWMA and TCC using the average run length (ARL) as the criterion, was presented by Khaliq *et al.* in 2016. The outcomes demonstrated that Tukey-EWMA is more efficient than TCC and EWMA combined. In order to track changes in the average parameter, Supchotthatree (2016) later proposed the Double Exponentially Weighted Moving Average-Tukey's control chart (DEWMA-TCC). It was discovered that this method was more efficient than TCC, DEWMA and EWMA-TCC. In order to identify changes in the variation of processes with symmetric and asymmetric distributions, Mongkoltawat proposed the Exponentially Weighted Moving Average-Tukey's control chart for Moving Range and the Exponentially Weighted Moving Average-Tukey's control chart for range (EWMAMR-TCC and EWMAR-TCC) in 2016. ARL was used as the criteria. At all change levels, the suggested control chart's efficiency was discovered to be superior to that of the EWMA and TCC. Eventually, in 2017, Tukey EWMA-CUSUM Chart (MEC-TCC) was proposed by Khaliq and Gul for robustness against non-normality and shift detection. Based on a comparative analysis, the suggested scheme outperforms its current counterparts, which include classical Shewhart, EWMA, CUSUM, Tukey and other variants like mixed EWMA-CUSUM, Tukey EWMA and Tukey CUSUM Charts. Furthermore, certain charts mentioned above are shown as exceptional cases in the suggested design. Two real-life data sets were used for real-life considerations: one was connected to air quality, and the other was about smartphone accelerometers. Nonparametric control charts have also been created and designed by numerous authors in a variety of contexts, such as Abbas *et al.* (2018), Chakraborti and Graham (2019), Riaz *et al.* (2019), Shafqat *et al.* (2020) and Mabude *et al.* (2022).

Nonparametric, or distribution-free control charts – control charts that do not take parameters into account were essential in helping to overcome these constraints. The nonparametric EWMA-Sign (EWMA-SN) Chart was initially proposed by Yang *et al.* (2011) for non-normal or unknown distributions. It is an effective method for identifying slight changes in the process position. Additionally, the data were converted to the standard normal distribution using the arcsine method, which was used to construct the ArcSine-EWMA Chart from the EWMA-SN chart in order to detect the error of the mean during the process. Liu [17] created the GWMA-Sign (GWMA-SN) Chart to improve the detection capability for slight process shifts. A distribution-free EWMA Chart based on the Wilcoxon Signed-Rank statistic was presented by Amin and Searcy (1991) to guarantee the advantage of their nonparametric chart against the classical $\overline{X}$-EWMA scheme under heavy-distributions. The MEWMA-Sign (MEWMA-SN) Chart was proposed by Aslam *et al.* (2020), using ARL as the criterion for measuring performance. The EWMA-SN chart was more effective at detecting changes than the EWMA-SN and EWMA Charts but could not detect process changes in the right-skewed distribution case. The EWMA-SN Chart was more effective at detecting changes than the EWMA-SN and EWMA Charts but could not detect process changes in the right-skewed distribution case, which was studied by Taboran and Sukparungsee in (2022). Petcharat and Sukparungsee (2023) developed the MEWMA-Signed-Rank (MEWMA-SR) chart, which combined the MEWMA Chart with nonparametric statistics known as Sign Rank. The performance of the MEWMA-SR Chart was compared with that of EWMA, EWMA-SN, EWMA-SR, MEWMA and MEWMA-SN Charts to detect a change in the process mean parameter.

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There are some nonparametric statistics applied to statistical control charts, which are easy to implement in many application areas, especially when the observations are of unknown distribution or the parameters cannot be estimated. The nonparametric statistics as Sign, Arc-Sine and Sign-Ranked and the mixed nonparametric control charts are presented as follows.

(1) Sign Statistics: SN

Assume that $X_{jt}, j = 1, 2, ..., n$ and $t = 1, 2, 3, ...$ represent the observation in the size (n) subgroup. Equation (20) can be used to indicate the difference between the observations and the target value *(T)*, or $X_{jt} - T$ within groups, if the known target value is to be monitored.

$$Y_{jt} = X_{jt} - T, t = 1, 2, 3, ..., j = 1, 2, ..., n. \tag{20}$$

The Sign statistic S_t can be determined as Equation (21):

$$SN_t = \sum_{j=1}^{n} I_{jt}. \tag{21}$$

Equation (21), I_{jt} can be written as Equation (22):

$$I_{jt} = \begin{cases} 1, & Y_{jt} > 0 \\ 0, & \text{otherwise} \end{cases}. \tag{22}$$

The total number of observations that fit the binomial distribution $(n, p_0 = 0.5)$ with a parameter for the control case is then the Sign statistic. The process proportion in the control process $p = p_0 = P(Y \leq T) = P(Y > T) = 0.5$ is represented by the value of $p = P(Y > 0)$. However, the procedure spirals out of control when $p_0 \neq 0.5$.

(2) Sign Rank Statistics: SR

Let W_{jt} denote the rank of the absolute difference $|X_{jt} - T|$ within the j^{th} subgroup. The sign rank statistics are defined as follows

$$SR_j = \sum_{j=1}^{n} I_{jt} W_{jt} \tag{23}$$

where $I_{jt} = \begin{cases} 1, when(X_{jt} - T) > 0 \\ 0, when(X_{jt} - T) = 0 \\ -1, when(X_{jt} - T) < 0 \end{cases}$

(3) Exponentially Weighted Moving Average–Sign Control Chart: EWMA-SN Chart

Yang *et al.* (2011) created the mixed EWMA chart based on the Sign statistic, known as the EWMA-SN Chart. The EWMA-SN statistic with a smoothing parameter $\lambda (0 < \lambda \leq 1)$, as shown in Equation (24):

$$Z_{SN_t} = \lambda SN_t + (1 - \lambda) Z_{SN_{t-1}}. \tag{24}$$

The mean and asymptotic variance are

$$E(Z_{SN_t}) = np$$

$$V(Z_{SN_t}) = npq\left(\frac{\lambda}{2 - \lambda}\right).$$

The EWMA-SN control limits are

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$$UCL/LCL = np \pm H_{11}\sqrt{\frac{\lambda}{2-\lambda}(npq)}$$

where H_{11} is the EWMA-SN chart's coefficient control limit corresponding to the wanted ARL_0.

(4) Exponentially Weighted Moving Average – Sign rank Control Chart: EWMA-SR Chart

Graham *et al.* (2011) presented a nonparametric EWMA mixed with a signed-rank statistic control chart. The EWMA-SR statistic as shown in Equation (25):

$$Z_{SR_t} = \lambda SR_t + (1-\lambda)Z_{SR_{t-1}}. \quad (25)$$

The mean and asymptotic variance are

$$E(Z_{SR}) = 0$$

$$V(Z_{SR}) = \frac{n(n+1)(2n+1)}{6}\left(\frac{\lambda}{2-\lambda}\right).$$

The EWMA-SR control limits are

$$UCL/LCL = \pm H_{12}\sqrt{\left(\frac{n(n+1)(2n+1)}{6}\right)\left(\frac{\lambda}{2-\lambda}\right)}$$

where H_{12} is the EWMA-SR chart coefficient control limit corresponding to the desired ARL_0.

3. Time series models

One of the widely used time series analysis for forecasting is Box–Jenkins method, which is a stationary time series data. In the case of non-stationary time series data, the data have to transform into stationary data before using the method. For data with non-stationary in mean, the data have to find the differencing between data. For seasonal data, the data also have to find the seasonal differencing. In addition, for non-stationary in variance, the data have to transform using logarithm.

3.1 Autoregressive integrated moving average model: ARIMA

(1) Stationary models

Autoregressive Moving Average model of order p and q: (ARMA *(p,q)*) is modeled as follow.

$$X_t = \delta + \phi_t X_{t-1} + \phi_2 X_{t-2} + \ldots + \phi_p X_{t-p} + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} - \ldots - \theta_q\varepsilon_{t-q}$$

where X_t is time series data at time t, δ is a constant, $\phi_1, \phi_2, \ldots, \phi_p$ are autoregressive parameters, $\theta_1, \theta_2, \ldots, \theta_q$ are Moving Average parameters, ε_t is an error at time t in a normal distribution with a mean of 0 and variance of σ^2.

(2) Nonstationary Models

If time series data does not have a stationary in mean and variance, it has to transform into a stationary before considering the models using ARIMA $(p,d,q)(P,D,Q)_L$ as follow.

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$$(1-B)^d\left(1-B^L\right)^D\left(1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p\right)\left(1-\Phi_L B^L-\Phi_{2L} B^{2L}-\ldots-\Phi_{PL} B^{PL}\right)X_t$$
$$=\mu+\left(1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q\right)\left(1-\Theta_L B^L-\Theta_{2L} B^{2L}-\ldots-\Theta_{QL} B^{QL}\right)\varepsilon_t$$

where X_t is time series data at time t, δ is a constant, ϕ_i is a autoregressive parameter, Φ_i is a Seasonal Autoregressive parameter, θ_i is a Moving Average parameter, Θ_i is a Seasonal Moving Average parameter, d is number of times to find a differences, D is number of times to find a seasonal differences, ε_t is an error at time t in a normal distribution with a mean of 0 and variance of σ^2.

3.2 Autoregressive fractionally integrated moving average model: ARFIMA

(1) ARFIMA*(p,d,q)* model

$$(1-B)^d\left(1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p\right)X_t=\mu+\left(1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q\right)\varepsilon_t \quad (26)$$

where X_t is time series data at time t, μ is a constant, $(1-B)^d$ is a fractional difference operator, where

$$(1-B)^d:=\sum_{p=0}^{\infty}\binom{d}{p}(-B)^p=1-dB-\frac{d(1-d)}{2!}B^2-\ldots.$$

ε_t is an error at time t in a normal distribution with a mean of 0 and variance of σ^2

B is a backward-shift operator, where $B^p X_t=X_{t-p}$ and $B^q\varepsilon_t=\varepsilon_{t-q}$

3.3 Autoregressive integrated moving average with exogenous variables model

ARIMAX model is as follow.

$$(1-B)^d\left(1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p\right)Y_t=\mu+\left(1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q\right)\varepsilon_t+\sum_{i=1}^{r}\omega_i X_{it}$$

where Y_t is an observation of time series data at time t, X_t is an observation of external parameter at time t, ϕ_i is an autoregressive parameter, θ_i is a moving average parameter, ε_t is an error at time t in a normal distribution with a mean of 0 and variance of σ^2, r is number of external parameters, d is number of times to find differences.

4. The method for evaluating average run length

Two restrictions on true alarm and false alarm times must be traded off in order to detect the change-point. While detecting small changes may require a much larger number of observations, detecting large changes in the parameter may only require a small number of observations. The ARL – the anticipated stopping time assuming that the process is under control is one of the most widely used criteria for selecting an alarm time (to declare that a process is out-of-control). The ARL values obtained from each control chart are the values used to compare performance if it is necessary to use a control chart that appropriate to the existing information. This is because selecting the most efficient control chart will enable the

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fastest detection of process changes. In other words, we will be able to detect process changes as quickly as possible by using the least average number of samples to prevent occurrence. There are two states of ARL: ARL when the process is in-control and ARL when the process is out of control. There are several methods for determining the ARL. Such as Monte Carlo Simulation (MC), Markov Chain Approach (MCA), Numerical Integral Equation (NIE) and Explicit Formula, etc.

4.1 Monte Carlo simulation (MC) method

The MC method is a generally precise technique for estimating ARL values. In general, the ARL value is taken from the method MC compares the results with other methods, however, there may be some limitations, such as the time required to estimate the results is quite long.

$$ARL = \frac{\sum_{i=1}^{N} RL_i}{N}$$

where RL_i is the number of observations used to check until the first observation outside the control limit is found in the *i*th observation simulation, and N is the number of simulations in each scenario. Many researchers studied evaluation by using simulation technique. Costa and Fichera (2017) investigated the economic statistical design of an Autoregressive Moving Average control chart for autocorrelated data by using simulation approach. A Shewhart control chart was suggested by Sales *et al.* (2020) for monitoring the mean in a first order Poisson mixed integer autoregressive process. Using Monte Carlo simulations, the efficacy of the proposed method is evaluated based on the ARLs. The approach is demonstrated in practice through the examination involving the surveillance of criminal activity data and network traffic. (see, Chakraborty and Khurshid (2011).

4.2 Markov Chain Approach (MCA)

One approach employed for evaluating the ARL of control charts is the Markov chain approach. As an illustration, Brook and Evans (1972) firstly proposed a Markov chain approach for calculating the probability distribution of the CUSUM chart. During the 1990s, Lucus and Saccucci estimated ARL values for EWMA control charts utilizing the MCA method. Later, the ARL of the EWMA control chart for monitoring the ZINB model via the Markov chain approach was presented by Chananet *et al.* (2015). Moreover, numerous researchers, Brook and Evans (1972), Woodall (1984), Fu *et al.* (2002), Calzada and Scariano (2003) and Shu and Jiang (2006) analyzed the Markov chain approach as a means of approximating ARL. The MCA may have limitations in terms of convergence. Consequently, ARL cannot continually be approximated from MCA. The use of the Monte Carlo simulation procedure is required.

A general overview of how the Markov Chain approach can be applied to estimate ARL:

(1) Define the Markov Chain: Specify the states of the Markov Chain, where each state represents a different process condition (e.g. in-control or out-of-control). The transition probabilities between states are determined by the control charting rules and the characteristics of the process.

(2) Transition Probability Matrix: Create a transition probability matrix that describes the probabilities of moving from one state to another in a single time step. This matrix is often based on the control limits and the nature of the control chart used.

(3) Determine the probabilities of a steady state: Solve for the steady-state probabilities of being in each state.

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(4) Approximation of the ARL: The ARL for the in-control condition is estimated using the inverse of the steady-state probability for the in-control condition. A longer ARL indicates better performance of the control chart.

Next, we will present the ARL value using the MCA method when the data is Zero-Inflated Poisson for one sided CUSUM control chart which is published in Chananet *et al.* (2015).

Assume X_t are a sequence of independent identically zero-inflated Poisson distributed (i.i.d.) random variables with parameter c and ω. The upper-sided CUSUM control chart is specifically designed to identify an upward movement in the mean. The statistic C_t is used for this purpose.

$$C_t = \max\{0, X_t - a + C_{t-1}\}, t \in \{1, 2, ...,\}$$

where a is a reference value and an initial value $C_0 = 0$. If the statistics C_t exceed the control limit (h), the process is issued a signal to out of control for the case of upper control chart.

If the statistics $C_t = i$, indicate that the control chart is in control states S_i, then each state can be represented as random walk over the states $S_0, S_1, ..., S_h$ where the S_h is an absorbing state to represents an out of control region above the control limit and S_0 is an initial state. The transition probability (P_{ij}) is the probability of moving from state i to state j where $j = 1, 2, ..., N$ in each step shown as follow:

$$\begin{aligned} P_{ij} &= P(C_t \in S_j | C_{t-1} \in S_i) \\ &= P(X - a + C_{t-1} = j | C_{t-1} = i) \\ &= P(X = a + j - i), i \neq h, 0 < j < h \end{aligned}$$

and $P_{i0} = P(X \leq a - i), P_{ih} = P(X \geq a + h - i)$ and $P_{hh} = 1$. If the statistics C_t are move to an absorbing state, it is unachievable to leave that state. As a result, the probability of exiting that state is zero, denoted by $P_{hh} = 1$,; however, this value is not relevant to the ARL calculation.

We can replace to the transition matrix $(\mathbf{P})$ and element of matrix (P_{ij}) as

$$\mathbf{P} = \begin{bmatrix} P_{11} & \cdots & P_{1n} & | & P_{1,n+1} \\ \vdots & \ddots & \vdots & | & \vdots \\ P_{n1} & \cdots & P_{nn} & | & P_{n,n+1} \\ --- & --- & --- & | & --- \\ 0 & \cdots & 0 & | & 1 \end{bmatrix} \text{ or } \mathbf{P} = \begin{bmatrix} \mathbf{R} & (\mathbf{I} - \mathbf{R})\mathbf{1} \\ \mathbf{0} & 1 \end{bmatrix}$$

where $\mathbf{R}$ represents the transition probabilities of going from one transient state to another state with $n \times n$ dimension, $\mathbf{I}$ is the $n \times n$ identity matrix, $\mathbf{1}$ is the $n \times 1$ column vector of ones and $\mathbf{0}$ is the $1 \times n$ row vector of zeros. The ARL, which depends on t in control states, can be derived using Equation (1) proposed by Lucas and Saccucci (1990), where (N) is a positive integer after which substitute $P(RL = i) = \mathbf{p}^T(\mathbf{R}^{i-1} - \mathbf{R}^i)\mathbf{1}$ into Eq. (27). The ARL is rewritable as

$$ARL(t) = \sum_{i=1}^{\infty} iP(RL = i) \tag{27}$$

$$\begin{aligned} ARL(t) &= \sum_{i=1}^{\infty} i\mathbf{P}^T\left(\mathbf{R}^{i-1} - \mathbf{R}^i\right)\mathbf{1} \\ &= \sum_{i=1}^{\infty} \mathbf{P}^T\mathbf{R}^{i-1}\mathbf{1} \\ &= \mathbf{P}^T(\mathbf{I} - \mathbf{R})^{-1}\mathbf{1} \end{aligned} \tag{28}$$

where $\mathbf{p}^T = (0, ..., 0, 1, 0, ..., 0)^T$ is the probability vector of an initial state with 1, corresponding to a specified state and zeros elsewhere.

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4.3 Numerical integral equation (NIE) method

It is a method for estimating ARL with high accuracy. Crowder (1987) studied the numerical integration method to determine ARL. For exponential moving average control charts in the form of Fredholm's integral equation. Champ and Rigdon (1991) studied this method with EWMA and CUSUM control charts. The NIE method of ARL have been proposed by many literatures which are intensively studied (see Peerajit *et al.* (2016), Karoon *et al.* (2021), Bualuang and Peerajit (2023), Saesuntia *et al.* (2023)). The limitation of using this method is that it can be used only for continuous distribution.

A general overview of how the Numerical Integral Equation (NIE) method can be applied to estimate ARL:

(1) Define the control chart model: Specify the control chart model, including the chart type (e.g. Shewhart, CUSUM, EWMA control charts) and the parameters associated with the control chart (e.g. control limits, sample size).

(2) Set up the Integral equation: Formulate the integral equation based on the probability density function (p.d.f.) or cumulative distribution function (c.d.f.) associated with the control chart statistic. This equation represents the probability of staying in the in-control state or transitioning to the out-of-control state.

(3) Discretize the equation: Discretize the integral equation to convert it into a set of algebraic equations. This often involves dividing the domain of the integral into discrete intervals and approximating the integral with appropriate numerical methods.

(4) Solve the discretized equations: Solve the system of discretized equations numerically. This may involve iterative numerical methods or other techniques suitable for solving systems of nonlinear equations.

(5) Estimate ARL: Estimate the ARL by calculating the reciprocal probability of staying in the in-control state.

Herein, the approach of estimating the ARL via the numerical integral equation (NIE) approach such as the Gaussian, midpoint, trapezoidal and Simpson's rules are as.

(1) Simpson's Rule

Given $y = f(x)$. When an interval $[a, b]$ is divided into 2m intervals with the interval width $h = \frac{b-a}{2m}$, Simpson's Rule with 2m intervals is as follow.

$$S(f,h) = \frac{h}{3}(f(a) + f(b)) + \frac{2h}{3}\sum_{k=1}^{m-1} f(x_{2k}) + \frac{4h}{3}\sum_{k=1}^{m} f(x_{2k-1}), k = 1, 2, \ldots, 2m$$

which is an approximation from a definite integral of $f(x)$ in the interval [a, b] as shown in

$$\int_a^b f(x)dx \approx S(f,h)$$

(2) Midpoint Rule

Given $y = f(x)$. When an interval $[a, b]$ is divided into m intervals with the interval width $h = \frac{b-a}{m}$, Midpoint Rule) with m intervals is as follow.

$$M(f,h) \approx h\sum_{k=1}^{m} f\left(a + \left(k - \frac{1}{2}\right)h\right), k = 0, 1, 2, \ldots, m$$

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which is an approximation from a definite integral of $f(x)$ in the interval [a, b] as shown in

$$\int_a^b f(x)dx \approx M(f,h)$$

(3) Trapezoidal Rule

Given $y = f(x)$. When an interval $[a, b]$ is divided into m intervals with the interval width $h = \frac{b-a}{m}$, Trapezoidal Rule with m intervals is as follow.

$$T(f,h) = \frac{h}{2}(f(a) + f(b)) + h\sum_{k=1}^{m} f(x_k), k = 0, 1, 2, .., m$$

which is an approximation from a definite integral of $f(x)$ in the interval [a, b] as shown in

$$\int_a^b f(x)dx \approx T(f,h)$$

(4) Gauss quadrature rule

An approximation from a definite integral of $f(x)$ in the interval [a, b] as shown in

$$\int_{-1}^{1} f(x)dx \approx \sum_{k=1}^{m} w_{n,i} f[z_{n,i}], w = w_{n,1}, w_{n,2}, \ldots, w_{n,n}$$

In the next section, we will present a technique for calculating the ARL utilizing Numerical Integral Equation (NIE) approaches for EWMA control chart.

For a positive one-sided case of EWMA control chart, given that $H_U = H$ and $H_L = 0$, a condition for Stopping Time (τ_H) is:

$$\tau_H = \inf\{\, t > 0: Z_t > H\}$$

when H is a control limit.

Normally, the in control process ARL (ARL_0) is $H_L \leq Z_t \leq H_U$ and out of control process $ARL(ARL_1)$ is $Z_t < H_L$,or $Z_t > H_U$. Define a $T(u)$ function is a follow:

$$T(u) = E_\infty(\tau_{L,U}), X_0 = u$$

where τ_{H_L,H_U} is stopping time and if a process is out of control, the condition for Z_1 is as follow:

$$Z_1 = (1-\lambda)\, Z_0 + \lambda\xi_1 > H_U \text{ or } Z_1 = (1-\lambda)\, Z_0 + \lambda\xi_1 > H_L$$

In case of $ARL = 1$, if Z_1 in control limit can be written as

$$H_L < (1-\lambda)\, Z_0 + \lambda\xi_1 < H_U$$

Therefore, an average $T(Z_1) = T((1-\lambda)\, Z_0 + \lambda\xi_1\,)$ is higher than observation values from an out of control process when detected. It can be rearranged as

$$\frac{H_L - (1-\lambda)\, Z_0}{\lambda} < \xi_1 < \frac{H_U - (1-\lambda)\, Z_0}{\lambda}$$

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A probability distribution is $f(\xi_1)$

$$P\left(\frac{H_L - (1-\lambda)Z_0}{\lambda} < \xi_1 < \frac{H_U - (1-\lambda)Z_0}{\lambda}\right) = \int_{\frac{H_L-(1-\lambda)Z_0}{\lambda}}^{\frac{H_U-(1-\lambda)Z_0}{\lambda}} f(x)dx$$

where $f(x)$ is a probability function.

Therefore, the initial value of $x_0 = u$ and gave $x = \xi_1$. A new equation to find $T(u)$ is as follow:

$$T(u) = 1 - \left(P\left[\frac{H_L - (1-\lambda)Z_0}{\lambda} < \xi_1 < \frac{H_U - (1-\lambda)Z_0}{\lambda}\right]\right) + \int_{\frac{H_L-(1-\lambda)Z_0}{\lambda}}^{\frac{H_U-(1-\lambda)Z_0}{\lambda}} (1 + T((1-\lambda)u + \lambda x))f(x)dx$$

$$= 1 + \int_{\frac{H_L-(1-\lambda)X_0}{\lambda}}^{\frac{H_U-(1-\lambda)X}{\lambda}} T((1-\lambda)u + \lambda x))f(x)dx,$$

When replacing the integral parameters, the new equation is

$$T(u) = 1 + \frac{1}{\lambda}\left(\int_{H_L}^{H_u} T(x)f\left(\frac{x - (1-\lambda)u}{\lambda}\right)dx\right)$$

and it considers only positive one-sided case when $X_t \geq 0$.

For a positive case, ξ_1 can be assumed that $H_L = 0$ and $H_U = H$, then

$$T(u) = 1 + \frac{1}{\lambda}\left(\int_{0}^{H_u} T(x)f\left(\frac{x - (1-\lambda)u}{\lambda}\right)dx\right)$$

It is necessary to use a mathematic method to solve $T(u)$ using the approximation of the limits by rules for finding the area of a point $\{a_j, j = 1, 2, ..., m\}$ in the interval $[0, H]$ and a set of weighted constants $\{w_j, j = 1, 2, ..., m\}$ The approximation of the integral is

$$\int_0^H W(x)g(x)dx \approx \sum_{j=1}^{m} w_j g(a_j),$$

where $W(x)$ and $g(x)$ are functions. Then $\widetilde{T}(u)$ can be approximated as

$$\widetilde{T}(a_i) = 1 + \frac{1}{\lambda}\sum_{j=1}^{m} w_j \widetilde{T}(a_j) f\left(\frac{a_j - (1-\lambda)a_i}{\lambda}\right), i = 1, 2, ..., m.$$

Then,

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$$T_j = \widetilde{T}(a_i), j = 1, 2, ..., m.$$

It can be written in a form of matrix **R** with m x m dimensions as follow

$$[R]_{ij} = \frac{1}{\lambda} w_j f\left(\frac{a_j - (1-\lambda) a_i}{\lambda}\right).$$

Linear algebra equation can be written in a matrix equation as

$$\mathbf{T} = \mathbf{1} + \mathbf{RT}$$

where 1_m vector has a dimension m x 1, **R** is the transition probabilities matrix and **T** is the linear equation matrix. Then, $T - RT = 1$ which is equal to $(1 - R)T = 1$. This formula can solve the mathematical problem for explaining T vector. Finally, a_j is instead of u. The ARL estimation using the NIE method is shown as follows:

$$\widetilde{T}(u) = 1 + \frac{1}{\lambda} \sum_{j=1}^{m} w_j \widetilde{T}(a_j) f\left(\frac{a_j - (1-\lambda) u}{\lambda}\right) \tag{29}$$

4.4 Explicit formula

This method uses advanced mathematics knowledge to help, such as the method of integral equations and central limit theorems to prove the value of ARL. The advantage of this method is that the ARL values obtained are accurate and do not take time to process because they are exact formulas. However, there may be limitations in that finding a successful formula depends on the form of the probability distribution. Consequently, methods for estimating ARL, such as Monte Carlo simulation methods, Markov chain method or the method of numerical integral equations, are still popular.

The next section presents the concept of deriving the explicit formula for EWMA control charts when the data follows an autoregressive process, as discussed in Sukparungsee and Areepong (2017). The explicit formula of *ARL* for EWMA control charts when the error is exponential distribution can be proved as follow.

Given the lower control limit $H_L = 0$ and the upper control limit $H_U = h$. Statistics for EWMA control chart is Z_1. Given that

$$0 < (1-\lambda)Z_{t-1} + \lambda\delta + \lambda\phi_1 X_{t-1} + ... + \lambda\phi_p X_{t-p} < h$$

When $F(u)$ is Average run length of AR(p) process with the initial value $Z_0 = u$. Then, function is

$$F(u) = \mathbb{E}_\infty(\tau) \geq T, Z_0 = u$$

Consider the function $F(u) = 1 + \int L(Z_1) f(\zeta_1) d\zeta_1$

$$F(u) = 1 + \int F((1-\lambda)u + \lambda\mu + \lambda\phi_1 X_{t-1} + ... + \lambda\phi_1 X_{t-p}) f(y) dy.$$

Replacing the parameters in the function $F(u)$

$$F(u) = 1 + \frac{1}{\lambda}\int_0^h F(y) f\left(\frac{y - (1-\lambda)u}{\lambda} - \delta - \phi_1 X_{t-1} - \phi_2 X_{t-2} - ...\right) dy.$$

Then, $F(u) = \frac{1}{\lambda\alpha}\int_0^h F(y)\, e^{-\frac{y}{\lambda\alpha}} e^{\left(\frac{(1-\lambda)u}{\lambda\alpha} + \frac{\delta + \phi_1 X_{t-1} + ... + \phi_p X_{t-p}}{\alpha}\right)} dy$

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Given $G(u) = e^{\left(\frac{(1-\lambda)u}{\lambda\alpha}+\frac{\delta+\phi_1 X_{t-1}+\ldots+\phi_p X_{t-p}}{\alpha}\right)}$

$$F(u) = 1 + \frac{G(u)}{\lambda\alpha}\int_0^h F(y)\, e^{-\frac{y}{\lambda\alpha}}dy\,;\; 0 \le u \le h$$

and $d = \int_0^h F(y)\, e^{-\frac{y}{\lambda\alpha}}dy$. Therefore, $F(u) = 1 + \frac{G(u)}{\lambda\alpha}d$

$$F(u) = 1 + \frac{1}{\lambda\alpha}e^{\left(\frac{(1-\lambda)u}{\lambda\alpha}+\frac{\delta+\phi_1 X_{t-1}+..+\phi_p X_{t-p}}{\alpha}\right)}d$$

Next finding a constant d where

$$d = \frac{(-\lambda\alpha)\left(e^{-\frac{h}{\lambda\alpha}}-1\right)}{1+\frac{1}{\lambda}e^{\left(\frac{\delta+\phi_1 X_{t-1}+..+\phi_p X_{t-p}}{\alpha}\right)}\left(e^{-\frac{h}{\alpha}}-1\right)}$$

Replaced the constant d in the equation $F(u)$. Then,

$$F(u) = 1 - \frac{e^{\left(\frac{(1-\lambda)u}{\lambda\alpha}+\frac{\delta+\phi_1 X_{t-1}+..+\phi_p X_{t-p}}{\alpha}\right)}\left(e^{-\frac{h}{\lambda\alpha}}-1\right)}{1+\frac{1}{\lambda}e^{\left(\frac{\delta+\phi_1 X_{t-1}+..+\phi_p X_{t-p}}{\alpha}\right)}\left(e^{-\frac{h}{\alpha}}-1\right)}$$

The explicit formula of *ARL* for in control process for the EWMA control chart is

$$ARL_0 = 1 - \frac{e^{\left(\frac{(1-\lambda)u}{\lambda\alpha_0}+\frac{\delta+\phi_1 X_{t-1}+..+\phi_p X_{t-p}}{\alpha_0}\right)}\left(e^{-\frac{h}{\lambda\alpha_0}}-1\right)}{1+\frac{1}{\lambda}e^{\left(\frac{\delta+\phi_1 X_{t-1}+..+\phi_p X_{t-p}}{\alpha_0}\right)}\left(e^{-\frac{h}{\alpha_0}}-1\right)} \quad (30)$$

The explicit formula of *ARL* for out-of-control process is

$$ARL_1 = 1 - \frac{e^{\left(\frac{(1-\lambda)u}{\lambda\alpha_1}+\frac{\delta+\phi_1 X_{t-1}+..+\phi_p X_{t-p}}{\alpha_1}\right)}\left(e^{-\frac{h}{\lambda\alpha_1}}-1\right)}{1+\frac{1}{\lambda}e^{\left(\frac{\delta+\phi_1 X_{t-1}+..+\phi_p X_{t-p}}{\alpha_1}\right)}\left(e^{-\frac{h}{\alpha_1}}-1\right)} \quad (31)$$

where $-1 \le \phi_i \le 1$ is an auto regression coefficient, α is a parameter for exponential distribution, δ is a constant, λ is a weight for EWMA control chart, h is an upper control limit.

Many researchers have examined the ARL through using explicit formulas (see Suriyakat *et al.* (2012), Petcharat *et al.* (2013), Sukparungsee and Areepong (2017), Phanyaem (2021), Phanyaem (2022), Suriyakat and Petcharat (2022), Petcharat (2022), Peerajit and Areepong (2022), Phanthuna and Areepong (2022), Supharakonsakun and Areepong (2023)).

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5. Real world applications

This section applies the explicit formulas for the ARL to practical applications. Following the subsequent steps, the ARL formula has been implemented using actual data.

(1) To estimate parameters from a dataset, it is necessary to include a time series model.

(2) To estimate the parameter of residuals that follow an exponential distribution.

(3) By utilizing the parameter values obtained from the previous two steps, we can calculate the ARL values.

(4) The ARL value obtained from (3) was compared with other control charts to perform a performance comparison.

(5) To identify variations in the mean of a process, it is necessary to calculate the upper control limit (UCL). Subsequently, the control chart statistics should be computed using actual data, and these statistics should be plotted on a graph to visualize any deviations.

Khamrod *et al.* (2024) examined the performance of a modified exponentially weighted moving average control chart with an exponentially weighted moving average (EWMA) control chart by deriving explicit formulas for the ARL of the MA process with exogenous variables. This study investigates two economics datasets: the closing stock price of PTT Public Company Limited, which incorporates the daily foreign exchange rate of THB/USD as an exogenous variable and the daily highest silver price, which is correlated with the highest crude oil price. The results of applying the ARL using the two real datasets and the explicit formulas indicate that the modified EWMA control chart outperformed the EWMA control chart in the given conditions. Recently, Sukparungsee and Areepong (2024) applied the explicit formulas for the ARL of an MAX*(q,r)* process on an Extended EWMA control chart and compared the performance with CUSUM and EWMA control charts using 65 real-world data observations of the gold price. The exogenous variable is the USD/THB exchange rate from March 2023 to May 2023. The model has an improvement pattern with two MAX processes, i.e. MAX(1,1) and MAX(2,1). Subsequently, explicit solutions for the ARL of the Homogenously Weighted Moving Average control chart under autoregressive with trend processing were proposed by Sunthornwat *et al.* (2024a, b). An assessment is made of the effectiveness of the explicit formulations for the ARL on the HWMA control chart compared to the EEWMA and CUSUM control charts, employing the quarterly copper price data from January to August 2023. The explicit formula approach is an appropriate alternative for practical implementations requiring the HWMA control chart to identify process changes. Sunthornwat *et al.* (2024a, b) examined the effectiveness of a homogeneous weighted Moving Average (HWMA) control chart in identifying process mean shifts of a minor to moderate type. The present study employs daily natural gas price data spanning the period of January 2, 2023, to April 4, 2023. Based on the outcomes, the HWMA control chart for the natural gas price dataset is more suitable than the Extended EWMA and CUSUM control charts. In addition, the mixed control chart is very sensitive to detect a small shift in econometric applications as presented by Sukparungsee *et al.* (2022). We have chosen real GDP growth (%) in the Lebanese economy data with a normal distribution between 1970 and 2003 for this application. The results illustrate the effectiveness of the Shewhart, MA, EWMA, MA-EWMA and EWMA-MA control charts in detecting the mean of the real GDP growth (%). In this case study, the Shewhart, MA and EWMA control charts were unable to identify any changes, but the performance of the MA-EWMA and EWMA-MA control charts was able to identify a mean change at 1976.

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6. Conclusion

In the past several centuries, the use of control charts is applied in many areas of work which has research related to economic fluctuations or in econometrics. By creating and inventing new control charts that are effective in detecting changes in the average value or process variation. However, the economic process sometimes does not know the distribution of data or could not estimate the parameter. It is necessary to find an efficient alternative for creating charts without parameter, so-called nonparametric control charts which can eliminate trend, seasonal or random influences.

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