Section 3.1: Editors' Overview of Part 3

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The following table contains a brief summary of the contents of each chapter in Part 3 of this book. This section contains extensions of the theory to the case of aggregation error, heterogeneous agents, and risk. The chapter also derives the discounted monetary capital stock implied by the aggregation theoretic measurement of the monetary service flow.

Part 3 Section Contents
Extensions of Index Number Theory

Chapter Number	*Chapter Title*	*Contents*
9	A Dispersion-Dependency Diagnostic Test for Aggregation Error: with Applications to Monetary Economics and Income Distribution	Introduces the use of Divisia second moments for cases in which the usual Divisia first moment is not adequate.
10	Exact Aggregation under Risk	Extends aggregation theory to the case of risk, so that first order conditions become Euler equations.
11	Monitoring Monetary Aggregates under Risk Aversion	Econometrically applies the extended theory of monetary aggregation under risk.
12	The CAPM Risk Adjustment for Exact Aggregation over Financial Assets	Extends index number theory to the case of risk, and derives the risk adjustment to user costs required for consistency with Euler equation models.
13	Stochastic Volatility in Interest Rates and Nonlinearity in Velocity	Explores inference errors regarding velocity and money demand behavior, when the risk adjustment to user costs is overlooked.

Chapter Number	Chapter Title	Contents
14	A Reply to Julio J. Rotemberg	Derives the economic capital stock of money implied by the Divisia index's service flow, and resolves the paradox produced by the need to pay the simple sum price for the stock's joint product.
15	Partition of M2+ as a Joint Product: Commentary	Explores the empirical behavior of the theoretical monetary capital stock and compares with the behavior of the simple sum aggregate, by untangling the joint product.

Chapter 9:

Chapter 9 considers the implications of violations of aggregation assumptions made in earlier chapters. Violations of assumptions regarding aggregation over economic agents are considered, as well as violations of assumptions regarding aggregation over goods and assets. It is found that consistency with theory no longer remains possible, if aggregation is limited to the use of the Divisia first moment (the "Divisia index"). Divisia second moments then appear from the theory, and only disappear again if the violated assumptions are reimposed.

How to use the Divisia second moments, when needed, is derived and illustrated in applications.

Chapter 10:

Chapter 10 extends the theory of microeconomic quantity and price aggregation to the case of risk. In the earlier literature on quantity and price aggregation, as used in Chapter 3, the theoretical existence of exact aggregates is proved through the use of nested two stage budgeting theorems and duality theory. The objective is to identify an aggregator function such that the economic agent can be proven to behave as if the aggregate is indistinguishable from an elementary good. But under risk, two stage budgeting theorems do not work, and most duality theory does not apply.

In Chapter 10 behavioral equivalence of a weakly separable aggregate is successfully proven under risk, without the use of two stage budgeting or duality. The resulting fundamental theorem of quantity aggregation is Theorem 4 for consumers in that chapter and Theorem B1 for firms in the chapter's appendix. Policy relevance of the exact aggregate is proven in Section 2.5 of the chapter. In addition, in Section 4 it is proven that no generalization of the Divisia index for exact tracking is needed if risk applies only to future prices and interest rates. Generalization is needed only when there is risk regarding contemporaneous prices or interest rates.

In Section 5 of the chapter, the CE ("currency equivalent") index of Rotemberg (1991) and Rotemberg, Driscoll, and Poterba (1995) is proven to be a nested special case of the Divisia index, when currency is strongly separable from other monetary assets.

Chapter 11:

Using the theory from Chapter 10 on the existence of an exact monetary aggregate under risk, Chapter 11 uses generalized method of moments to estimate the parameters of the resulting Euler equations. Tastes are specified by a nested constant elasticity of substitution utility function in goods and money. Substitution of the econometrically estimated parameters back into the monetary quantity aggregator function produces the estimated exact monetary aggregate under risk. The simple sum and Divisia indexes (unadjusted for risk) are compared in terms of their ability to track the estimated exact aggregator function.

Chapter 12:

In Chapter 11, the monetary quantity aggregate produced from aggregation theory depends upon the parametric specification of the aggregator function and the choice of econometric estimator. The objective of index number theory is to track unknown aggregator functions nonparametrically. Chapter 12 extends the Divisia index through the introduction of a risk adjustment such that the resulting extended Divisia index exactly tracks the unknown aggregator function, when contemporaneous interest rates are not known with certainty and economic agents are risk averse. The ordinary Divisia index is shown to be a special case attained under perfect certainty.

The risk adjusted user costs are defined in Definition 1 and derived and provided in Theorem 1. The resulting risk-adjusted Divisia monetary aggregate is derived and provided in Theorem 2. If the same weights and the same formula are used to aggregate over user costs, rather than over monetary quantities, Theorem 2 provides the risk adjusted Divisia user cost index. If the simplifying assumptions of consumption capital asset pricing theory are accepted, the risk adjustment becomes equation (27).

In the latter sections of that chapter, the magnitude of the adjustment is investigated by solving the Euler equations numerically.

Chapter 13:

Having determined the correct extension of aggregation theory and index number theory for risk, it is interesting to ask about the nature of the inference errors that could be produced by ignoring the extension and using methods based upon the assumption of perfect certainty. Chapter 13 investigates the induced appearance of velocity instability that could be produced by making that error, and finds that some of the literature's findings of velocity instability could have been generated by failure to use proper risk adjustment in velocity function stability tests.

An interesting and useful formula first defined and derived in this paper is the aggregation-theoretic exact interest rate index that is dual to the exact monetary aggregate. The formula can be found in equation (5) in this chapter. The paper also establishes the relationship between the exact interest rate aggregate and the exact user cost price aggregate, which measures aggregate foregone interest (opportunity cost), rather than aggregate interest received.

Chapter 14:

The prior chapters all deal with monetary services as flows, rather than the implied economic capital stock of money produced by discounting that flow to present value. Chapter 14 derives the economic monetary capital stock. The resulting formula is provided in equation (2). The simple sum monetary asset stock is proven to be a joint product equal to the sum of the discounted expected investment yield of the monetary assets and the economic monetary stock defined by equation (2). That economic money stock, measuring monetary wealth, is shown to equal the discounted present value of the expected Divisia index's monetary service flow. The CE index is proven to be a special case attained under stationary expectations.

Chapter 15:

Chapter 15 empirically applies the decomposition of the simple sum joint product stock into its economic monetary capital stock and the discounted investment yield. Since many monetary assets now produce substantial interest yield, the gap between the economic monetary capital stock and the simple sum stock is found to be large. It is found that misusing the simple sum stock in an economic model requiring the economic monetary capital stock can produce misleading results.