Activity Cruciality as Measure of Network Schedule Structure Resilience

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Abstract

Purpose – Construction projects operate within a risky environment. It materialises as delays, which must be prevented or mitigated to avoid becoming amplified into late completion. But previous research has largely ignored how structural complexity of the underlying network schedules shapes their resilience.

Design/Methodology/Approach – This research hypothesizes that schedule structure plays a vital role in its ability to absorb or propagate delays. The impact of activity-level local risk factors is represented via activity duration distributions, i.e. probability density functions. The impact of project-level global risk factors is more challenging because they arise via interactions between multiple activities.

Findings – Modelling resilience to local and global risk factors can employ a matrix approach. Simulation shows that delay amplification depends on local structure, not global complexity.

Research Limitations/Implications – Criticality had merely relied upon a single deterministic analysis of a network schedule to categorize activities as having zero or nonzero float from fixed relative duration a dependency structure. Repeated probabilistic analysis with sampled durations gives criticality indices of activities. This research limits itself to network schedules with point-wise relations between activities.

Practical Implications – Managers can use this knowledge to develop schedules that protect their expected project duration with a suitable structural complexity.

Originality/Value – Contributions to the body of knowledge are as follows: It converts the dependency structure into a reachability matrix and adds a correlation matrix to capture how the predecessor performance may impact its successors. It correlates criticality of activities with structural complexity indices. And it ranks activities objectively by their cruciality, i.e. potential delay propagation.

Keywords Schedules, Risk, Delays, Structure, Correlation, Resilience

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1. Introduction

Construction projects operate within a risky environment. It materialises as delays, which must be prevented or mitigated to avoid becoming amplified into late completion. Modelling construction projects with network schedules, which are the *de facto* industry standard across North America (Galloway, 2006), comprise activities as building blocks. They are linked by arrows that represent sequencing constraints for technical or administrative reasons. In network schedules, a local delay can impact the project duration if the activity is on the critical path, i.e. has zero idle time after its scheduled timeframe as flexibility (float). Even if it is on a noncritical side path and has some float, any delay that exceeds such float makes it critical itself. Clearly, how activities behave in network schedules depends on two fundamental factors: their own local planned and actual durations and their global relations with predecessors and successors. Absolute durations matter less than relative durations compared to other activities because whether an activity is deemed critical is not based on an inherent delay risk of its work, but by comparing it to concurrent activities on parallel paths. And upstream activities can impact downstream ones on any sequential paths.

2. Literature Review

Structural complexity of network schedules is therefore a root cause of their potential resilience to delays, and, in turn, their expected performance toward a desired on-time project completion. But recent studies mostly ignore how structural complexity of the underlying network schedules shapes their resilience. Project complexity also continued to be understood qualitatively as "difficult" based on the individual tasks within an activity and the respective required skills, e.g. by Sinha *et al.* (2006). Yet this research will seek a strictly quantitative basis for studying schedule complexity. Suitable complexity measures should draw on the mathematical area of graph theory (Latva-Koivisto, 2001). A pertinent previous study criticised simplistic metrics that had averaged predecessor and successor links per activity. It developed a logarithmic metric that counts actual versus possible links and ignores redundant links (Nassar and Hegab, 2006). But further work assessed project complexity simply by number of links per activity (Nassar, 2010). Valadares Tavares *et al.* (1999) identified the challenge that inspired this research as the "analysis and classification of the shape or morphology of each project network; … [and] the relationship between the morphology and the uncertainty concerning the total duration of the project" (*ibid.*, pp. 510–511). Attempting to capture the overall morphology, i.e. network structure, they defined six indicators to track "activity count, relative longest path length, network width in each sequence step (termed 'progressive level'), and relative link density" (Lucko *et al.*, 2018, p. 762). Path length was in sequence steps. They generated random networks and plotted their project duration over individual indicators. The most widely accepted complexity index for networks, specifically schedules, appears to be the restrictiveness estimator (RT). It has the intuitive feature of being exactly 0.0 for parallel and 1.0 for serial networks (Schwindt 1995). It captures structural intricacies via its reachability matrix between all activity pairs (Su *et al.* 2016), which makes it unaffected by any redundant links. Said matrix can be generated from the dependency matrix that lists all direct links.

Another concept that merits revisiting is criticality. Criticality had merely relied upon a single deterministic analysis of a network schedule to categorize activities as having a zero or nonzero float from fixed relative duration a dependency structure. A criticality index for each activity can be calculated from a Monte Carlo simulation of a schedule with probabilistic durations, either as percentage of runs in which it was critical or more accurately as percentage of days being critical to simulated duration (Tang and Mukherjee, 2012). But such static and dynamic views completely sidestepped the major challenge of explicitly considering the structure itself and only indirectly recorded the overall output of its behaviour under simulated delays.

3. Methodology

3.1. Goal

This research hypothesises that schedule structure plays a vital role in its ability to absorb or propagate delays. The goal is to quantify how delay resilience is rooted in a complex structure of network schedules. This is notwithstanding other mitigation strategies such as accelerating activities that are falling behind. Managers can use the results to analyse and refine such link structures for better protection from delays.

3.2. Objectives

Towards this goal, three objectives are set: firstly, converting dependency into reachability and correlating the timeliness of predecessor activities with any successors. Secondly, defining cruciality of an activity with respect to negatively impacting another as the product of its dynamic criticality index, its reachability and its correlation with said other activity finish. Of course, the most important other activity is the last one, which constitutes the project finish. Thirdly, applying the newly defined cruciality to trace the cumulative effect of potential delays along paths in network schedules. This explores the relation between schedule structure (complexity index) and the potential delay propagation between its activities (cruciality index).

This research acknowledges that a plethora of internal and external risks exist for construction activities, e.g. physical size, custom design, materials interfaces, site conditions, delivery access, meteorological events, productive resources, schedule duration, capital cost, human factors, management practices and owner requirements, but assumes that they can be reduced to probability duration distribution functions.

3.3. Algorithm

This research limits itself to network schedules with point-wise relations between activities. The impact of activity-level local risk factors is represented via activity duration distributions, i.e. probability density functions. The impact of project-level global risk factors is more challenging, because they arise via interactions between multiple activities. To explore how schedule structure affects its performance, the methodology flowchart with five modules is developed as per Figure 1.

The *activity input module* collects basic data of name and probability density function for their durations. Modelling resilience to local and global risk factors can employ a matrix approach. The descriptions of the following modules explain their respective matrices. A Monte Carlo simulation with 1,000 runs creates randomised duration for each activity. The *schedule structure module* generates different structures as distinct sets of incoming and outgoing links between activities. Their reachability matrix (where $R = [r_{ij}]_{n \times n}$ is an n by n matrix, where $r_{ij} = 1$ if activity i and j are reachable, else $r_{ij} = 0$) is determined as well to later calculate the cruciality matrix and RT. Reachability from any activity finish to the last activity finish (i.e. project finish) is always 1.0, because by definition a proper network schedule has exactly one start node and one finish node that bookend all other activities. But for other activity pairs, the reachability could be 0.0 if they are located on parallel paths. Random durations from the *activity input module* are combined with relations from the *schedule structure module*, the *CPM module* yields the earliest finishes of activities. These will be different for each individually randomized schedule. Activity-on-node and polygon plots of the network are created by the *graphical output module* so users can verify them. Repeated probabilistic analysis with sampled durations gives criticality indices of activities. And this module also outputs the correlation coefficient matrix $[\rho_{ij}]_{n \times n}$ (where ρ_{ij} is the correlation coefficient between the earliest finishes of activities i and j). Of course, correlation

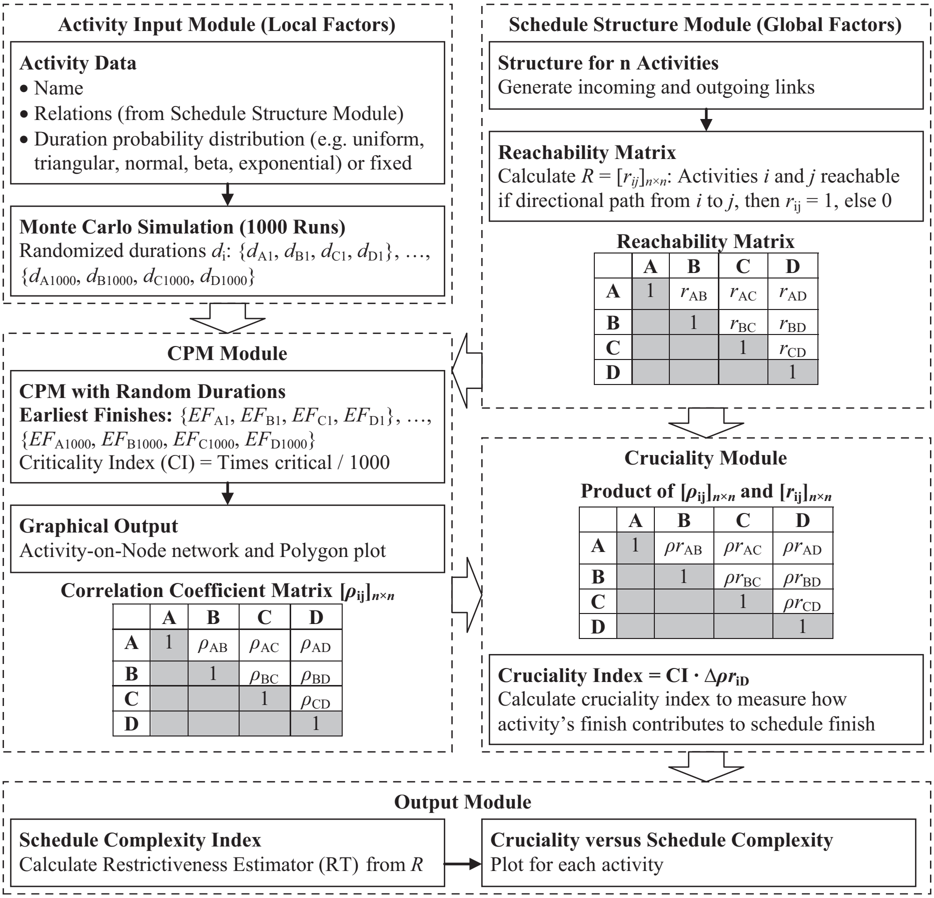

Figure 1. Flowchart

ρ_{iD} of any activity i with the last activity, which represents the project duration D itself, is of particular interest. Next the *cruciality module* individually multiplies cells in the correlation coefficient matrix and the reachability matrix to yield the correlation coefficient ρ_{ij} of activities i and j only if they are directly reachable ($r_{ij} \neq 0$). Note that the product $\rho_{ij} \times r_{ij}$ is a cumulative value. It comprises correlation of all predecessors of activity i with j. Multiplying its non-cumulative value with the CI of activity I now finally gives the cruciality index CI $\Delta\rho r_{iD}$ to measure how the activity finish contributes to the schedule finish, i.e. project duration. Finally, the output module evaluates the performance (cruciality index) of each activity within different structures by calculating the aforementioned RT (Schwindt 1995) from directly from the reachability matrix of each schedule and matching it with the cruciality index of every activity within it.

4. Analysis

Figures 2 through 8 show the networks for schedules comprising Activities 1, 2 and 3 between the start and finish (labelled S and F). They use a triangular probability distribution {2, 3 and 4} to sample their random durations. Note that example schedules #4 in Figures 5 and #6 in Figure 7 have a direct link from activity 3 to F and 2 to F, respectively. Arranging activities as corners of a polygon gives an alternative view of the link structure without rearranging the network. The computer implementation can generate both plots.

Table 1 provides the cruciality calculations for these seven example schedules, along with their density (ratio of existing to possible links, Lancichinetti *et al.*, 2010), and RT. Criticality indices are multiplied with noncumulative correlations. Of particular interest are merges in the network structure, which are in italics in Table 1. Their criticality index is 1.0,

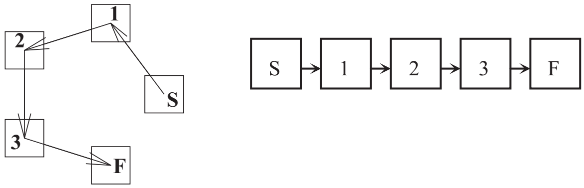

Figure 2.
#1, RT = 1.00

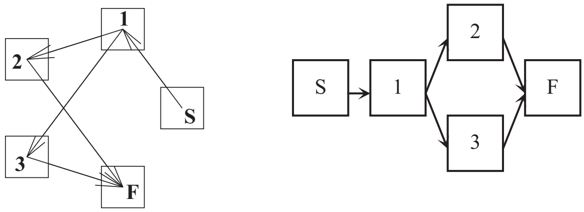

Figure 3.
#2, RT = 0.67

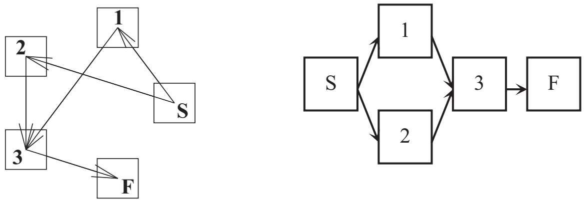

Figure 4.
#3, RT = 0.67

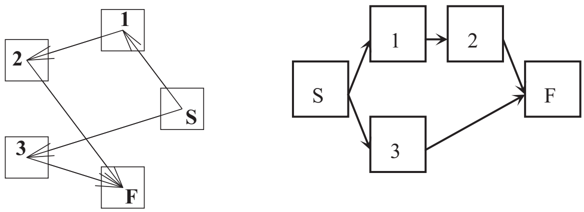

Figure 5.
#4, RT = 0.33

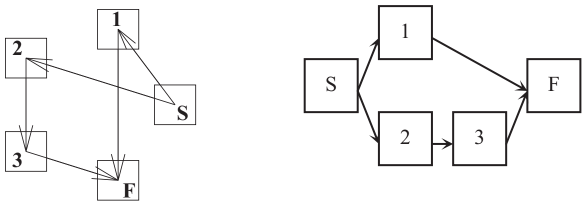

Figure 6.
#5, RT = 0.33

because while either predecessor can be critical in a simulation run, their joint successor will always inherit criticality from one of them. But in larger networks with multiple merges on parallel paths, this will not necessarily still be the case because the critical path may bypass a merge on a path that has some float. The correlation index j here always refers to the last activity finish. Of interest is the last column that adds cruciality indices of all activities. The sum is always one because non-cumulative correlations can be added if activities are sequential, and if they are parallel the CI gives a weighted adjustment to only those cases when they are actually critical, which then can be added as well.

Plotting cruciality over the schedules sorted from left to right from high to low RT in Figure 9, the distinct impact of serial or schedule structure becomes visible. Schedule #1 is fully serial and the cruciality is highest for Upstream Activity 1 and lowest for downstream activity 3. Schedule #7 is fully parallel and the three activities have almost identical crucialities except for minor fluctuations from the simulation. In #2 activity 1 dominates and its successors 2 and 3 have almost no impact. In #3, Activities 1 and 2 are parallel, so that #3 has a much larger impact. In #4 the serial Activities 1 and 2 eclipse the parallel path that contains only activity 3. In #5 the same happens for activities 2 and 3 parallel to 1. And in #6 the same happens for activities 1 and 3 parallel to 2. Table 2 summarizes numeric values from Table 1; note that each column of Table 2 sums to 1.0 (and each equivalent row of Table 1). Overall, the following observations can be extracted: 1. In serial structures the predecessor cruciality is larger than the successor cruciality. 2. Merge activities where parallel paths join have a larger cruciality than their successors, e.g. Activity 3 in Schedule #3. And 3. Cruciality is not sensitive to RT because it is a complexity index for the global structure. Future research should thus focus on local structural clusters for characterizing and quantifying their behaviour. Activities can now be ranked both by their individual and cumulative cruciality – not merely by their static criticality! – within the given network structure to receive managerial attention. They can be prioritized in planning to protect the desired production by resource allocation. They can be identified during execution for being accelerated to contain localized delays and prevent them from growing into a global problem.

Since all activities have the same distribution of {2, 3, 4}, any differences in results can be attributed to the differences in network structure. Scatter plots in Figures 10 through 16 show the project finish over the activity finish as blue (Activity 1), red (Activity 2), and green

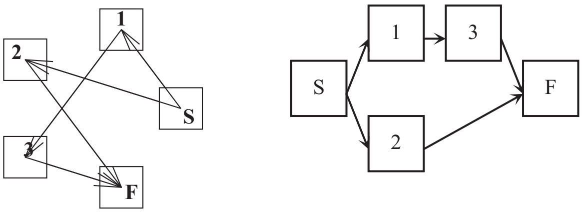

Figure 7.
#6, RT = 0.33

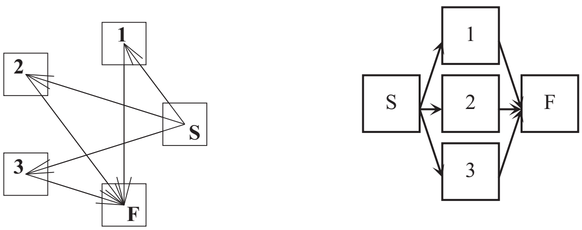

Figure 8.
#7, RT = 0.00

Table 1.
Calculations

			CI					$\Delta\rho_{ij}$					$CI \cdot \Delta\rho_{ij}$					
#	Density	RT	S	1	2	3	F	S	1	2	3	F	S	1	2	3	F	Sum
1	0.4	1.00	1	1	1	1	1	0	0.602	0.223	0.175	0	0	0.602	0.223	0.175	0	1.000
2	0.5	0.67	1	1	0.483	0.517	*1* = 0.483 + 0.517	0	0.788	0.051	0.046	0.161	0	0.788	0.024	0.024	0.078	1.000
												0.166					0.086	
3	0.5	0.67	1	0.482	0.518	*1* = 0.482 + 0.518	1	0	0.426	0.390	0.574	0	0	0.205	0.202	0.277	0	1.000
											0.610					0.316		
4	0.5	0.33	1	1	1	0	*1* = 1 + 0	0	0.735	0.265	0.019	0	0	0.735	0.265	0	0	1.000
												0.981					0	
5	0.5	0.33	1	0	1	1	*1* = 0 + 1	0	0.033	0.708	0.292	0.967	0	0	0.708	0.292	0	1.000
												0					0	
6	0.5	0.33	1	1	0	1	*1* = 0 + 1	0	0.716	0.045	0.284	0.955	0	0.716	0	0.284	0	1.000
												0					0	
7	0.6	0.00	1	0.338	0.327	0.335	*1* = 0.338 + 0.327 + 0.335	0	0.482	0.437	0.475	0.518	0	0.163	0.143	0.159	0.175	1.000
												0.563					0.184	
												0.525					0.176	

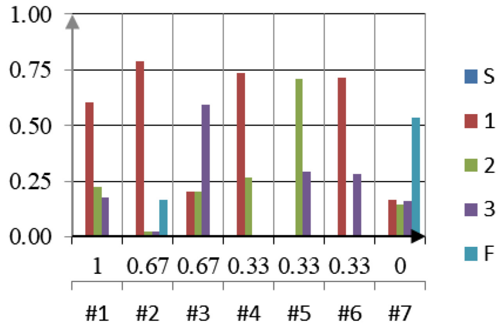

Figure 9.
Cruciality over RT

Table 2.
Crucialities

Schedule	#1	#2	#3	#4	#5	#6	#7
RT	1	0.67	0.67	0.33	0.33	0.33	0
S	0	0	0	0	0	0	0
1	0.602	0.788	0.205	0.735	0	0.716	0.163
2	0.223	0.024	0.202	0.265	0.708	0	0.143
3	0.175	0.024	0.593	0	0.292	0.284	0.159
F	0	0.164	0	0	0	0	0.535

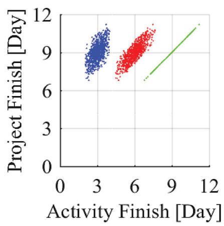

Figure 10.
#1 Scatterplot

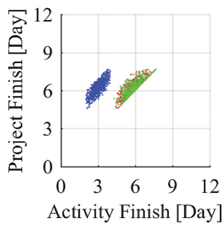

Figure 11.
#2 Scatterplot

(Activity 3) clusters from all 1,000 simulations runs for that particular schedule structure. Note that the last activity finish always has correlation 1.0 with the project finish because they are the same. This is proven by Figure 13, whose Network #4 has Activities 1 and 2 parallel to 3, which thus is delegated to a non-critical path, so that the finish of Activity 2 becomes the effective project finish. The pattern of Figure 10 reflects the serial structure of schedule #1 and Figure 16 shows the equal impact of the parallel activities on the project finish in Schedule #7. Figure 11 reflects that Activities 2 and 3 are parallel and both last in #2, and *vice versa*, are both not last in Figure 12 of #3. Figures 14 and 15 (#5 and #6) break the otherwise symmetric schedule structure by connecting Activities 1 and 2, respectively, directly with F. This causes the clusters of the non-last activities to not overlap fully. Considering the complexity of these examples, which ranges from RT = 1.0 (serial) for #1 to

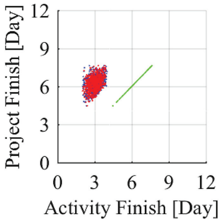

Figure 12.
#3 Scatterplot

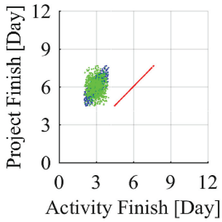

Figure 13.
#4 Scatterplot

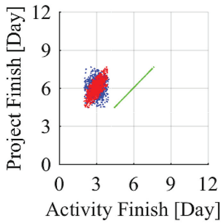

Figure 14.
#5 Scatterplot

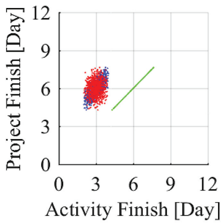

Figure 15.
#6 Scatterplot

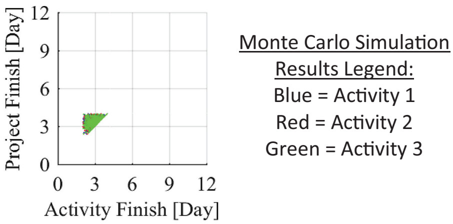

Figure 16.
#7 Scatterplot

0.0 for #7 (parallel), no clear pattern emerges, because the overall reduction in project finish from Figure 10 through Figure 16 is simply caused by the increasing parallel nature of the networks that have the same number of activities. Simulation shows that delay amplification depends on local structure, not global complexity.

5. Conclusions

This research has examined the resilience of network schedules to delays by generating and simulating a range of network structures and defined and measured cruciality as a fusion of its behaviour and ability to impact other activities. Contributions to the body of knowledge are: It converts the dependency structure into a reachability matrix and adds a correlation matrix to capture how the predecessor performance may impact its successors. It correlates criticality of activities with structural complexity indices. And it ranks activities objectively by their cruciality, i.e. potential delay propagation. Managers can use this knowledge to develop schedules that protect their expected project duration with a suitable structural complexity.

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