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Corporate bankruptcy prediction: re-estimating classical accounting-based models with a sample from Portugal

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Abstract

Purpose – We aimed to test the predictive accuracy of three widely used accounting-based corporate bankruptcy prediction models (Altman's Z"-score, Ohlson's model and Zmijewski's model) and re-estimate the models' coefficients to improve accuracy and help small and medium-sized companies (SMEs) better assess their corporate clients' credit risk in a European context.

Design/methodology/approach – We estimated corporate bankruptcy risk for a comprehensive sample of 9,371 companies from Portugal (of which 370 were bankrupt) using Altman's Z"-score as well as Ohlson's and Zmijewski's models. We compared the models' predictions to each company's actual status (bankrupt or non-bankrupt). We re-estimated the coefficients of the three models using a logit regression methodology to improve the accuracy of predicting bankrupt companies. We tested the new equation coefficients both in-sample and out-of-sample and conducted a robustness test on 228 companies from nine other European countries.

Findings – The original models still demonstrate an overall accuracy (percentage of correctly predicted bankrupt and non-bankrupt companies) above 80%. However, they produce high errors in predicting bankruptcy only (Type I errors of 27.03% in Altman's model, 22.97% in Ohlson's model and 16.76% in Zmijewski's model). While maintaining overall accuracy above 80%, the re-estimated equations reduced these errors to 13.24% in Altman's model and 15.68% in Ohlson's model. However, it failed to reduce Zmijewski's model errors, which increased to 18.38%. The reduction in errors compared to the original models was also evident in out-of-sample testing. We conclude that Ohlson's re-estimated model provides more balanced results regarding Type I errors (15.68%) and overall accuracy (84.35%).

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Originality/value – This article contributes to the literature on corporate bankruptcy prediction by re-estimating the coefficients of three widely used accounting-based models with data from Portugal, showing that the new equations perform well within their market context. We believe that it will help SMEs measure the bankruptcy risk of their corporate clients with readily available accounting information.

Keywords Altman's Z"-score, Ohlson's model, Zmijewski's model, Corporate bankruptcy prediction, Logit regression

Paper type Research article

Introduction

In the European Union (EU), the post-COVID-19 monetary tightening cycle started in mid-2022. The European Central Bank's primary refinancing interest rate increased from 0 to 4.5% in fourteen months, directly impacting Euribor, the primary indexing reference for floating-rate loans in Euros. The impact of interest rate increases will be asymmetric in EU countries, depending on the weight of floating-rate loans in the total outstanding credit amount. Portugal's central bank (Bank of Portugal, 2022) estimated that the rise of interest rates would increase the number of companies in vulnerable situations from 18% in 2021 to 26% in 2023 (an increase not seen since the sovereign debt crisis in 2012), leading to a potential increase in corporate bankruptcies. Recent data from the Portuguese government indicates that 2024 registered the highest increase in collective dismissals over the past decade (CNN Portugal, 2024). A research report (Atradius, 2024) forecasts that the highest increases in worldwide bankruptcies will be recorded in Italy, Singapore, the Netherlands, Portugal, Poland, and the United States. We see predominantly European countries in this bankruptcy forecast.

Bankruptcy serves a paradoxical role in a market economy. It acts as a necessary purge, weeding out inefficient businesses that waste valuable resources. This allows more innovative and adaptable companies to take their place, fostering overall economic health. However, as economist Schumpeter (2013) termed it, this creative destruction comes at a significant cost. The negative impacts of bankruptcy extend beyond the immediate closure of the failing firm. Creditors face financial losses, employees lose jobs and income, and suppliers can be left with unpaid debts. This financial disruption can ripple outwards, creating a domino effect as struggling suppliers, in turn, default on their obligations. In extreme cases, a single bankruptcy can trigger a wave of insolvencies, destabilizing entire industries and communities (Battiston, Gatti, Gallegati, Greenwald, & Stiglitz, 2007). Therefore, research into predicting bankruptcy risk becomes critical for mitigating these cascading effects. Authorities can implement preventative measures by identifying companies on the brink of financial collapse. This allows struggling businesses to take corrective actions, potentially avoiding bankruptcy altogether. Moreover, early detection safeguards creditors from significant losses, promoting a more stable financial environment for future investments.

There remains an ongoing debate in the literature regarding the most effective methodology for predicting corporate bankruptcies; i.e. whether it is market-based models or accounting-based models. The debate over the most effective models for predicting business failures is likely to continue in the short to medium term. Researchers frequently propose new techniques, and failure events are subject to many variables, particularly for studying corporate bankruptcy. We believe that for small and medium-sized enterprises (SMEs) that have accounts receivable in business-to-business industries, it is essential to have an accessible tool for measuring the default risk of their corporate customers. Smaller countries, such as Portugal (a member of the European Union), have a limited number of large corporations with credit ratings and even fewer listed on the stock market that can be tracked using market-based models. As of 2024, Euronext Lisbon (Portugal's stock exchange) had only 52 stock-listed companies. For players and regulators in these countries, it is more important to test and improve the accuracy of accounting-based models than market-based models.

In light of the rising risk of corporate bankruptcy, we are interested in helping corporate credit decision-makers more effectively assess the bankruptcy risk of their corporate clients. Banks and large corporations have access to credit rating services and internal systems that monitor the bankruptcy risk of their clients. However, SMEs often cannot afford these services. Nevertheless, today, access to accounting information of any company is available at affordable prices, and researchers developed several simple accounting-based prediction models in the second half of the twentieth century to predict bankruptcy risk. The most famous and cited models are Altman's Z"-score, Ohlson's model, and Zmijewski's model. We hypothesized that these models are still valid in the post-COVID-19 context and that their coefficients can be re-estimated to fine-tune their forecast accuracy.

Therefore, we aimed to (1) evaluate the current predictive accuracy of three widely-used accounting-based bankruptcy prediction models (Altman's Z"-score, Ohlson's model, and Zmijewski's model) using recent data from Portugal, and (2) re-estimate their coefficients using logistic regression to improve prediction accuracy, particularly in reducing Type I errors that impose the highest costs on business creditors who fail to identify bankrupt clients. We developed new equations for each model based on a large sample of 9,371 Portuguese companies. We tested them out-of-sample, finding that these new equations reduced the errors in predicting bankruptcies with high overall accuracy. We also conducted a robustness test using a sample of 228 companies from nine other European countries, confirming the reduction in prediction errors compared to the original three models. However, we found that while the original models achieved overall accuracy above 80%, their high Type I error rates may limit their practical utility for SMEs seeking to assess corporate client bankruptcy risk, particularly where misclassifying a bankrupt company as healthy could result in significant financial losses.

The remainder of the article proceeds as follows. Section 2 reviews the literature. Section 3 presents the models, the data, and the estimation method. Section 4 shows the results and discusses them, comparing the original and the re-estimated models' accuracy. Section 5 presents a robustness test. Finally, Section 6 presents conclusions.

Literature review

Scholars have conducted bankruptcy prediction research in various fields, countries, and sectors (Bellovary, Giacomino, & Akers, 2007; Altman, Iwanicz-Drozdowska, Laitinen, & Suvas, 2017; Barboza, Kimura, & Altman, 2017; Hacibedel & Qu, 2023; Yu, Li, & Liu, 2024, among others). They used accounting information and financial ratios to predict corporate bankruptcies in the 1960s with a univariate model (Beaver, 1966). Altman (1968) developed the use of accounting ratios for bankruptcy predictions by introducing the multivariate discriminant analysis methodology to predict failures, naming the model's output as the Z-score. The model incorporated ratios representing liquidity, retained earnings, operating profitability, solvency, and efficiency. If the Z-score of a given industrial company was below a pre-determined threshold (the distress zone), there was a high probability of bankruptcy in the following year. There was also a grey zone (uncertain results) and a safe zone (healthy companies with very low risk of bankruptcy).

Altman, Hartzell, and Peck (1998) developed a model for non-listed and non-manufacturing companies that is analogous to the original, which one can still apply to manufacturing companies. Scholars further developed this new model, known as the Emerging Market Score model, by incorporating bond rating equivalents to estimate new threshold scores for the distress zone, grey zone, and safe zone in determining bankruptcy probabilities (Altman *et al.*, 1998, 2019; Altman, 2005). The authors named the new model Z"-score. While various studies have advocated various financial distress prediction methods, the modified Altman Z"-score model remains popular due to its user-friendliness. This user-friendliness stems from its reliance on readily available financial ratios applicable across diverse companies and countries (Manaseer & Al-Oshaibat, 2018; Ningsih & Permatasari, 2018; Goh, Mat Roni, & Bannigidadmath, 2022; Achbah & Fréchet, 2024). However, it is crucial to acknowledge that the model's simplicity might also be a limitation. More complex methods, incorporating additional factors or utilizing advanced analytics, could offer enhanced predictive accuracy (Wu, Ma, & Olson, 2022). Evaluating the model's performance across different industries would be a valuable contribution to the literature on financial distress prediction. Although bankers and other credit analysts still widely use the Z"-score due to Basel II and Basel III requirements (Agarwal & Taffler, 2008; Salina, Zhang, Jiao, & Hassan, 2024), several recent research articles show that the model is losing some accuracy (Grice & Ingram, 2001; Bandyopadhyay, 2006; Bauer & Agarwal, 2014; Cındık & Armutlulu, 2021). Nevertheless, Reisz and Perlich (2007) demonstrated that in the short term (one year), the Z"-score outperformed market-based models in predicting corporate bankruptcy. However, they found that the Z"-score loses some of its accuracy in longer forecast horizons.

Ohlson (1980) innovated by estimating the probability of corporate bankruptcy (ranging from zero to one) through a logistic regression model, thereby bypassing the need for a balanced sample of bankrupt and nonbankrupt companies, as required by the previous methodology (multivariate discriminant analysis). The sample used to estimate the model's coefficients comprised only industrial stock-listed companies. Begley, Ming, and Watss (1996) tested the reliability of Altman's and Ohlson's models with data from the 1980s, finding that, overall, the Ohlson model tends to outperform Altman's model, producing substantially fewer Type I errors and a similar number of Type II errors. These authors demonstrated that the models' ratio coefficients change over time, likely due to fluctuations in credit market conditions and bankruptcy laws. They re-estimated the coefficients of both models, confirming that after the re-estimation, the Ohlson model, using logit regression – a more robust technique than discriminant analysis – produced better estimates. Recent tests of Ohlson's model have demonstrated its continued reliability in predicting bankruptcy (Lawrence, Pongsatat, & Lawrence, 2015). Later, Zmijewski (1984) contributed to the field by applying *probit* regression methodology to the accounting-based ratio analysis. He also collected a sample of stock-listed companies, excluding those in the financial services industry. This approach has also been shown to outperform Altman's Z"-score (Wu, Gaunt, & Gray, 2010; Bauer & Agarwal, 2014), consolidating the trend of using binary regression techniques to estimate accounting-based bankruptcy models.

Altman *et al.* (2017) followed the trend of using logit regression to estimate new coefficients for the Z"-score model, employing a sample of European Union companies, and found modest improvements in predictability. However, their study reported relatively low areas under the ROC curve (AUC-ROC) across countries and acknowledged that these accuracy levels have practical limitations for bankruptcy prediction applications. They also found that the model performs better on a one-year horizon and that the company's size is statistically significant in predicting bankruptcy. Nevertheless, Li (2023) used logistic regression spines to study the accuracy of liquidity ratios (a ratio present in most accounting-based prediction models) in predicting bankruptcy, finding a non-linear relation between the liquidity ratio and corporate failure that, when considered, provides a predictive power of the liquidity ratio even in more extended forecast periods.

Das, Hanouna, and Sarin (2009) compared the relative performance of accounting-based models to market-based information models in predicting corporate bankruptcy, finding that accounting-based models, particularly Altman's Z"-score, have higher explanatory power than market-based models. Recent research on this topic has focused on improving estimation techniques (Wosnitza, 2023) and developing machine-learning models to predict financial distress (Barboza *et al.*, 2017; Hacibedel & Qu, 2023). Alanis, Chava, and Shah (2023) found that stock market variables drive forecast performance, concluding that market-based models are superior but that accounting ratios become more relevant for bankruptcy forecasting during crisis periods. Recently, Wu *et al.* (2022) combined the established Altman Z-Score model with a multi-layer perceptron artificial neural network (MLP-ANN). This hybrid model aims to alert stakeholders early to a company's deteriorating financial health, enabling them to take timely actions and potentially mitigate potential losses. According to these authors, integrating the Altman Z-Score, a well-validated metric for predicting financial distress, with the non-linear learning capabilities of an MLP-ANN potentially leads to a more robust and adaptable framework.

Other recent research has focused on the predictive accuracy of corporate financial distress (or bankruptcy) of alternative measures, such as corporate social responsibility (Li, Sun, & Zhang, 2024). Nevertheless, a study about the state-of-the-art bankruptcy prediction models by Soukal *et al.* (2024) confirmed Altman's Z"-score as the most popular model. As of today, a search in

Google Scholar shows that scholars cited Altman's (1968) original Z-score research article 26,434 times, Ohlson's (1980) 10,006 times, and Zmijewski's (1984) 4,717 times, making these three models the most researched and tested accounting-based corporate bankruptcy prediction models.

Although several studies on corporate bankruptcy prediction using empirical data from Portugal have been conducted (Pindado & Rodrigues, 2004; Laitinen & Suvas, 2013; Pacheco, Madaleno, Correia, & Maldonado, 2022), most employ self-developed model equations incorporating various financial ratios. Altman *et al.* (2017) conducted the most extensive study of a classic model, including data from Portugal, where they re-estimated the Z"-score model with an extensive global sample of companies and afterward tested it at the national level.

We were interested in contributing to the literature about accounting-based bankruptcy models in a European context by developing an empirical study in Portugal, a country with a corporate sector especially exposed to increased bankruptcy risk, reviewing, and re-estimating the three most popular bankruptcy models (Altman's model, Ohlson's model, and Zmijewski's model) to help practitioners better understand their corporate clients' bankruptcy risk. We were also interested in testing whether the re-estimated coefficients of these models could be successfully applied to predicting bankruptcies in other European countries.

Methodology

Altman Z"-score model

We based the Z"-score model that we applied to predict bankruptcy in both privately held and stock-listed companies, and both manufacturing and non-manufacturing companies, on the following equation (Altman *et al.*, 1998):

$$Z'' = +3.25 + 6.56(\mathrm{X1}) + 3.26(\mathrm{X2}) + 6.72(\mathrm{X3}) + 1.05(\mathrm{X4}) \tag{1}$$

in which:

X1 = Working Capital/Total Assets,

X2 = Retained Earnings/Total Assets,

X3 = Operating Income/Total Assets.

X4 = Book Value of Equity/Total Liabilities.

According to Altman (2005), the model has been computed to produce a Z"-score close to zero for bankrupt companies. Altman used bond rating equivalents to classify companies with a Z"-score lower than 4.15 as having a B rating equivalent, thus being in the distress zone and having a high chance of bankruptcy within one year. Later, Altman *et al.* (2019) updated this threshold to 4.03. The data we collected spans from 2016 to 2021, and as such, we used a Z"-score value lower than 4.03 to indicate bankruptcy risk. Studies with more recent data should update this threshold to the current bond market situation.

Ohlson model

The Ohlson (1980) model is based on the bankruptcy probability function P, with $0 \leq P \leq 1$, which the logistic function can represent:

$$\mathrm{P}\,(\mathrm{dummy} = 1) = \left(1 + \exp\,(-y_{\mathrm{i}})^{-1}\right), \text{where } y_{\mathrm{i}} = \Sigma_j\, \beta_{\mathrm{j}} \mathrm{X}_{\mathrm{ij}} = \beta' \,\mathrm{X}_{\mathrm{i}} \tag{2}$$

X_{i} is a vector of predictors for the i_{th} observation, and β is a vector of unknown parameters. P is increasing in y, and y equals log $(P/(1-P))$. The model was as follows:

$$\begin{aligned} \mathrm{Y} = {} & -1.32 - 0.407(\mathrm{SIZE}) + 6.03(\mathrm{TLTA}) - 1.43(\mathrm{WCTA}) \\ & + 0.0757(\mathrm{CLCA}) - 1.72(\mathrm{OENEG}) - 2.37(\mathrm{NITA}) - 1.83(\mathrm{FUTL}) \\ & + 0.285(\mathrm{INTWO}) - 0.521(\mathrm{CHIN}) \end{aligned} \tag{3}$$

in which:

SIZE = log(Total Assets/GDP price-level index) base value 100 for 2016,
TLTA = Total Liabilities divided by Total Assets,
WCTA = Working Capital divided by Total Assets,
CLCA = Current Liabilities divided by Current Assets,
OENEG = 1 if Total Liabilities exceed Total Assets (0 otherwise),
NITA = Net Income divided by Total Assets,
FUTL = Funds provided by operations (EBITDA) divided by Total Liabilities,
INTWO = 1 if Net Income was negative for the last two years (0 otherwise),
CHIN = $(NI_t - NI_{t-1})/(|NI_t| + |NI_{t-1}|)$, where NI_t is the Net Income for the most recent period.

The output of equation (3) represents the log-odds ratio (Y), and the more negative it is, the lower the probability of the company going bankrupt. To determine the probability of bankruptcy, one needs to convert Y into a value between 0 and 1 by computing equation (2), above. When the obtained P (probability) is larger than 0.5, there is a high chance that a bankruptcy process will be initiated within one year.

Zmijewski model

The Zmijewski model is as follows (Zmijewski, 1984):

$$Z = -4.336 - 4.513(\mathrm{ROA}) + 5.679(\mathrm{FINL}) + 0.004(\mathrm{LIQ}) \tag{4}$$

in which:

ROA = Net Income/Total Assets,
FINL = Total Debt/Total Assets,
LIQ = Current Assets/Current Liabilities.
With a probability function of:
$P\ (\text{dummy} = 1) = \Phi(Z)$
Where $\Phi(Z)$ is Z's standard normal cumulative distribution function.

The probability of bankruptcy is also measured on a scale from 0 to 1, where a probability larger than 0.5 represents a higher probability of bankruptcy within one year.

Data

We collected the data from the SABI (Iberian Balance Sheet Analysis System) database of Bureau Van Dijk, a Moody's Analytics service, which classifies companies according to NACE codes and allows for selection based on their status, including active, bankrupt, in liquidation, suspended, and others. We excluded financial companies (including banking and insurance), resulting in 805,775 companies from 2006 to 2021. We selected all companies with at least ten employees and total assets exceeding 2 million euros (excluding those defined as micro-enterprises under European Union legislation). We excluded these companies because, in Portugal, due to tax reasons, many individual professionals operate through limited liability companies, and there are real estate asset-holding vehicle companies with no relevant activities that can distort the analysis. We also excluded the subsidiaries from the sample. This reduced the sample size to 20,681 companies, of which 16,557 had a status of "active" and 783 had a status of "bankruptcy" (the remaining companies were suspended, ceased, or had a similar status). We are only interested in studying the accuracy of the models in predicting bankruptcy, not distress, as many distressed companies do not ultimately become bankrupt (Gilbert, Menon, & Schwartz, 1990). Although the bankruptcy dates of the bankrupt companies spanned from 2006 to 2021, there were multiple periods with missing accounting data for the models' ratios, and several recent start-up companies have only one or two years of accounting information. To exclude these start-up companies and obtain a sample with complete data, we included another filter that selected only active companies with complete accounting data from 2016 to 2021, resulting in 9,001 companies with six years of observations.

Regarding bankrupt companies, we also searched for companies that went bankrupt between 2016 and 2021. However, we relaxed the size criteria to require at least one year of total assets exceeding 2 million euros in the sample years, as we expected that failing companies would see their total assets decrease over time. Under these criteria, we identified 370 bankrupt companies in the sample over the six years, with at least one year of complete accounting data. Moreover, we know that the Z”-score is more potent in predicting bankruptcy in one-year horizons. Therefore, following Altman *et al.* (2017), we limited the analysis of the financial ratios of bankrupt and non-bankrupt companies to their last year’s available accounting report. We acknowledge that our approach, which utilizes the most recent available financial report, has inherent limitations. Companies approaching bankruptcy may have missing or delayed financial reports in the period preceding failure, and the time gap between the accounting year’s closing and the actual bankruptcy event can vary substantially. While this represents the practical reality of information availability for credit assessment, it may introduce some uncertainty in the temporal relationship between financial indicators and bankruptcy outcomes.

The sample comprises 370 bankrupt companies and 9,001 non-bankrupt companies, resulting in a bankrupt-to-non-bankrupt ratio of 4.1% (twenty-four non-bankrupt companies for every one bankrupt company). This compares with the 1.5% ratio used by Altman *et al.* (2017), the 5.1% ratio used by Ohlson (1980), and the 4.8% ratio used by Zmijewski (1984).

We extracted the NACE with two-digit core codes of these companies, finding 81 sectors. We further grouped them into families of two-digit codes centered on the most representative sector for improved analysis. Table 1, below, presents the sectoral distribution of the sample companies aggregated by the most representative sectors. We aggregated the sectors with fewer than 5% of bankruptcies in the sample in the “Other” category.

The Manufacturing sector represented the largest sector of the sample, both in non-bankrupt and bankrupt companies, followed by Construction and Transportation. Construction had the highest bankruptcy rate, followed by the Mining and Quarrying sector, and the Other Services sector came in third place. This context may affect the estimation of coefficients because different sectors have distinct typical operating conditions and varying mean accounting ratios.

The total sample had 9,371 companies, of which 370 went bankrupt from 2017 to 2022 (one year after the last account filing). Table 2 shows the number of bankrupt companies with financial reports per year.

The number of bankruptcies in our sample increased steadily from 2016 to 2021, except for 2020 when loan repayments were suspended due to COVID-19 legislation. However, this

Table 1. Sectoral distribution of the sample

Sector	Non-B.	% Non-B.	Bankrupt	% Bankrupt	Bankruptcy rate
Construction (41–43)	652	6.6%	52	12.0%	7.4%
Mining and Quarry (05–09)	66	0.7%	5	1.1%	7.0%
Other services (94–96)	29	0.3%	2	0.5%	6.5%
Transportation (49–53)	439	4.4%	30	6.9%	6.4%
Manufacturing (10–33)	3,142	31.6%	197	45.3%	5.9%
Other	4,673	56.4%	84	34.2%	1.8%
Total	9,001	100%	370	100%	3.9%

Note(s): NACE two-digit codes in parentheses after the sector names represent the European Statistical Classification of Economic Activities. The table shows the sectors with bankruptcy rates above 5% (sample bankrupt companies divided by the total number of companies in each sector), with all remaining sectors grouped as “Other”. Non-B and Bankrupt show the number of companies in each sector that are not bankrupt and bankrupt, respectively. The following percentages are their weight in their category

Source(s): Authors’ own elaboration

Table 2. Number of bankrupt companies per year

Year	2016	2017	2018	2019	2020	2021	Total
Bankruptcies	53	50	63	69	62	73	370

Source(s): Authors' own elaboration

trend reflects our sample composition rather than necessarily representing overall Portuguese bankruptcy patterns, as our selection criteria may influence the observed distribution.

Estimation

We wanted to identify whether it is possible to predict accuracy by re-estimating the coefficients of the three models with recent, country-specific accounting data. Theoretically, we based the models on groups of accounting-based ratios (liquidity, profitability, solvency, etc.), which we will maintain, as testing other ratios is not the purpose of this study. We aimed to re-estimate the existing models under a specific market and updated accounting context.

Although the overall accuracy of the estimation (the percentage of the sample with a correct prediction) is important, the real problem for companies in business-to-business activities is Type I errors (bankrupt companies that are not correctly predicted). A company's bankruptcy can affect its suppliers' profitability, potentially leading to the supplier's bankruptcy if the risk is not adequately managed. We believed that re-estimating the models' coefficients to reduce Type I errors would be helpful for Chief Financial Officers of European SMEs.

We did not re-estimate the models using multivariate discriminant analysis like Altman initially did. It is a less reliable technique than probit or logit regression because it relies on the assumption that the variables follow a normal distribution and that the samples of bankrupt and non-bankrupt companies have equal covariances. Regarding the Probit regression used by Zmijewski, it assumes that the variables are normally distributed and that there is a linear relationship between the predictors and the latent variable, despite evidence showing a non-linear relation between the liquidity ratio and corporate bankruptcy (Li, 2023). Logit regression is a more robust technique as it does not require the data to be normally distributed. Edward Altman has favored the logit regression in his latest research papers about the Z"-score (Altman *et al.*, 2017; Altman, 2018).

Nevertheless, it is common knowledge that logistic regression is not effective with imbalanced data, especially when the minority class is the object of interest, as is the case in our sample, where there are 370 bankrupt companies and 9,001 non-bankrupt ones. In these cases, the conditional probability of belonging to the interest class can be underestimated (My & Ta, 2023), resulting in high Type I errors (i.e. failing to predict companies that will go bankrupt). One of the most common methods to address this problem is resampling, which involves either adding records to the minority class (oversampling) or deleting them from the majority class (undersampling). It has been found that oversampling outperforms undersampling (Mohammed, Rawashdeh, & Abdullah, 2020).

We ran the re-estimation with the imbalanced sample to test the output. However, the results led to Type I errors that were much higher than those of the original models, rendering the new coefficients useless for improving bankruptcy prediction. We will proceed with oversampling the bankrupt company to match the number of non-bankrupt companies, thereby reducing the likelihood of a high Type I error in re-estimating the three models.

Results and discussion

Altman Z"-score model descriptive statistics

Table 3 shows the descriptive statistics of the Z"-score model.

Table 3. Altman's Z"-score model descriptive statistics

Predictor Company	X1 Non-B.	Bankrupt	X2 Non-B.	Bankrupt	X3 Non-B.	Bankrupt	X4 Non-B.	Bankrupt
Median	0.263	−0.051	0.214	−0.109	0.045	−0.046	0.835	0.042
Mean	0.257**	−0.241**	0.207**	−0.446**	0.061**	−0.164**	2.288**	0.036
	(0.004)	*(0.045)*	*(0.005)*	*(0.066)*	*(0.001)*	*(0.023)*	*(0.153)*	*(0.027)*
F-test	0.000		0.000		0.000		0.000	
t-test	0.000		0.000		0.000		0.000	

Note(s): Non-B. means non-bankrupt companies. Bankrupt means bankrupt companies. X1 = Working Capital/Total Assets; X2 = Retained Earnings/Total Assets; X3 = Operating Income/Total Assets; X4 = Book Value of Equity/Total Liabilities. Standard errors in parentheses. ** represents statistical significance at the 1% level, indicating whether the means differ significantly from zero. The *F*-test is the *p*-value of the equality of variances test, used to determine which *t*-test should be used. The *t*-test is the *p*-value of the two-tailed equality test of the means with unequal variance, applied to test whether the Non-Bankrupt and Bankrupt samples have different means

Source(s): Authors' own elaboration

We ran the *F*-test to find out if the samples of non-bankrupt and bankrupt companies had unequal variances, which they did. With that information, we ran a paired samples' *t*-test with unequal variance to determine if the means of the non-bankrupt companies' ratios had statistically significant differences from those of bankrupt companies, and we found that they did. As expected, the first three ratios were positive for non-bankrupt firms and negative for bankrupt firms, showing that having negative working capital (X1), negative retained earnings (X2), and negative operating income (X3) are good predictors of bankruptcy. For bankrupt firms, the equity-to-liabilities ratio (X4) approached zero (mean = 0.036, not significantly different from zero), reflecting capital structures where equity tends to zero. Non-bankrupt firms maintained significantly higher ratios (mean = 2.288**), confirming that adequate capitalization is crucial for financial stability. These results are similar to those of Altman *et al.* (2017), with a large sample (2,640,778) of companies from 28 European countries, the US, Colombia, and China, with data collected from 2007 to 2010. Similar to Altman *et al.* (2017), non-bankrupt firms exhibited medians and means of X1, X2, and X3 that were close to each other, indicating symmetry in the distribution. However, bankrupt firms showed medians of these three ratios that are larger than the means, indicating a left-skewed distribution. This is consistent with a sample where some companies experienced significant financial losses, resulting in highly negative ratios.

Summarizing, the results of this study are broadly consistent with those of Altman *et al.* (2017), which were based on a much larger, global dataset. Both studies demonstrate that non-bankrupt companies have significantly higher medians and means for the first three ratios (Working Capital/Total Assets, Retained Earnings/Total Assets, and Operating Income/Total Assets) than bankrupt companies. This suggests that maintaining positive working capital, retained earnings, and operating income can significantly improve a company's financial health and reduce the risk of bankruptcy. As expected, the Book Value of Equity/Total Liabilities ratio (X4) exhibits a lower median for bankrupt firms compared to non-bankrupt firms.

Ohlson's model descriptive statistics

In Table 4, we found the descriptive statistics of Ohlson's model.

Again, we can see from the *F*-test that the samples of non-bankrupt and bankrupt companies have unequal variances, and the *t*-test for the samples shows that non-bankrupt companies have statistically significant differences from bankrupt companies for all ratios. As expected, all ratios were statistically significantly different from zero. Similar to Altman's model, negative working capital (WCTA) and negative income (NITA, FUTL, and INTWO –

Table 4. Ohlson's model descriptive statistics

Predictor	SIZE		TLTA		WCTA		CLCA		OENEG		NITA		FUTL		INTWO		CHIN	
Company	Non-B.	Bankrupt	Non-B.	Bankrupt	Non-B.	Bankrupt	Non-B.	Bankrupt	Non-B.	Bankrupt	Non-B.	Bankrupt	Non-B.	Bankrupt	Non-B.	Bankrupt	Non-B.	Bankrupt
Median	8.756	8.297	0.545	0.956	0.263	−0.050	0.565	1.091	0.000	0.000	0.034	−0.059	0.083	−0.042	0.000	0.000	0.159	0.000
Mean	8.991**	8.399**	0.546**	1.248**	0.257**	−0.239**	0.866**	2.062**	0.025**	0.423**	0.045**	−0.181**	0.273*	−0.094**	0.079**	0.216**	0.180**	−0.144**
	(0.011)	(0.050)	(0.003)	(0.056)	(0.004)	(0.045)	(0.042)	(0.195)	(0.002)	(0.026)	(0.001)	(0.024)	(0.126)	(0.010)	(0.002)	(0.021)	(0.006)	(0.030)
F-test	0.036		0.000		0.000		0.000		0.000		0.000		0.000		0.000		0.000	
t-test	0.000		0.000		0.000		0.000		0.000		0.000		0.000		0.000		0.000	

Note(s): Non-B. means non-bankrupt companies. Bankrupt means bankrupt companies. SIZE is the log(Total Assets/GDP price-level index) with base value 100 for 2016, TLTA is the Total Liabilities divided by Total Assets, WCTA is the Working Capital divided by Total Assets, CLCA is the Current Liabilities divided by Current Assets, OENEG is one if Total Liabilities exceed Total Assets (zero otherwise), NITA is the Net Income divided by Total Assets, FUTL is the Funds provided by operations (EBITDA) divided by Total Liabilities, INTWO is one if Net Income was negative for the last two years (zero otherwise), CHIN is $(NI_t - NI_{t-1})/(|NI_t| + |NI_{t-1}|)$, where NI_t is the Net Income for the most recent period. Standard errors in parentheses. ** represents statistical significance at the 1% level, indicating whether the means differ significantly from zero. The *F*-test is the *p*-value of the equality of variances test, used to determine which *t*-test should be used. The *t*-test is the *p*-value of the two-tailed equality test of the means with unequal variance, applied to test whether the Non-Bankrupt and Bankrupt samples have different means

Source(s): Authors' own elaboration

in this case, where “1” represents negative income) as well as declining net income (CHIN) correlated with bankrupt companies. Some ratios and indicators also showed medians close to means in non-bankrupt companies and skewed distributions in bankrupt companies (TLTA, WCTA, CLCA, NITA). Regarding the results of other studies applying Ohlson’s model, Grice and Dugan (2003) found that bankrupt companies showed a working capital (WCTA), unlike our results. The remaining ratios had similar values.

Zmijewski model descriptive statistics

Table 5 shows the descriptive statistics of the Zmijewski model.

Moreover, Zmijewski’s model showed a statistically significant difference between the ratios of non-bankrupt and bankrupt companies. Negative income (ROA), higher leverage (FIN), and lower liquidity (LIQ) correlated with bankrupt companies. Nevertheless, the liquidity ratio of non-bankrupt companies was not statistically different from zero. This was caused by several companies with very low current liabilities, producing extremely high values of this ratio (more than 10% of the non-bankrupt companies show a liquidity ratio higher than 5). We may see this in the large mean (2.999) of the liquidity ratio of non-bankrupt companies, compared to the much lower median (1.768) and having a high standard error (3.354). The inverse of this ratio (Ohlson’s model CLCA) had a bottom of zero, avoiding this situation. The estimations for the non-bankrupt companies were similar to the results of Grice and Dugan (2003), who found a mean ROA of 0.038, a mean FINL of 0.564, and a mean LIQ of 2.179. Regarding the bankrupt companies, they found a ROA of −0.175, a FINL of 0.890, and a LIQ of 3.12. Nevertheless, their results had large standard deviations.

Models’ results and accuracy comparison

We computed the Z”-scores and Probabilities for all company-year observations of bankrupt and non-bankrupt companies. Table 6 compares the models’ accuracy, including the cut-offs and the Type I and Type II errors for each model.

All three models presented an accuracy above 80%, an acceptable level for coefficients estimated more than 25 years ago. Ohlson’s model achieved the highest overall accuracy (88.19%), primarily due to its very low Type II error rate (i.e. classifying a non-bankrupt company as bankrupt). Accuracy considered both bankrupt and non-bankrupt correct predictions as a percentage of the total sample, giving the largest group (non-bankrupt companies) more weight in this measure. The estimated accuracy of Altman’s model (83.89%) was better than the 76.25% estimated by Cındık and Armutlulu (2021) in a study with a small sample (80) of Turkish companies. Grice and Dugan (2003) found an accuracy of 81.3% for

Table 5. Zmijewski’s model descriptive statistics

Predictor Company	ROA Non-B.	 Bankrupt	FINL Non-B.	 Bankrupt	LIQ Non-B.	 Bankrupt
Median	0.034	−0.055	0.545	0.956	1.768	0.905
Mean	0.045**	−0.185**	0.546**	1.250**	2.992	1.208**
	(0.001)	(0.027)	(0.003)	(0.056)	(3.354)	(0.122)
F-test	0.000		0.000		0.000	
t-test	0.000		0.000		0.000	

Note(s): Non-B. means non-bankrupt companies. Bankrupt means bankrupt companies. ROA is Net Income/Total Assets, FINL is Total Debt/Total Assets, and LIQ is Current Assets/Current Liabilities. Standard errors in parentheses. ** represents statistical significance at the 1% level, indicating whether the means differ significantly from zero. The *F*-test is the *p*-value of the equality of variances test, used to determine which *t*-test should be used. The *t*-test is the *p*-value of the two-tailed equality test of the means with unequal variance, applied to test whether the Non-Bankrupt and Bankrupt samples have different means

Source(s): Authors’ own elaboration

Table 6. Original models' accuracy

Model Company	Altman Non-B.	Bankrupt	Ohlson Non-B.	Bankrupt	Zmijewski Non-B.	Bankrupt
Sample	9,001	370	9,001	370	9,001	370
Predicted	7,591	270	7,979	285	7,377	308
Cut-off	≥4.03	<4.03	≤0.5	>0.5	≤0.5	>0.5
False	1,410	100	1,022	85	1,624	62
Type I error		27.03%		22.97%		16.76%
Type II error	15.66%		11.35%		18.04%	
Accuracy	83.89%		88.19%		82.01%	

Note(s): Non-B. is the number of non-bankrupt companies. Bankrupt is the number of bankrupt companies. Type I error is the percentage of bankrupt companies classified by the model as non-bankrupt, and Type II error is the percentage of non-bankrupt companies classified as bankrupt. Accuracy is the percentage of correctly predicted types in the total sample

Source(s): Authors' own elaboration

Zmijewski's model, which is slightly lower than our results, but a notably lower accuracy of 39.8% for Ohlson's model. However, these authors' tests on Ohlson's model accuracy had a different sampling methodology, which could distort the comparative analysis. Zmijewski's model has the lowest Type I error (16.76%), but the highest Type II error (18.04%), leading to the lowest accuracy. The Z"-score performed poorly in terms of Type I error (27.03%), but still presented an overall accuracy similar to the other two models, which is remarkable if we consider that multiple discriminant analysis served to estimate the model's coefficients.

Re-estimated models

Models' re-estimation. After processing the oversampling, which increased the number of bankrupt companies in the sample to match the number of non-bankrupt samples, we applied logit regression to re-estimate the three models using the same independent variables, estimating the new coefficients to predict bankruptcy. The dependent variable in the three models was binary, with zero (0) for non-bankrupt companies and one (1) for bankrupt companies. We considered the values of the variables from last year's available financial accounts. Table 7 presents new estimated coefficients.

Altman's re-estimated model had all estimated coefficients statistically significant, and the coefficients' signs were negative, as expected, suggesting that higher relative working capital, reserves, profitability, and equity will result in lower odds of bankruptcy. The estimates for Altman's model had the same signs as those of Altman *et al.* (2017), but the coefficients differed, suggesting that different samples yielded distinct estimates.

Moreover, Ohlson's re-estimated model had the highest pseudo-R-squared value and the highest Likelihood Ratio (LR), suggesting that it is the best model fit for this dataset. Nevertheless, three of the nine new estimated coefficients, plus the constant, were not statistically significant. This aligns with Ohlson's (1980) observation that there is no strong theoretical basis for selecting all nine variables. These non-significant variables may also be redundant or irrelevant in the context of Portuguese companies. With nine independent variables, Ohlson's model carries an increased risk of multicollinearity due to the possible redundancy of ratios in explaining the dependent variable. We tested multicollinearity, finding that the estimated models did not have problems. The three estimated models exhibited a low mean variance inflation factor (VIF), with all variables having a VIF of less than 5.

In the case of Zmijewski's re-estimated model, the estimated coefficients were similar to those of the original model, although we estimated them using a different methodology. The

Table 7. Logit re-estimation of the models' coefficients

Models	Altman		Ohlson		Zmijewski	
	Constant	0.877** (0.030)	Constant	−0.174 (0.232)	Constant	−4.049** (0.082)
	X1	−0.192* (0.076)	SIZE	−0.422** (0.024)	ROA	−4.260** (0.220)
	X2	−0.650** (0.075)	TLTA	5.167** (0.133)	FINL	5.279** (0.103)
	X3	−8.314** (0.289)	WCTA	−0.155 (0.090)	LIQ	−0.014 (0.007)
	X4	−1.933** (0.063)	CLCA	0.005 (0.006)		
			OENEG	−0.110 (0.098)		
			NITA	−2.169** (0.563)		
			FUTL	−3.095** (0.320)		
			INTWO	−0.179** (0.068)		
			CHIN	−0.727** (0.042)		
Pseudo R^2	0.430		0.463		0.395	
LR statistic	10733.32**		11706.84**		9844.81**	
Deviance	14222.75		13681.56		15111.26	
Mean VIF	1.57		1.73		1.23	

Note(s): Standard errors in parentheses. * and ** represent statistical significance at the 5% and 1% levels, respectively

Source(s): Authors' own elaboration

signs of the three coefficients were also as expected. However, liquidity (LIQ) was not statistically significant, just as it was not in the original estimate (Zmijewski, 1984). This ratio is the inverse of Ohlson's model CLCA (current liabilities to current assets), which is also not statistically significant in the re-estimated Ohlson's model. This could have resulted from the specificity of the Portuguese market, where companies may operate with lower levels of liquidity due to extended periods of outstanding sales that can be financed through factoring or discounting. However, it could have also resulted from the logistic regression estimation problem identified by Li (2023), it fails to produce statistically significant estimates because the relationship between the liquidity ratio and bankruptcy is non-linear.

To preserve the theoretical model, we retained all variables from the initial models, including those with no statistical significance. We tested the accuracy of the re-estimated models without the non-statistically significant variables, and it did not improve.

The new equations were:

$$\begin{aligned}\text{Altman's Log} - \text{Odds} = {} & 0.877 - 0.192(\text{X1}) - 0.650(\text{X2}) - 8.314(\text{X3}) \\ & - 1.933(\text{X4})\end{aligned} \tag{5}$$

in which $P(dummy = 1) = \frac{1}{1 + e^{-(0.877-0.192(\text{X1})-0.650(\text{X2})-8.314(\text{X3})-1.933(\text{X4})}}$

$$\begin{aligned}\text{Ohlson's Log} - \text{Odds} = {} & -0.174 - 0.422(\text{SIZE}) + 5.167(\text{TLTA}) - 0.155(\text{WCTA}) \\ & + 0.005(\text{CLCA}) - 0.110(\text{OENEG}) - 2.169(\text{NITA}) \\ & - 3.095(\text{FUTL}) - 0.179(\text{INTWO}) - 0.727(\text{CHIN})\end{aligned} \tag{6}$$

in which

$$P(dummy = 1) \frac{1}{1 + \mathrm{e}^{-(-0.174-0.422(\mathrm{SIZE})+5.167(\mathrm{TLTA})-0.155(\mathrm{WCTA})+0.005(\mathrm{CLCA})-0.110(\mathrm{OENEG})-2.169(\mathrm{NITA})-3.095(\mathrm{FUTL})-0.179(\mathrm{INTWO})-0.727(\mathrm{CHIN})}}$$

$$\text{Zmijewski's Log} - \text{Odds} = -4.049 - 4.260(\mathrm{ROA}) + 5.279(\mathrm{FINL}) - 0.014(\mathrm{LIQ}) \qquad (7)$$

in which

$$P(dummy = 1) = \frac{1}{1 + \mathrm{e}^{-(-4.049-4.260(\mathrm{ROA})+5.279(\mathrm{FINL})-0.014(\mathrm{LIQ})}}$$

Table 8 shows the accuracy of the re-estimated models tested in the original, imbalanced sample.

The Type I errors (bankrupt companies classified as non-bankrupt) have been reduced in the first two of the three models, with Altman's re-estimated model reducing the Type I error to 13.24% from 27.03% and Ohlson's re-estimated model reducing to 15.68% from 22.70%. However, inversely, the Type II errors increased in the first two of the three models due to oversampling the bankrupt observations, which reduced the weight of the non-bankrupt companies in the estimation sample. The logit re-estimated Zmijewski's model was worse regarding Type I errors and better regarding Type II errors, suggesting that estimation through probit might be more accurate than through logit. The probit model is based on the normal distribution, having narrower tails than the logit model. Thus, it classifies more observations as bankrupt (1) when probabilities are near 0.5, thereby reducing Type I error.

The accuracy of Altman's and Ohlson's models dropped due to oversampling. However, as we aimed to, Altman's and Ohlson's logit re-estimates made better bankruptcy predictions (also known as the recall). The area under the receiving operating characteristic curve (AUC-ROC) had high values (0.91 and 0.92), indicating that all three models effectively distinguished between bankrupt and non-bankrupt companies. Figures 1–3 show visual representations.

Altman *et al.* (2017) used a large sample from 31 countries, of which 29 were European, to re-estimate Altman's Z"-score with a logit regression. The results of their estimation for Portugal in-sample yielded an AUC-ROC of 0.741, which was lower than our model's re-

Table 8. Logit re-estimated models' accuracy in-sample

Model Company	Altman Non-B.	Bankrupt	Ohlson Non-B.	Bankrupt	Zmijewski Non-B.	Bankrupt
Sample	9,001	370	9,001	370	9,001	370
Predicted	7,294	321	7,592	312	7,463	302
Cut-off	≤0.5	>0.5	≤0.5	>0.5	≤0.5	>0.5
False	1,707	49	1,409	58	1,538	68
Type I error		13.24%		15.68%		18.38%
Type II error	18.96%		15.65%		17.09%	
Accuracy	81.26%		84,35%		82.86%	
AUC-ROC	0.91		0.92		0.91	

Note(s): AUC-ROC means the Area Under the ROC Curve. Non-B. is the number of non-bankrupt companies. Bankrupt is the number of bankrupt companies. Type I error is the percentage of bankrupt companies classified by the model as non-bankrupt, and Type II error is the percentage of non-bankrupt companies classified as bankrupt. Accuracy is the percentage of correctly predicted types in the total sample

Source(s): Authors' own elaboration

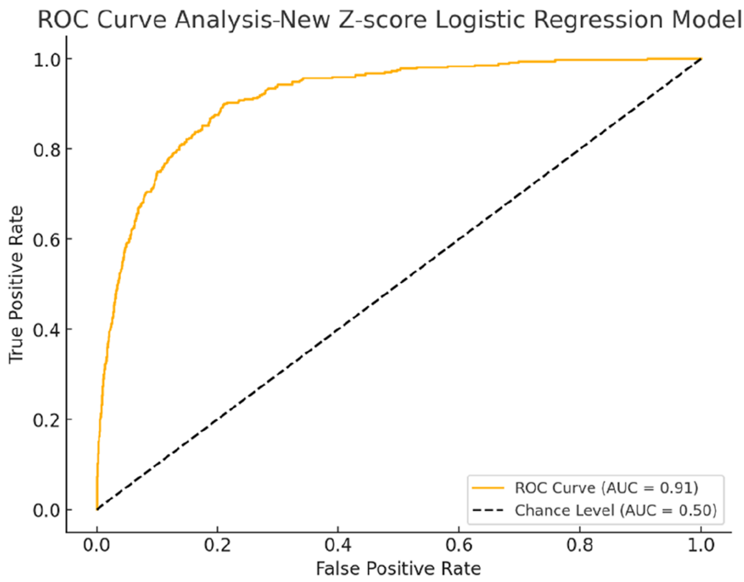

Figure 1. Re-estimated Z”-score’s area under the ROC curve. Source: Authors’ own elaboration

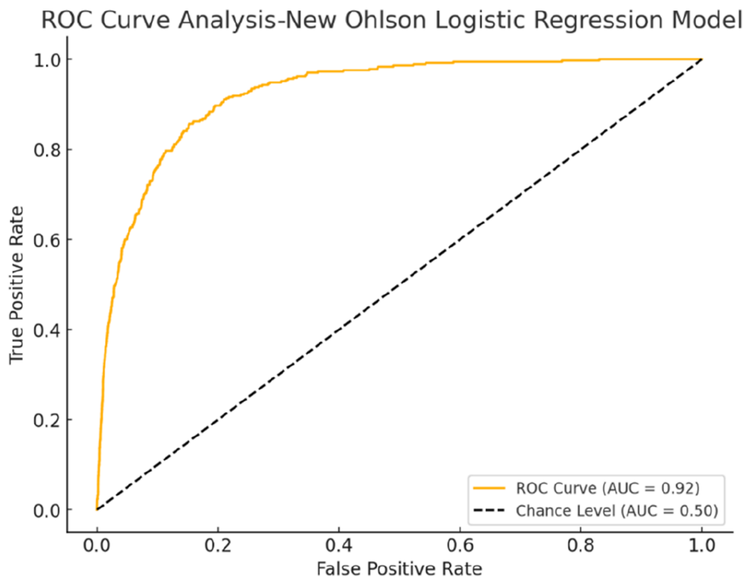

Figure 2. Re-estimated Ohlson’s area under ROC curve. Source: Authors’ own elaboration

estimates. In the following section, we present how we tested these new equations in different samples.

Out-of-sample models accuracy testing. We repeated the data collection process described in the *Data* section, selecting companies with active and bankrupt status in 2023 and collecting

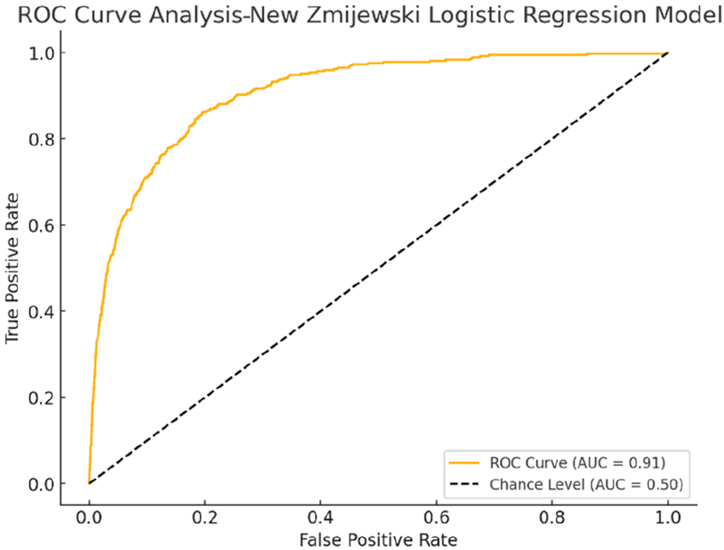

Figure 3. Re-estimated Zmijewski's area under ROC curve. Source: Authors' own elaboration

accounting data for 2022. We found 9,337 active and 87 bankrupt companies. This represents the most significant number of bankruptcies in our sample years (see Table 1), consistent with our expectation of increased bankruptcy risk in the post-COVID economic environment. We applied the equations from the previous section to predict corporate bankruptcies using this out-of-sample data and measured the accuracy of the logit models. Table 9 presents the results.

As expected, the Type I errors were higher than in the in-sample test, but the Type II errors were much lower. This is a consequence of the out-of-sample imbalance, as the bankruptcies were only for one year (2023), amounting to 87, a much smaller number than the in-sample, where they were 370. This, coupled with an increase in non-bankrupt companies, most of which were the same as those in the in-sample, results in the non-bankrupt class representing 99.08% of the sample. Logistic regression is known to be biased toward the majority class, which can increase accuracy to higher levels than those found in the in-sample accuracy test.

Table 9. Logit re-estimated models' accuracy out-of-sample

Model Company	Altman Non-B.	Bankrupt	Ohlson Non-B.	Bankrupt	Zmijewski Non-B.	Bankrupt
Sample	9,337	87	9,337	87	9,337	87
Predicted	8,310	66	8,264	70	7,963	71
Cut-off	≤0.5	>0.5	≤0.5	>0.5	≤0.5	>0.5
False	1,027	21	1,073	17	1,374	16
Type I error		24.14%		19.54%		18.39%
Type II error	11.00%		11.49%		14.72%	
Accuracy	88.88%		88.43%		85.25%	
AUC-ROC	0.91		0.93		0.92	

Note(s): AUC-ROC means the Area Under the ROC Curve. Non-B. is the number of non-bankrupt companies. Bankrupt is the number of bankrupt companies. Type I error is the percentage of bankrupt companies classified by the model as non-bankrupt, and Type II error is the percentage of non-bankrupt companies classified as bankrupt. Accuracy is the percentage of correctly predicted types in the total sample

Source(s): Authors' own elaboration

The results of the out-of-sample accuracy test for the original models followed the same pattern as the in-sample results, with Type I errors higher than the re-estimated Altman's and Ohlson's models but lower Type II errors. Zmijewski's original model has lower Type I errors than the re-estimated model in the out-of-sample test. The overall accuracy of the original models was lower than that of their re-estimated versions. We decided not to publish the results due to the lack of available space, but we can share them upon request.

The AUC-ROC was high, surpassing the values reported by Altman *et al.* (2017) for their logit regression estimates, both for Portugal (0.741) and for the combined data from all 31 countries (0.748). As we aimed to test whether expanding these re-estimates to predict bankruptcy in other European countries was possible, in the next section, we present the results of the robustness test on a sample of European companies.

Robustness test

We repeated the process outlined in section *Data*, but in the Orbis database (also a Moody's Analytics service). The extraction process was more complex than in SABI due to differences in financial reporting (although less significant within the European Union) and differences in reporting bankruptcy. We included bankruptcy and insolvency proceedings, as several European countries register bankruptcy under that classification. We searched for companies within the European Union (excluding Portugal) that were incorporated before 2016 and that declared bankruptcy between 2016 and 2023, with at least 10 employees and two years of reported accounts. We found only 83 such companies. Of these, only 38 had complete data for the ratio calculations. The countries were Belgium (5), Denmark (1), Spain (2), Finland (3), France (12), Hungary (2), the Netherlands (2), Romania (2), and Sweden (9). We then searched for non-bankrupt companies in the same countries to create a country-based homogeneous sample. Unfortunately, some countries lacked accounting information, which prevented the construction of a perfectly matched sample of bankrupt and non-bankrupt companies per country. We ranked the available companies according to total assets (larger than 2 million euros). We picked companies from the smaller to the larger until we ran out of companies in one of the countries, resulting in a sample of 190 non-bankrupt companies, primarily SMEs, with the following distribution by countries: Belgium (25), Denmark (7), Spain (13), Finland (15), France (60), Hungary (6), The Netherlands (16), Romania (1), and Sweden (47). The global sample had 228 companies, with one bankrupt company for every five non-bankrupt companies. We present the results of the accuracy test in Table 10.

We also computed the results of the original models applied to the same sample for comparison. Table 11 shows the results.

Table 10. Logit re-estimated models' accuracy in the European sample

Model Company	Altman Non-B.	Bankrupt	Ohlson Non-B.	Bankrupt	Zmijewski Non-B.	Bankrupt
Sample	190	38	190	38	190	38
Predicted	138	24	136	28	136	25
Cut-off	≤0.5	>0.5	≤0.5	>0.5	≤0.5	>0.5
False	52	14	54	10	54	13
Type I error		36.84%		26.32%		34.21%
Type II error	27.37%		28.42%		28.42%	
Accuracy	71.05%		71.93%		70.61%	
AUC-ROC	0.69		0.74		0.72	

Note(s): AUC-ROC means the Area Under the ROC Curve. Non-B. is the number of non-bankrupt companies. Bankrupt is the number of bankrupt companies. Type I error is the percentage of bankrupt companies classified by the model as non-bankrupt, and Type II error is the percentage of non-bankrupt companies classified as bankrupt. Accuracy is the percentage of correctly predicted types in the total sample

Source(s): Authors' own elaboration

Table 11. Original models' accuracy in the European sample

Model Company	Altman Non-B.	Bankrupt	Ohlson Non-B.	Bankrupt	Zmijewski Non-B.	Bankrupt
Sample	190	38	190	38	190	38
Predicted	142	20	148	27	132	25
Cut-off	≥4.03	<4.03	≤0.5	>0.5	≤0.5	>0.5
False	48	18	42	11	58	13
Type I error		47.37%		28.95%		34.21%
Type II error	25.26%		22.11%		30.53	
Accuracy	71.05%		76.75%		68.86%	
AUC-ROC	n.a		0.76		0.72	

Note(s): AUC-ROC means the Area Under the ROC Curve. Non-B. is the number of non-bankrupt companies. Bankrupt is the number of bankrupt companies. Type I error is the percentage of bankrupt companies classified by the model as non-bankrupt, and Type II error is the percentage of non-bankrupt companies classified as bankrupt. Accuracy is the percentage of correctly predicted types in the total sample

Source(s): Authors' own elaboration

Our re-estimated coefficients yielded lower Type I errors than the original models and were comparable in accuracy. The exception was Zmijewski's original model, which produced the same errors as the re-estimated version. Nevertheless, the errors were higher than those predicted using the out-of-sample Portuguese companies, suggesting that each country should use estimated coefficients based on its national companies.

The re-estimated model that produces the lowest Type I error and highest AUC-ROC was Ohlson's model. The original Ohlson model showed better AUC-ROC than our re-estimates, but produced slightly higher Type I errors. Altman's model continues to produce high Type I errors, suggesting that the chosen ratios in the equation were less relevant for bankruptcy prediction than those of Zmijewski and Ohlson. Our AUC-ROC of Altman's re-estimated model (0.69) was lower than Altman *et al.* (2017) estimates (0.74), but we worked with a much smaller sample in this robustness test.

Our robustness test revealed important limitations in the cross-country applicability of the re-estimated models. The accuracy dropped significantly when applied to other European countries, with Type I errors remaining high (26.32 to 36.84%). This suggests that country-specific institutional, legal, and economic factors substantially influence the performance of bankruptcy prediction models, limiting the direct transferability of coefficients estimated in one country to others. Moreover, the small sample size in our robustness test (228 companies, with only 38 bankruptcies) meant that individual firm classifications could significantly impact overall accuracy measures, and researchers should interpret our conclusions regarding European applicability with caution.

Conclusion

This study aimed to test the accuracy of three world-famous accounting-based corporate bankruptcy prediction models (Altman's Z"-score, Ohlson's model, and Zmijewski's model) to check if they can still serve to measure corporate default risk in a market (Portugal) that can soon be affected by a wave of bankruptcies and to re-estimate those models to better predict bankruptcy, either in Portugal or in other European countries.

We applied the models to a large sample of 9,371 Portuguese companies. We ran the three models using their original coefficients, all of which were estimated in the 1980s and 1990s. We expected the coefficients to change over time and differ across companies of different countries and sizes. We wanted to know if they could still be applied to assess the bankruptcy probability of companies of almost all sizes and sectors. The three models still exhibit very reasonable accuracy levels above 80%, although they report high Type I errors (failing to identify a company that will enter bankruptcy within the next year).

We re-estimated the coefficients of all three models using logit regressions and tested the re-estimates with an out-of-sample analysis of Portuguese companies. Although the overall accuracy of the three models reduced slightly, we successfully reduced the Type I errors, which was our primary objective. Logit did not improve Zmijewski's model estimations, suggesting that the original methodology (probit regression) might be more accurate for accounting-based prediction models. We leave this hypothesis as a suggestion for future research.

We further tested the re-estimated models on 228 companies from nine European countries, demonstrating that our re-estimates outperform the original models in predicting bankruptcies. Ohlson's model achieves the best balance between reducing Type I errors and overall accuracy, whether in its original form or re-estimated.

Using available accounting information, our results can help small and medium-sized enterprises better assess their customers' credit risk in an environment with higher corporate bankruptcy rates. Nevertheless, although the newly estimated equations with accessible financial ratios contribute to a better assessment of corporate credit risk, we concluded that while the original models demonstrated overall accuracy above 80%, their high Type I error rates limit their practical utility for bankruptcy prediction, where misclassifying bankrupt companies imposes significant costs. Our re-estimated models, particularly Ohlson's, offer a more balanced approach with improved bankruptcy detection capabilities (Type I error reduced to 15.68%) while maintaining a reasonable overall accuracy. However, the limited cross-country transferability observed in our robustness test suggests that country-specific re-estimation may be necessary for optimal performance.

As limitations, we acknowledge that our reliance on the last available financial report introduces temporal uncertainty, as the gap between financial reporting and bankruptcy events can vary substantially. We also acknowledge that sectoral factors may influence model performance, as different industries exhibit varying financial ratio patterns. For example, retail sector firms typically operate with low or negative working capital, which affects the working capital ratios used in all three models (X1 in Altman's model, WCTA in Ohlson's model, and the inverse in Zmijewski's LIQ ratio). Future research should examine how industry characteristics affect the accuracy of bankruptcy predictions and consider sector-adjusted coefficient estimation. Moreover, we acknowledge that we tested the re-estimated model accuracy out-of-sample only one year after the end of the estimation period. This could introduce some bias, especially in the non-bankruptcy prediction measurement, as most sample companies are similar. Furthermore, our European robustness test sample was relatively small (228 companies with 38 bankruptcies), limiting the generalizability of cross-country findings. Finally, while our re-estimated models improve Type I error rates for Portuguese companies, the limited cross-country transferability suggests that optimal performance requires country-specific calibration.

By addressing this limitation and testing whether probit regression outperforms logit regression in reducing Type II errors, as our findings suggest, future studies can significantly enhance the accuracy and reliability of accounting-based bankruptcy prediction models, providing valuable tools for financial analysts, investors, and policymakers.

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