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# Implied volatility modeling and forecasting: evidence from China

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## Abstract

**Purpose** – Testing several approaches for implied volatility modeling and forecasting.
**Design/methodology/approach** – Comparative empirical study with four traded options.
**Findings** – Non-parametric higher-order spline is better than parametric stochastic volatility inspired (SVI) in China.
**Research limitations/implications** – Our results imply that even though popular on Wall Street, SVI seems not to be utilized by traders and market-makers in China.
**Practical implications** – Traders may consider higher-order spline as a better method for implied volatility modeling and forecasting.
**Originality/value** – Propose to model and forecast implied volatility via the fifth-order spline interpolation as a first; initiates studies of the empirical performance of SVI and the fifth-order spline models in implied volatility modeling and forecasting.

**Keywords** SVI, 5th-order spline, Volatility modeling and forecasting,
In-sample and out-of-sample pricing errors, Volatility arbitrage, China's options
**Paper type** Research paper

## 1. Introduction

Implied volatility, which primarily relies on the Black-Scholes-Merton (BSM) model (Black and Scholes, 1973; Merton, 1973), has both theoretical and practical implications. Theoretically, the study of implied volatility can lead to better pricing models for options and a better understanding of market information contents (Jiang and Tian, 2005) and completeness (Almeida and Freire, 2022). In practice, implied volatility estimating is vital in options trading for pricing and hedging (i.e. risk management) (Muzzioli, 2010). Market makers, who facilitate the smooth functioning of options markets, quote implied volatilities in trading and utilize them in hedging risks of short option positions (Hull, 2015).

Implied volatility has been studied extensively for the US options market (Breeden and Litzenberger, 1978; Gatheral, 2006; Figlewski, 2010; Derman and Miller, 2016). In contrast, such research for China's options market, which has a relatively short history but is developing rapidly (Jiao *et al.*, 2021), is just getting started. For modeling implied volatility, both industry-oriented and academia-oriented approaches exist (Homescu, 2011). There

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seems to be a disconnection between industry and academia, however, because the two groups seem to focus on different methods without too many overlaps. For these reasons, we propose to investigate the empirical performance of both practical-oriented and academic-oriented approaches in China's options market.

Among many approaches to implied volatility modeling, our study chooses two archetypal representatives: the practical-oriented stochastic volatility inspired (SVI) model and the academically popular 5th-order spline interpolation. Originated at Merrill Lynch in 1999 and popular on Wall Street, SVI was initially publicized by Gatheral (2004). Subsequent developments are summarized in a seminar paper by Gatheral and Jacquier (2014). The SVI model, providing a functional form, is parametric [1]. However, most research on SVI focuses primarily on the arbitrage-free parametrization of the surface SVI (SSVI) (Zhao and Hodges, 2013; Guo *et al.*, 2016; Corbetta *et al.*, 2019; Martini and Mingone, 2022, 2023) and recently the extended SSVI (eSSVI) (Hendriks and Martini, 2019; Mingone, 2022), while overlooking the empirical performance of SVI in implied volatility modeling and forecasting.

In contrast, Figlewski (2010) proposes a nonparametric approach, the 5th-order spline interpolation. This spline approach, leading to smooth volatility curves, is used primarily to extract empirical risk-neutral densities. Later, it is employed to study empirical pricing kernels by Linn *et al.* (2018) and Jiao *et al.* (2021). In addition to extracting risk-neutral densities, we propose that the 5th-order spline interpolation can also be used to model and forecast implied volatilities, which seems to be overlooked by previous studies [2]. With a smooth volatility curve, we can compute both in-sample and out-of-sample pricing errors for options (a.k.a. implied volatility modeling and forecasting).

Based on the reasons given above, we initiate the study of the empirical performance of implied volatility modeling and forecasting. For our empirical analysis, we choose China's options market [3]. Given that traders and market makers in China are relatively inexperienced and unsophisticated, it is an open question whether SVI, even though well-established and popular on Wall Street, is employed in practice. Therefore, we believe it is interesting to learn whether SVI is suitable for China, where the options market is emerging as a significant rival to the US options market. Further, it can be fascinating to compare SVI (industrial and parametric) with the 5th-order spline (academic and nonparametric).

Methodologically, we make several innovations. First, we propose to utilize risk-neutral tests to rule out possible arbitrage in the spline-smoothed volatility curves. We focus on curves instead of surfaces in this initial study because, arguably, it is more important for market makers to avoid being arbitraged for price quotes of options with the same maturity but different strike prices. Second and more importantly, we propose to gauge volatility modeling by in-sample pricing errors and volatility forecasting by out-of-sample pricing errors. It is worth noting that price-based errors, being model-independent, are better than model-dependent (implied) volatility-based errors. Innovatively, we use only arbitrage-free data for assessing pricing errors. It is meaningless, we argue, to compare pricing performance if prices in the dataset admit arbitrage. Finally, we propose a volatility arbitrage strategy and use the resultant P&L as an additional measure to evaluate volatility modeling. The arbitrage trade employs three out-of-the-money (OTM) calls and three OTM puts closest to at-the-money (ATM) and utilizes the Derman-Miller arbitrage (2016).

As a first, our paper carries out a comprehensive empirical study of implied volatility modeling and forecasting for China's new market. We choose four representative options in China for the empirical study (for our reasons, see Section 3.1.1). They are China 50 ETF options (stock portfolio as underlying, European), CSI 300 index options (stock index with known cash dividends as underlying, European), soybean meal futures options (futures as underlying, American), and copper futures options (futures as underlying, European). We include daily data for 2021 and use only out-of-money options. To assess volatility modeling, we use in-sample pricing errors and P&Ls of a volatility arbitrage strategy (see Section 2.4 later). We find that with lower pricing errors and smaller arbitrage P&Ls, the 5th-order spline appears to be a better model for options traded in China. Next, we gauge volatility forecasting by out-of-sample pricing performance. In addition to the 5th-order spline, we forecast volatility by employing linear interpolation on implied variance and interpolating cubic spline on implied volatility. Empirically, we find that the three splines outperform SVI for all four options. Our results imply that even though popular on Wall Street, SVI seems not to be utilized by traders and market-makers in China.

Our study makes four contributions to the derivatives literature [4]. First, we propose to model and forecast implied volatility via the 5th-order spline interpolation, which has been used primarily to extract empirical risk-neutral densities (Figlewski, 2010). Note that implied volatility modeling was studied previously in Gatheral (2004) and Gatheral and Jacquier (2014), for example. Second, our paper initiates studies of the empirical performance of SVI and the 5th-order spline models in implied volatility modeling and forecasting for the options market in China. Previous studies mainly focus on the arbitrage-free parametrization of the surface SVI (Zhao and Hodges, 2013; Guo *et al.*, 2016; Corbetta *et al.*, 2019; Hendriks and Martini, 2019; Nagy and Ormos, 2019; Mingone, 2022; Martini and Mingone, 2022, 2023). Third, to measure the performance of implied volatility modeling, we propose to use in-sample pricing errors and volatility arbitrage P&Ls. Note that volatility errors are found in Koopman *et al.* (2005), Ederington and Guan (2005), Gospodinov *et al.* (2006), Becker (2007), Seo and Kim (2015), and Jeon *et al.* (2020). Fourth, we suggest assessing the performance of implied volatility forecasting by analyzing out-of-sample pricing errors. Most current literature relies on in-sample pricing errors, which may perform poorly in gauging the forecasting of implied volatility. Recently, Liu *et al.* (2021) and Vrontos *et al.* (2021) used machine learning to study the forecasting of implied volatility in the US; however, their out-of-sample pricing result is limited.

The remainder of the paper is structured as follows. Section 2 introduces the paper's methodology, including the surface SVI model, the 5th-order spline model, two interpolating spline methods, and a volatility arbitrage strategy. Data and computational details are presented in Section 3. In Section 4, we discuss the empirical results, including in-sample errors, volatility arbitrage P&Ls, and out-of-sample errors. Finally, Section 5 concludes the paper with remarks.

## 2. Methodology

In this section, we briefly reintroduce two representative approaches to volatility modeling. The stochastic volatility inspired (SVI) model, which dates back to Merrill Lynch in 1999, is widely used on Wall Street (Gatheral and Jacquier, 2014). In academic studies, the 5th-degree spline smoothing of implied volatility curves is proposed (Figlewski, 2010) and utilized to construct the empirical risk-neutral density (Linn *et al.*, 2018; Jiao *et al.*, 2021).

### *2.1 SVI*

According to Gatheral and Jacquier (2014), many arbitrage-free SVI surface parametrizations are possible. Our unreported exploratory study shows that the raw SVI and SVI jump-wings (SVI-JW) parameterizations are somewhat unstable numerically. Therefore, we choose to utilize the surface SVI in this paper.

*SVI parametrization* We reintroduce the surface SVI by directly following Gatheral and Jacquier (2014). First, the total implied variance is defined in Definition 4.1 of Gatheral and Jacquier (2014) as follows:

$$w(k,\theta_\tau) = \frac{\theta_\tau}{2}\left\{1 + \rho\phi(\theta_\tau)k + \sqrt{[\phi(\theta_\tau)k + \rho]^2 + (1-\rho^2)}\right\} \tag{1}$$

where $k = \ln\left(\frac{K}{F_0}\right)$ is the log forward moneyness (with strike $K$ and forward price for stocks or futures price for futures $F_0$), $\theta_\tau := \sigma^2_{BSM}(0,\tau)\tau$ is the at-the-money (ATM) [5] total implied variance [6], $\sigma_{BSM}(k,\tau)$ is the BSM implied volatility (with maturity $\tau$). Note that $-1 < \rho < 1$.

Further, Gatheral and Jacquier (2014) provide several functional forms for $\phi(\theta)$ [7]. Our exploratory study shows that the following power-law function in Example 4.2 of Gatheral and Jacquier (2014),

$$\phi(\theta) = \eta\theta^{-\gamma} \tag{2}$$

(with $\eta > 0$ and $0 < \gamma < 1$) and the Heston-like function in their Example 4.1 are both robust. In this paper, we choose the above power-law function in our implementation.

Finally, Theorem 4.2 of Gatheral and Jacquier (2014) provides the following conditions for the absence of butterfly (convex) arbitrage in the surface SVI model:

$$\begin{aligned}\theta\phi(\theta)(1+|\rho|) &< 4\\ \theta\phi^2(\theta)(1+|\rho|) &\leq 4\end{aligned}$$

This paper focuses on volatility curves instead of the surface, so we do not try to eliminate calendar spread arbitrages on the surface.

*SVI calibration* Given a group of BSM implied total variances, we interpolate linearly to obtain the at-the-money $\theta_\tau$ and calibrate the power-law parameter set $\{\rho,\eta,\gamma\}$ by minimizing the following objective function:

$$\underset{\{\rho,\eta,\gamma\}}{argmin}\sum_j\left[w(k_j,\theta_\tau) - \sigma^2_{BSM}(k_j,\tau)\tau\right]^2$$

where $j$ indexes the given moneyness of the implied total variances.

We propose a straightforward minimization scheme here. With a set of random guesses for $\{\rho,\eta,\gamma\}$ as initial values, we minimize the above objective function via the Nelder-Mead simplex algorithm. Then, we apply the parameter constraints, $-1 < \rho < 1, \eta > 0$, and $0 < \gamma < 1$, and no butterfly arbitrage conditions (3–4). If an optimal parameter set satisfies both the parameter constraints and no-arbitrage conditions, it is deemed acceptable and kept. Following Jiao *et al.* (2021), to mitigate the trap of local minima, we generate 100 sets of random initial parameter values [8], carry about the minimization, check constraints and no-arbitrage conditions, and take the optimal set with the lowest value for the objective function to be our calibrating result.

### *2.2 The spline model*

Figlewski (2010) proposes to use the 5th-order spline interpolation of implied volatility to recover the risk-neutral density from market option prices. This approach is preferred because, in addition to leading to smooth volatility curves, it can also yield continuous and smooth risk-neutral densities. This approach was later adopted by Linn *et al.* (2018) and adapted by Jiao *et al.* (2021) [9]. Our reintroduction follows Jiao *et al.* (2021).

*The 5th-order spline* Given a group of BSM implied volatilities, we employ the 5th-order spline interpolation to smooth the implied volatility curve on moneyness [10]. We place one knot at-the-money (i.e. moneyness of one) with boundary knots at the lowest and highest moneyness observed [11].

We briefly reintroduce the spline function for the convenience of readers. The fitted volatility function is as follows:

$$\widehat{\sigma}(m) = \sum_{j=1}^{b} c_j B_j(m) \tag{3}$$

where $m$ is moneyness (i.e. strike price divided by stock price), $B_j(m)$ is the $j$th B-spline basis function, and $b$ is the number of basis functions. The coefficients $c_j$ are obtained by the following minimization:

$$\underset{\{c_j, j=[1,b]\}}{argmin} \sum_{i=1}^{n} \left[ \sigma(m_i) - \sum_{j=1}^{b} c_j B_j(m_i) \right]^2$$

where $m_i$ is the market moneyness of $i$th traded options. In our case, $b = 6$.

*Risk-neutral tests* With the smoothed volatility curve, we compute finely-spaced BSM European call prices on the curve. Further, we numerically estimate the risk-neutral densities within the observed moneyness range, using Equation (4) of Jiao *et al.*, (2021):

$$q_m = e^{r\tau} \frac{c(m + \Delta_m) + c(m - \Delta_m) - 2c(m)}{(\Delta_m)^2}$$

where $r$ is the risk-free rate, $c$ is the BSM call price, and $m$ is the moneyness. Following Jiao *et al.* (2021), we set $\Delta_m$ to be 0.0001.

If any of the estimated $q_m$ turns out to be negative, a 5th-order spline interpolation without knots is carried out again. Finally, the smoothed volatility curve is accepted if no negative densities exist (Figlewski, 2010; Linn *et al.*, 2018; Jiao *et al.*, 2021). For CSI 300 index options, ten curves are discarded out of 1,362 curves. For copper futures options, only one curve is removed out of 596 curves.

SVI is a parametric volatility model, while the spline-interpolated volatility curve is nonparametric. Therefore, the spline model can also be used to arbitrage volatility. When forecasting volatilities outside the observed moneyness range, we set the volatility to be the nearest observed volatility.

### *2.3 Interpolating splines*

The 5th-order spline model, which does not require any observed data points to be on the spline curve, may perform poorly in forecasting volatilities. For forecasting, we include two other interpolating splines, which go through all the observed data points, as benchmarks. The simplest is undoubtedly the linear interpolation on implied variance (i.e. the squared implied volatility) [12]. Further, interpolating cubic spline on implied volatility is also widely used [13].

By definition, the in-sample pricing error of interpolating splines is zero. As a result, we only need to consider out-of-sample pricing errors for interpolating splines. Again, in forecasting volatilities outside the observed moneyness range, we set the volatility to be the nearest observed volatility.

The four approaches introduced above have pros and cons. With Formula (1), SVI is trivial to compute for any given moneyness and can be made arbitrage-free. For the 5th-order spline, it is not easy to estimate variance for moneyness outside the observed moneyness range. Finally, the two other interpolating splines can be used to forecast, but not model, volatilities.

### *2.4 Volatility-arbitrage strategy*

Academic studies usually focus on volatility forecasting or out-of-sample pricing. In industry, traders may be more interested in volatility modeling-based arbitrage opportunities to make money. A simple volatility-arbitrage strategy is suggested by Derman and Miller (2016). If an option's BSM implied volatility is higher (lower) than the model volatility, one could sell (buy) the option and hedge with the underlying at the model volatility. If the abnormal market volatility reverts to the mean (model), the trade is closed to make a profit.

In this paper, we propose a strategy utilizing only liquid options. For each trading day, we select three OTM calls and three OTM puts closest to at-the-money. Among these six options, we choose the option having the largest discrepancy between the BSM implied and model volatilities and apply the Derman-Miller arbitrage [14]. The arbitrage position is held for one trading day and then liquidated.

We compute the one-day P&L of this volatility-arbitrage strategy as is commonly done in the literature (Hull, 2015; Liu *et al.*, 2022). Ignoring the time value of money, we can write the arbitrage P&L as follows:

$$e_1 = I \times [v_1(\tau - \delta) - v_0(\tau) + \Delta_0(S_0 - S_1)] \tag{4}$$

where $I$ is 1 (−1) for longing (shorting) options, $v_0$ is the market option price on day 0, $\Delta_0$ is the option delta computed with the model volatility on day 0, $S_t$ is the underlying price on day $t$, $\tau$ is the maturity of the option on day 0, and $\delta$ equals one trading day. For simplicity, we assume that short sales are allowed and there are no transaction costs.

## 3. Empirical studies

### *3.1 Data*

*3.1.1 Data source.* At the beginning of our research in 2021, the Chinese market traded 24 options. This paper chooses four representative options from four exchanges for the empirical studies [15]. They are European options on China 50 ETF (50ETF options), European CSI 300 stock index option, American soybean meal futures option, and European copper futures options. The 50ETF option from Shanghai Stock Exchange is the first exchange-traded option in China, while the CSI 300 index option from China Financial Futures Exchange is the first stock index option in China. The soybean meal futures option from Dalian Commodity Exchange is China's first commodity futures option. Finally, the copper futures option from Shanghai Futures Exchange is China's first European commodity futures option. Appendix provides additional details about these options.

The data in our paper are obtained from the WIND database. For consistency, we utilize daily data for 2021 for China 50 ETF options, CSI 300 index options, and soybean meal futures options. The sample interval for the copper futures options is from November 2020 to October 2021 however, because this option became American-style as of November 2021. The underlying closing prices [16] are also from WIND.

Note that the daily data provides only closing prices for options. Fortunately, WIND also provides daily bid-ask mid-price implied volatilities for options. With the implied volatility, we can recover the option's mid-price. In our empirical study, we choose the option prices with fewer cases of butterfly arbitrage. As a result, the mid-prices for 50ETF and CSI 300 index options are utilized, while the closing prices for the two commodities futures options are employed.

*3.1.2 Recovering mid-price.* The bid-and-ask mid-price is commonly utilized in empirical studies of traded options (Bakshi *et al.*, 1997; Barone-Adesi *et al.*, 2008). Even though WIND does not provide bid or ask prices, it does the mid-price implied volatility. Therefore, to recover the mid-price, we need to reverse the computation of WIND.

There are three issues with the approach of WIND. First, WIND defines the days to maturity as $T - t + 1$, where $T$ is the expiry date, and $t$ is the trading date, which unfortunately counts the days to maturity by one more day. Second, WIND uses the yield to maturity of a one-year (Chinese) treasury bond as the risk-free rate, which ignores the different maturities of traded options. Third, For the CSI 300 index, WIND sets the dividend yield to zero. This may be incorrect because the components of the CSI 300 index pay cash dividends.

There are two more details to note. WIND prices American options via binomial trees with the Cox-Ross-Rubinstein parameterization (Cox *et al.*, 1979) and 100 steps. Further, WIND sets the dividend yield to zero for China 50 ETF. We follow the WIND approach to compute the mid-price from the implied volatility in WIND.

*3.1.3 Implied volatility.* With either the closing prices or the recovered mid-prices of options, we can now estimate "better" implied volatilities and use them in our empirical study.

Unlike WIND, we correctly define the days to maturity as $T - t$, where $T$ is the expiry date, and $t$ is the valuation date. Further, Following Jiao *et al.* (2021), we use Shibor rates as risk-free interest rates. Shibor rates are converted to the continuously-compounded zero-coupon rate. Then, for any given maturity, the risk-free rate is obtained by linear interpolating the converted continuously-compounded zero-coupon rate.

European options are priced via BSM formulas. Following Jiao *et al.* (2021), we do not consider dividends in pricing 50ETF options. According to Liu and Qiao (2017), the CSI 300 index is more appropriately treated as an investment asset with known cash dividends because companies in China pay dividends predominantly between May and August. As a result, we utilized the modified BSM formulas (2-3) from Guo and Liu (2019) to price CSI 300 index options. Please also see Appendix for in-depth information. For American options, we compute prices via binomial trees with the Trigeorgis (1991) parameterization, which is numerically better than the Cox-Ross-Rubinstein parameterization, and 500 steps.

*3.1.4 Data filters.* Following roughly common practice (Bakshi *et al.*, 1997; Jiao *et al.*, 2021), we filter the options data using the following seven criteria. First, delete options with zero volatility from WIND. Second, delete options with prices that are less than ten times the minimum tick of the option [17]. Third, delete options with a trading volume of 100 for 50ETF options and zero for the other three options. Fourth, delete options with less than ten days to maturity. Fifth, delete at-the-money and in-the-money options because typically, only out-of-the-money (OTM) options are used in SVI. Sixth, remove a trading day if there is any butterfly (or convex) arbitrage among the OTM calls or OTM puts [18]. Seventh, keep a trading day if there are at least six puts and at least one call, at least six calls and at least one put, or at least three calls and at least three puts after the previous six filters. Finally, note that for 50ETF options, we do not include options that are adjusted for dividends, as is done in Jiao *et al.* (2021).

We filter the mid-prices and closing prices separately. With mid-price, we end up with 7,333 50ETF options, 23,797 CSI 300 index options, 3,017 soybean meal futures options, and 5,004 copper futures options. With the closing price, we obtain 7,295 50ETF options, 17,563 CSI 300 index options, 14,074 soybean meal futures options, and 11,442 copper futures options. Arguably, the more options, the better for empirical studies. Therefore, we estimate implied volatilities for 50ETF and CSI 300 index options from mid-prices but copper and soybean meal futures options from closing prices.

For 50ETF and CSI 300 index options, a call auction session usually sets the closing price in the last three minutes of each trading day. Therefore, it stands to reason that traders can place any bid or ask price without worrying about being arbitraged. Consequently, one can expect more arbitrage opportunities in the closing prices, which seems to be confirmed by the results reported in the previous paragraph.

### *3.2 Computed variables*

Following Bakshi *et al.* (1997), Barone-Adesi *et al.* (2008), and Liu *et al.* (2022), we categorize options by moneyness and maturity. Moneyness is defined as the ratio of the strike price to the underlying price. Moneyness between 0.85 and 1 for puts (between 1 and 1.15 for calls) is grouped into OTM, while moneyness below 0.85 for puts (above 1.15 for calls) does into deep OTM. Maturity is grouped by calendar days to maturity: 10–90 (short), 91–180 (medium), and 181 and up (long) for soybean meal futures options, or 10–90 (short) and 91 and up (long) for the three other options.

*3.2.1 Implied volatility.* Tables 1–4 present the four options' descriptive statistics on price, implied volatility, moneyness, and maturity. Let's discuss these tables one by one. Table 1 shows 3,361 50ETF puts and 3,972 50ETF calls. Overall, option prices are between

| | Short (10–90 days) | | | | Long (>90 days) | | | |
|---|---|---|---|---|---|---|---|---|
| | Price | $\sigma$ (%) | MN | Mat | Price | $\sigma$ (%) | MN | Mat |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| Mean | 0.0044 | 31.66 | 0.8188 | 45 | 0.0210 | 25.06 | 0.8164 | 139 |
| Min | 0.0010 | 20.54 | 0.7478 | 14 | 0.0085 | 21.53 | 0.7478 | 97 |
| Max | 0.0130 | 43.90 | 0.8499 | 89 | 0.0688 | 29.77 | 0.8499 | 225 |
| SD | 0.0027 | 4.75 | 0.0239 | 19 | 0.0106 | 1.75 | 0.0247 | 27 |
| *N* | | 182 | | | | 145 | | |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| Mean | 0.0268 | 21.46 | 0.9316 | 42 | 0.0951 | 23.08 | 0.9274 | 145 |
| Min | 0.0010 | 13.76 | 0.8502 | 12 | 0.0076 | 15.50 | 0.8502 | 91 |
| Max | 0.1607 | 38.15 | 0.9997 | 90 | 0.2742 | 28.39 | 0.9997 | 237 |
| SD | 0.0279 | 3.20 | 0.0391 | 20 | 0.0581 | 2.64 | 0.0420 | 40 |
| *N* | | 2,171 | | | | 863 | | |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| Mean | 0.0302 | 20.55 | 1.0713 | 42 | 0.1076 | 20.24 | 1.0726 | 145 |
| Min | 0.0010 | 12.97 | 1.0003 | 12 | 0.0152 | 15.27 | 1.0003 | 91 |
| Max | 0.1640 | 36.78 | 1.1498 | 90 | 0.2725 | 24.53 | 1.1498 | 237 |
| SD | 0.0285 | 3.18 | 0.0417 | 20 | 0.0566 | 1.97 | 0.0424 | 40 |
| *N* | | 2,139 | | | | 843 | | |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| Mean | 0.0043 | 28.06 | 1.2176 | 50 | 0.0289 | 22.72 | 1.2056 | 139 |
| Min | 0.0010 | 19.05 | 1.1501 | 13 | 0.0062 | 19.12 | 1.1501 | 91 |
| Max | 0.0184 | 48.06 | 1.3942 | 90 | 0.0900 | 27.48 | 1.2945 | 210 |
| SD | 0.0031 | 5.22 | 0.0544 | 20 | 0.0172 | 1.56 | 0.0360 | 35 |
| *N* | | 624 | | | | 366 | | |

**Note(s):** OTM: out-of-the-money. 50ETF: China 50 ETF. MN: moneyness, the ratio of an option's strike price to its underlying price. Mat: maturity (in calendar days). Max: maximum. Min: minimum. SD: standard deviation. *N*: number of options. Price: computed using the implied volatility from the Wind database via Black-Scholes-Merton formulas. $\sigma$: the Black-Scholes-Merton-implied volatility of "Price" re-computed in this paper. The options data is filtered by nine criteria described in Section 3.1

**Source(s):** Authors' own work

**Table 1.**
Description of 50EFT options from January 4, 2021 to December 31, 2021

| | Short (10–90 days) | | | | Long (>90 days) | | | |
|---|---|---|---|---|---|---|---|---|
| | Price | $\sigma$ (%) | MN | Mat | Price | $\sigma$ (%) | MN | Mat |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| Mean | 5.4 | 29.78 | 0.7951 | 58 | 39.4 | 26.89 | 0.7865 | 177 |
| Min | 2.0 | 19.94 | 0.5539 | 18 | 4.9 | 19.38 | 0.6199 | 91 |
| Max | 23.6 | 56.58 | 0.8498 | 90 | 154.9 | 38.93 | 0.8496 | 347 |
| SD | 3.4 | 5.02 | 0.0526 | 16 | 31.1 | 3.52 | 0.0541 | 68 |
| *N* | 743 | | | | 1,208 | | | |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| Mean | 50.2 | 22.34 | 0.9304 | 51 | 175.8 | 23.67 | 0.9234 | 220 |
| Min | 2.0 | 12.36 | 0.8500 | 10 | 8.7 | 17.05 | 0.8500 | 91 |
| Max | 260.3 | 43.09 | 0.9999 | 90 | 519.8 | 31.66 | 0.9998 | 361 |
| SD | 47.6 | 3.23 | 0.0390 | 22 | 95.5 | 2.60 | 0.0417 | 79 |
| *N* | 7,442 | | | | 4,858 | | | |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| Mean | 52.3 | 18.99 | 1.0647 | 51 | 155.7 | 16.12 | 1.0707 | 222 |
| Min | 2.1 | 10.67 | 1.0001 | 10 | 13.6 | 9.98 | 1.0003 | 91 |
| Max | 232.1 | 41.62 | 1.1498 | 90 | 437.9 | 25.09 | 1.1498 | 361 |
| SD | 44.1 | 3.51 | 0.0420 | 22 | 74.1 | 2.26 | 0.0443 | 79 |
| *N* | 4,634 | | | | 2,913 | | | |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| Mean | 7.4 | 25.94 | 1.2050 | 54 | 42.1 | 19.95 | 1.2064 | 182 |
| Min | 2.0 | 16.98 | 1.1500 | 10 | 4.5 | 14.21 | 1.1500 | 91 |
| Max | 46.6 | 47.28 | 1.3470 | 90 | 172.0 | 29.59 | 1.3470 | 342 |
| SD | 6.7 | 4.42 | 0.0432 | 19 | 28.2 | 2.62 | 0.0409 | 63 |
| *N* | 837 | | | | 1,162 | | | |

**Note(s):** OTM: out-of-the-money. MN: moneyness, the ratio of an option's strike price to its underlying price. Mat: maturity (in calendar days). Max: maximum. Min: minimum. SD: standard deviation. *N*: number of options. Price: computed using the implied volatility from the Wind database via Black-Scholes-Merton formulas. $\sigma$ (%): the Black-Scholes-Merton-implied volatility of "Price" re-computed in this paper. The options data is filtered by nine criteria described in Section 3.1

**Source(s):** Authors' own work

**Table 2.**
Description of CSI 300 index options from January 4, 2021 to December 31, 2021

0.001 and 0.2742, implied volatilities range from 12.97% to 48.06%, minimum (maximum) moneyness is 0.7478 (1.3942), and maturities go from 12 to 237. Most puts and calls (3,034 and 2,982, respectively) belong to the OTM group. For the same moneyness groups, there are more options within the short maturity group. Noticeably, deep OTM-short maturity options have higher average implied volatilities; the average implied volatilities show smiles for both long and short maturities, but the curvature of the smile for short maturity is more pronounced. Similar patterns are reported for S&P 500 index option in Barone-Adesi *et al.* (2008).

For CSI 300 index options, there are 14,251 and puts 9,546 calls (Table 2). The option mid-prices, implied volatilities, and maturities range from 2 to 519.8, 9.98% to 56.58%, and 10 to 361, respectively. The minimum (maximum) moneyness is 0.5539 (1.3470). The average implied volatilities also show smiles.

Soybean meal futures options consist of 6,597 filtered puts and 7,477 filtered calls (Table 3). The closing prices, implied volatilities, and maturities vary widely between 5 and 221.5, 7.97% and 70.98%, and 10 and 324, respectively. The minimum (maximum) is 0.6570 (1.3140). Similar to Table 1, there are way more OTM calls and puts. The average implied volatilities again show smiles, and the smiles flatten with longer maturities.

**Table 3.** Description of soybean meal futures options from January 4, 2021 to December 31, 2021

| | Short (10–90 days) | | | | Medium (91–180 days) | | | | Long (>180 days) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Price | $\sigma$ (%) | MN | Mat | Price | $\sigma$ (%) | MN | Mat | Price | $\sigma$ (%) | MN | Mat |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | | | | | |
| Mean | 6.5 | 24.65 | 0.8235 | 78 | 9.4 | 22.41 | 0.8056 | 130 | 18.1 | 18.85 | 0.8124 | 229 |
| Min | 5.0 | 19.71 | 0.7342 | 41 | 5.0 | 17.27 | 0.6570 | 91 | 6.0 | 15.11 | 0.6910 | 181 |
| Max | 13.0 | 34.88 | 0.8499 | 90 | 29.0 | 37.67 | 0.8499 | 180 | 46.5 | 27.47 | 0.8498 | 304 |
| SD | 1.6 | 2.46 | 0.0231 | 10 | 3.9 | 3.38 | 0.0377 | 25 | 7.7 | 2.20 | 0.0317 | 35 |
| *N* | 158 | | | | 697 | | | | 300 | | | |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | | | | | |
| Mean | 31.5 | 18.82 | 0.9399 | 54 | 53.4 | 17.62 | 0.9246 | 132 | 74.3 | 16.69 | 0.9200 | 226 |
| Min | 5.0 | 11.76 | 0.8501 | 10 | 5.5 | 14.10 | 0.8500 | 91 | 10.0 | 7.97 | 0.8501 | 181 |
| Max | 146.5 | 33.39 | 0.9997 | 90 | 181.5 | 26.03 | 0.9997 | 180 | 221.5 | 20.77 | 0.9997 | 324 |
| Std | 26.4 | 2.43 | 0.0385 | 22 | 36.3 | 1.40 | 0.0422 | 26 | 40.7 | 0.98 | 0.0413 | 34 |
| *N* | 2,209 | | | | 2,164 | | | | 1,069 | | | |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | | | | | |
| Mean | 37.8 | 21.64 | 1.0687 | 51 | 73.1 | 19.67 | 1.0757 | 131 | 100.4 | 18.53 | 1.0789 | 222 |
| Min | 5.0 | 13.93 | 1.0003 | 10 | 13.0 | 14.87 | 1.0003 | 91 | 31.5 | 14.84 | 1.0003 | 181 |
| Max | 160.5 | 70.98 | 1.1499 | 90 | 181.5 | 37.01 | 1.1500 | 180 | 213.0 | 24.09 | 1.1500 | 324 |
| SD | 27.9 | 3.86 | 0.0413 | 22 | 34.8 | 2.22 | 0.0429 | 26 | 35.8 | 1.41 | 0.0416 | 34 |
| *N* | 2,643 | | | | 2,245 | | | | 1,031 | | | |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | | | | | |
| Mean | 11.5 | 27.78 | 1.1852 | 59 | 24.5 | 23.26 | 1.1967 | 133 | 45.7 | 20.74 | 1.1810 | 222 |
| Min | 5.0 | 19.75 | 1.1501 | 22 | 5.0 | 17.96 | 1.1502 | 91 | 15.0 | 17.56 | 1.1503 | 181 |
| Max | 36.0 | 41.21 | 1.2861 | 90 | 66.5 | 30.49 | 1.3140 | 180 | 78.0 | 25.21 | 1.2620 | 297 |
| SD | 6.5 | 3.76 | 0.0277 | 17 | 10.7 | 2.20 | 0.0343 | 24 | 13.8 | 1.42 | 0.0251 | 29 |
| *N* | 436 | | | | 884 | | | | 238 | | | |

**Note(s):** OTM: out-of-the-money. MN: moneyness, the ratio of an option's strike price to its underlying price. Mat: maturity (in calendar days). Max: maximum. Min: minimum. SD: standard deviation. *N*: number of options. Price: computed using the implied volatility from the Wind database via Black-Scholes-Merton formulas. $\sigma$ (%): the Black-Scholes-Merton-implied volatility of "Price" re-computed in this paper. The options data is filtered by nine criteria described in Section 3.1

**Source(s):** Authors' own work

Table 4 shows 7,682 puts and 3,760 calls for copper futures options. The closing prices, implied volatilities, and maturities change from 10 to 4,878, 4.66% to 94.10%, and 10 to 323, respectively. The minimum (maximum) moneyness is 0.5296 (1.2616). Again, there are more OTM puts and calls. The average implied volatilities once again show smiles.

*3.2.2 SVI parameters.* For each trading day and each maturity, we try to search for a set of surface SVI parameters; If no parameter set is found, we give up searching for an SVI model curve. The optimal surface SVI parameter set is then converted to the SVI-JW parameter set via Lemma 4.1 of Gatheral and Jacquier (2014):

$$v_\tau = \frac{\theta_\tau}{\tau}$$

$$\psi_\tau = 0.5\sqrt{\theta_\tau}\phi(\theta_\tau)\rho$$

$$p_\tau = 0.5\sqrt{\theta_\tau}\phi(\theta_\tau)(1-\rho)$$

$$c_\tau = 0.5\sqrt{\theta_\tau}\phi(\theta_\tau)(1+\rho)$$

| | Short (10–90 days) | | | | Long (>90 days) | | | |
|---|---|---|---|---|---|---|---|---|
| | Price | $\sigma$ (%) | MN | Mat | Price | $\sigma$ (%) | MN | Mat |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| Mean | 43.6 | 36.56 | 0.7575 | 49 | 150.7 | 25.92 | 0.7489 | 147 |
| Min | 10.0 | 19.75 | 0.5296 | 10 | 14.0 | 16.31 | 0.5701 | 91 |
| Max | 326.0 | 94.10 | 0.8500 | 90 | 842.0 | 46.34 | 0.8500 | 323 |
| SD | 37.0 | 10.97 | 0.0765 | 20 | 123.1 | 5.53 | 0.0647 | 44 |
| $N$ | 1,865 | | | | 1,565 | | | |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| Mean | 516.0 | 22.41 | 0.9264 | 42 | 1,054.0 | 19.77 | 0.9102 | 145 |
| Min | 10.0 | 12.35 | 0.8500 | 10 | 112.0 | 4.66 | 0.8500 | 91 |
| Max | 3310.0 | 41.56 | 0.9999 | 90 | 4,370.0 | 25.86 | 0.9998 | 323 |
| SD | 543.9 | 3.90 | 0.0422 | 22 | 753.0 | 2.31 | 0.0387 | 56 |
| $N$ | 3,477 | | | | 775 | | | |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| Mean | 699.1 | 22.03 | 1.0623 | 42 | 1,572.4 | 19.98 | 1.0722 | 154 |
| Min | 10.0 | 13.80 | 1.0001 | 10 | 248.0 | 13.97 | 1.0001 | 91 |
| Max | 3,310.0 | 40.39 | 1.1499 | 90 | 4,878.0 | 39.59 | 1.1494 | 323 |
| SD | 589.3 | 3.18 | 0.0398 | 22 | 750.5 | 2.92 | 0.0354 | 60 |
| $N$ | 2,763 | | | | 689 | | | |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| Mean | 78.6 | 26.70 | 1.1815 | 41 | 459.9 | 21.27 | 1.1746 | 137 |
| Min | 10.0 | 18.84 | 1.1502 | 11 | 72.0 | 16.64 | 1.1508 | 91 |
| Max | 654.0 | 42.77 | 1.2601 | 89 | 892.0 | 25.47 | 1.2616 | 293 |
| SD | 79.1 | 4.42 | 0.0240 | 18 | 212.0 | 2.10 | 0.0242 | 62 |
| $N$ | 261 | | | | 47 | | | |

**Note(s):** OTM: out-of-the-money. MN: moneyness, the ratio of an option's strike price to its underlying price. Mat: maturity (in calendar days). Max: maximum. Min: minimum. SD: standard deviation. $N$: number of options. Price: computed using the implied volatility from the Wind database via Black-Scholes-Merton formulas. $\sigma$ (%): the Black-Scholes-Merton-implied volatility of "Price" re-computed in this paper. The options data is filtered by nine criteria described in Section 3.1

**Source(s):** Authors' own work

**Table 4.**
Description of copper futures options from November 2, 2020 to October 29, 2021

$$\tilde{v}_\tau = \frac{\theta_\tau}{\tau}\left(1-\rho^2\right)$$

These five parameters can be interpreted as follows. $v_\tau$ is the observed ATM variance, $\psi_\tau$ is the ATM skew, $p_\tau$ is the slope of the put (left) wing of the curve, $c_\tau$ is the slope of the call (right) wing of the curve, and $\tilde{v}_\tau$ is the minimum variance of the curve.

We average the SVI-JW parameters across trading days within maturity groups and summarize the resultant means and standard deviations in Table 5. Panel A of Table 5 shows the five parameters for 50ETF options. Only 84.8% of the trading-expiry days' combinations in the long-maturity group yield optimal parameters. The mean minimum variances are very close to the mean ATM variances, implying that the parameter $\rho$ is small. The average skewnesses are negative, so the minimum variance corresponds to $k > 0$ or is located on the right of the ATM variance. Further, the mean skewness for the long-maturity group is bigger absolutely, implying a steeper curve for the put wing.

| Maturity | | $v_\tau$ | $\psi_\tau$ | $p_\tau$ | $c_\tau$ | $\widetilde{v}_\tau$ | N | V (%) |
|---|---|---|---|---|---|---|---|---|
| *A: China 50 ETF options* | | | | | | | | |
| 10–90 days | Mean | 0.0367 | −0.0015 | 0.4367 | 0.4337 | 0.0360 | 437 | 98.6 |
| | SD | 0.0094 | 0.0506 | 0.1347 | 0.1426 | 0.0094 | | |
| >90 days | Mean | 0.0418 | −0.0571 | 0.5322 | 0.4179 | 0.0400 | 139 | 84.8 |
| | SD | 0.0086 | 0.0757 | 0.2426 | 0.1520 | 0.0086 | | |
| All | Mean | 0.0379 | −0.0149 | 0.4597 | 0.4299 | 0.0370 | 576 | 94.9 |
| | SD | 0.0095 | 0.0624 | 0.1719 | 0.1450 | 0.0093 | | |
| *B: CSI 300 index options* | | | | | | | | |
| 10–90 days | Mean | 0.0351 | −0.0541 | 0.5419 | 0.4336 | 0.0338 | 587 | 92.4 |
| | SD | 0.0127 | 0.0703 | 0.1962 | 0.1522 | 0.0126 | | |
| >90 days | Mean | 0.0337 | −0.1429 | 0.7181 | 0.4322 | 0.0306 | 270 | 37.1 |
| | SD | 0.0099 | 0.0906 | 0.2331 | 0.1610 | 0.0101 | | |
| All | Mean | 0.0347 | −0.0821 | 0.5974 | 0.4332 | 0.0327 | 857 | 62.9 |
| | SD | 0.0119 | 0.0876 | 0.2239 | 0.1549 | 0.0120 | | |
| *C: Soybean meal futures options* | | | | | | | | |
| 10–90 days | Mean | 0.0339 | 0.0885 | 0.4019 | 0.5788 | 0.0322 | 308 | 97.2 |
| | SD | 0.0080 | 0.0641 | 0.1154 | 0.1736 | 0.0072 | | |
| 91–180 days | Mean | 0.0328 | 0.0995 | 0.4090 | 0.6079 | 0.0310 | 277 | 97.9 |
| | SD | 0.0071 | 0.0625 | 0.1103 | 0.1595 | 0.0068 | | |
| >180 days | Mean | 0.0297 | 0.1091 | 0.3581 | 0.5763 | 0.0277 | 162 | 97.0 |
| | SD | 0.0035 | 0.0604 | 0.1070 | 0.1720 | 0.0035 | | |
| All | Mean | 0.0326 | 0.0970 | 0.3950 | 0.5891 | 0.0308 | 747 | 97.4 |
| | SD | 0.0071 | 0.0632 | 0.1133 | 0.1686 | 0.0066 | | |
| *D: Copper futures options* | | | | | | | | |
| 10–90 days | Mean | 0.0400 | 0.0016 | 0.5981 | 0.6013 | 0.0393 | 332 | 89.2 |
| | SD | 0.0094 | 0.0803 | 0.1914 | 0.2304 | 0.0091 | | |
| >90 days | Mean | 0.0403 | 0.0647 | 0.4079 | 0.5372 | 0.0385 | 209 | 93.3 |
| | SD | 0.0131 | 0.0979 | 0.1527 | 0.2712 | 0.0120 | | |
| All | Mean | 0.0410 | 0.0260 | 0.5246 | 0.5766 | 0.0390 | 541 | 90.8 |
| | SD | 0.0110 | 0.0927 | 0.2000 | 0.2487 | 0.0103 | | |

**Note(s):** SVI-JW: SVI jump-wings parameters converted from the SSVI parameters (see Section 4.x for details). $v_\tau$: ATM variance, the squared implied ATM volatility. $\psi_\tau$: ATM skew. $p_\tau$: slope of the left/put wing. $c_\tau$: slope of the right/call wing. $\widetilde{v}_\tau$: minimum implied variance. Maturity: in calendar days. Sample periods for options on China 50 ETF, CSI 300 index and soybean meal futures are between January 4, 2021 and December 31, 2021. Copper futures options data are between November 2, 2020 and October 29, 2021. SD: standard deviation. $N$: number of optimal SVI curves. V: $N$ as a percentage of all possible trading-expiry days combinations within the samples

**Source(s):** Authors' own work

**Table 5.** Means and standard deviations of estimated SVI-JW parameters within the sample periods

Similar findings can be seen for CSI 300 index options (Panel B of Table 5), but the put slopes are higher than those in Panel A, implying stepper left wings. It is puzzling that only roughly one-third (37.1%) of the trading-expiry days' combinations in the long-maturity group yield optimal parameters, which may be worth further investigation in another research.

Next, soybean meal futures options seem to differ from 50ETF and CSI 300 index options (Panel C of Table 5). The average skewnesses are positive, so the minimum variance corresponds to $k < 0$ or is located on the left of the ATM variance. Further, the call slopes are bigger, implying steeper curves for the right wing. Finally, the long-maturity group for copper futures options is similar to soybean meal futures options, while the two wings for the short-maturity group are somewhat symmetrical.

**Table 6.**
In-sample pricing errors of SVI and 5th-order spline models for 50ETF options from January 4, 2021 to December 31, 2021

| Model | Short (10–90 days)<br>MPE (%) | MAE (%) | RMSE | *N* | Long (>90 days)<br>MPE (%) | MAE (%) | RMSE | *N* |
|---|---|---|---|---|---|---|---|---|
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| SVI | −7.75 | 16.98 | 0.0008 | 182 | 7.13 | 8.14 | 0.0024 | 140 |
| Spline | −0.68 | 3.29 | 0.0003 | 182 | −1.79 | 3.59 | 0.0009 | 145 |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| SVI | 4.41 | 9.76 | 0.0038 | 2,139 | −3.82 | 8.31 | 0.0139 | 746 |
| Spline | −0.31 | 4.10 | 0.0028 | 2,171 | −1.28 | 5.07 | 0.0094 | 863 |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| SVI | 9.88 | 12.18 | 0.0038 | 2,113 | 1.61 | 5.39 | 0.0081 | 712 |
| Spline | 1.45 | 3.93 | 0.0026 | 2,139 | 2.37 | 5.16 | 0.0103 | 843 |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| SVI | −3.31 | 20.46 | 0.0010 | 623 | −2.23 | 5.81 | 0.0018 | 281 |
| Spline | −1.48 | 7.50 | 0.0004 | 624 | −1.04 | 6.39 | 0.0028 | 366 |

**Note(s):** OTM: out-of-the-money. SVI: the SSVI model of Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Short/Long: maturity in calendar days. MPE: $\frac{1}{n}\sum(P^{mod}-P^{mkt})/P^{mkt}$. MAE: $\frac{1}{n}\sum\left|P^{mod}-P^{mkt}\right|/P^{mkt}$. RMSE: $\sqrt{\frac{1}{n}\sum(P^{mod}-P^{mkt})^2}$. $P^{mkt}$: market option price. $P^{mod}$: Black-Scholes-Merton option price with the volatility from the SVI or spline model. $N$: number of options priced by the SVI or spline model

**Source(s):** Authors' own work

**Table 7.**
In-sample pricing errors of SVI and 5th-order spline models for CSI 300 index options from January 4, 2021 to December 31, 2021

| Model | Short (10–90 days)<br>MPE (%) | MAE (%) | RMSE | *N* | Long (>90 days)<br>MPE (%) | MAE (%) | RMSE | *N* |
|---|---|---|---|---|---|---|---|---|
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| SVI | 20.77 | 32.87 | 2.65 | 694 | 25.63 | 28.12 | 12.82 | 879 |
| Spline | 5.82 | 11.09 | 1.28 | 743 | 16.24 | 16.71 | 12.13 | 1,203 |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| SVI | 3.86 | 15.18 | 12.21 | 6,885 | −2.03 | 14.48 | 28.86 | 1,860 |
| Spline | 4.80 | 9.68 | 7.43 | 7,442 | 4.65 | 12.52 | 27.09 | 4,795 |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| SVI | 2.79 | 13.97 | 9.15 | 4,310 | −3.43 | 14.65 | 27.36 | 1,084 |
| Spline | −1.22 | 10.67 | 9.50 | 4,634 | 2.82 | 17.21 | 36.22 | 2,884 |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| SVI | −15.04 | 22.75 | 1.59 | 791 | −15.92 | 18.90 | 9.23 | 460 |
| Spline | −6.32 | 14.27 | 1.83 | 837 | −17.04 | 22.99 | 14.44 | 1,162 |

**Note(s):** OTM: out-of-the-money. SVI: the SSVI model of Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Short/Long: maturity in calendar days. MPE: $\frac{1}{n}\sum(P^{mod}-P^{mkt})/P^{mkt}$. MAE: $\frac{1}{n}\sum\left|P^{mod}-P^{mkt}\right|/P^{mkt}$. RMSE: $\sqrt{\frac{1}{n}\sum(P^{mod}-P^{mkt})^2}$. $P^{mkt}$: market option price. $P^{mod}$: Black-Scholes-Merton option price (modified for known cash dividends) with the volatility from the SVI or spline model. $N$: number of options priced by the SVI or spline model

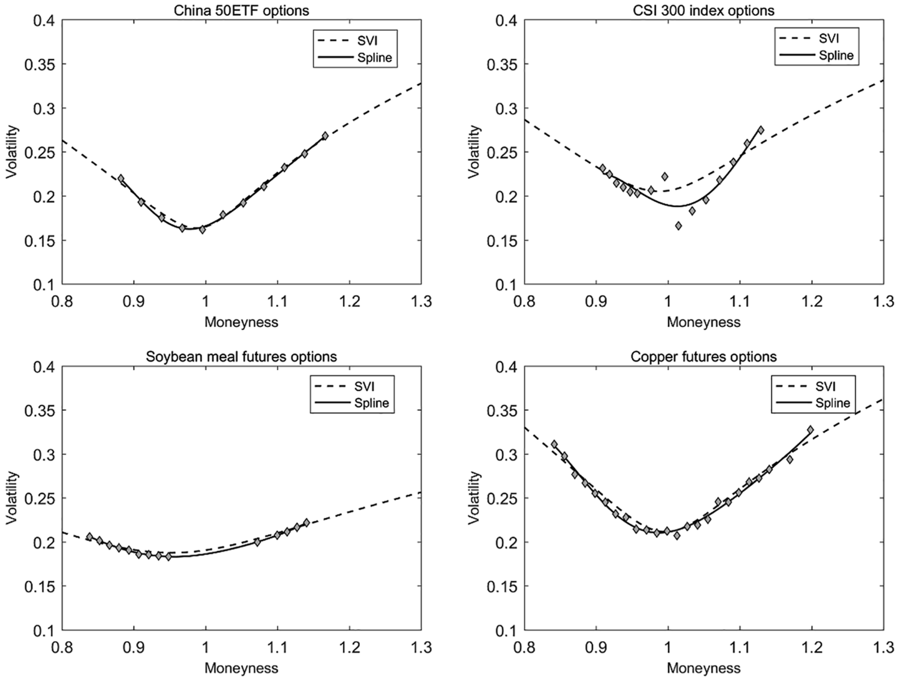
**Source(s):** Authors' own work

## 4. Empirical results

We empirically test the performance of implied volatility modeling via in-sample pricing errors and the volatility arbitrage P&L. For testing volatility forecasting, we utilize out-of-sample pricing errors. The computational details are presented, and the results are discussed in this section.

### *4.1 Implied volatility modeling*

*4.1.1 In-sample pricing errors.* In the empirical study of the SVI model, we only include options on days on which an optimal SVI parameter set is available (see Table 5). Following Bakshi *et al.* (1997) and Barone-Adesi *et al.* (2008), we propose to use three measures to assess pricing errors: the mean pricing error (MPE), the mean absolute pricing error (MAE), and the root mean square error (RMSE). It is worth noting that price-based errors, which are widely used in empirical studies of options, provide straightforward magnitudes for gauging pricing errors. Volatility-based errors, however, are challenging to interpret. More importantly, if we view the BSM model as merely a mapping tool, the BSM implied volatility depends on BSM, while an option price does not. Therefore, price-based errors are arguably better measures than volatility-based errors.



**Note(s):** The trading date and expiry date are June 30, 2021 and July 28, 2021 (China 50 ETF options), June 30, 2021 and August 6, 2021 (CSI 300 index options), July 6, 2021 and November 5, 2021 (soybean meal futures options), and July 6, 2021 and July 26, 2021 (copper futures options), respectively. Data points are marked by diamonds

**Source(s):** Authors' own work

**Figure 1.**
SVI vs 5th-order spline for four kinds of options

**Table 8.**
In-sample pricing errors of SVI and 5th-order spline models for soybean meal futures options from January 4, 2021 to December 31, 2021

| Model | Short (10–90 days)<br>MPE (%) | MAE (%) | RMSE | *N* | Medium (91–180 days)<br>MPE (%) | MAE (%) | RMSE | *N* | Long (>180 days)<br>MPE (%) | MAE (%) | RMSE | *N* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | | | | | |
| SVI | −6.92 | 13.17 | 1.06 | 234 | −1.01 | 11.27 | 1.88 | 826 | −0.07 | 4.88 | 1.80 | 570 |
| Spline | 0.44 | 8.78 | 0.84 | 158 | −0.19 | 5.46 | 0.75 | 697 | 0.37 | 4.52 | 1.05 | 300 |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | | | | | |
| SVI | 3.03 | 5.86 | 2.44 | 2,221 | 4.36 | 5.90 | 3.52 | 2,217 | 1.64 | 3.27 | 3.75 | 1,092 |
| Spline | 0.13 | 3.72 | 1.89 | 2,209 | 0.39 | 2.44 | 1.99 | 2,164 | 0.39 | 2.29 | 2.72 | 1,069 |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | | | | | |
| SVI | 1.08 | 5.16 | 2.26 | 2,520 | 1.96 | 3.71 | 3.57 | 2,191 | 0.59 | 2.56 | 3.47 | 767 |
| Spline | 0.02 | 2.90 | 1.84 | 2,643 | 0.12 | 1.62 | 2.44 | 2,245 | 0.06 | 1.37 | 2.18 | 1,031 |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | | | | | |
| SVI | 0.24 | 8.76 | 1.49 | 389 | −0.94 | 4.92 | 2.08 | 669 | 0.16 | 3.14 | 2.66 | 163 |
| Spline | −0.13 | 3.93 | 0.60 | 436 | −0.22 | 2.30 | 0.92 | 884 | 0.41 | 1.97 | 1.60 | 238 |

**Note(s):** OTM: out-of-the-money. SVI: the SSVI model of Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Short/Long: maturity in calendar days. MPE: $\frac{1}{n}\sum(P^{mod}-P^{mkt})/P^{mkt}$. MAE: $\frac{1}{n}\sum\left|P^{mod}-P^{mkt}\right|/P^{mkt}$. RMSE: $\sqrt{\frac{1}{n}\sum(P^{mod}-P^{mkt})^2}$. $P^{mkt}$: market option price. $P^{mod}$: option price via binomial tree (with the Trigeorgis parametrization and 500 steps) with the volatility from the SVI or spline model. *N*: number of options priced by the SVI or spline model
**Source(s):** Authors' own work

The in-sample pricing errors are shown in Tables 6–9 for the four kinds of options. For 50ETF options, three observations can be made (Table 6). First, except for deep OTM call - Long, spline has much smaller errors in all three measures. For MAE, the largest error is only 7.50% (deep OTM call - Short) for spline but 20.46% (deep OTM call - Short) for SVI. Second, SVI is particularly bad for the short-maturity groups. Third, the spline is more robust for pricing more options.

CSI 300 index options (Table 7) are quite different from 50ETF options. First, the MAE errors are larger overall for CSI 300 index options. The smallest errors are 9.68% (spline) and 13.97% (SVI), respectively. Second, the errors for the two maturity groups are roughly the same. Again, spline outperforms SVI in six out of eight categories. Further, spline prices much more options than SVI. Interestingly, SVI is more robust for put options, while spline is more robust for call options.

Table 8 presents the in-sample pricing errors for soybean meal futures options. The MAE errors are roughly the same as those of 50ETF options, with the largest errors at 13.17% (SVI) and 8.78% (spline), respectively. Amazingly, spline outperforms SVI in all twelve categories.

Finally, SVI shows the largest MAE, 44.49% (deep OTM put - Short), for copper futures options (Table 9). MAE errors are larger for the short-maturity groups. Finally, spline outperforms SVI in all eight categories.

We can see the difference between SVI and spline more clearly in Figure 1. As expected, SVI curves show nice "smiles." On the other hand, the spline follows the data points more closely but can go only as far as the observed data. In all four graphs for the four kinds of options, spline does fit the market implied volatilities better. In accordance with previous discussions, SVI fits the observed implied volatilities rather poorly for CSI 300 index options.

In summary, spline outperforms SVI in in-sample pricing [19]. This implies that spline fits the market-implied volatilities better in most cases. Further, we can speculate that spline should lead to small arbitrage profits, which we will discuss next.

**Table 9.**
In-sample pricing errors of SVI and 5th-order spline models for copper futures options from November 2, 2020 to October 29, 2021

| | Short (10–90 days) | | | | Long (>90 days) | | | |
|---|---|---|---|---|---|---|---|---|
| Model | MPE (%) | MAE (%) | RMSE | *N* | MPE (%) | MAE (%) | RMSE | *N* |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| SVI | 25.5 | 44.49 | 35.39 | 811 | 8.83 | 21.76 | 68.36 | 1,363 |
| Spline | −0.29 | 10.77 | 5.49 | 1,865 | 0.17 | 3.81 | 6.24 | 1,565 |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| SVI | 27.61 | 28.70 | 77.72 | 2,806 | 9.77 | 11.76 | 117.41 | 695 |
| Spline | 1.75 | 5.34 | 25.60 | 3,477 | 0.19 | 1.99 | 40.58 | 774 |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| SVI | 6.65 | 8.23 | 46.31 | 2,451 | 5.41 | 6.44 | 141.38 | 635 |
| Spline | −0.38 | 3.83 | 32.74 | 2,763 | 0.14 | 1.21 | 54.63 | 683 |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| SVI | −0.13 | 17.20 | 17.49 | 220 | 4.84 | 8.02 | 93.72 | 53 |
| Spline | 0.24 | 6.50 | 4.90 | 261 | −0.09 | 1.23 | 6.24 | 45 |

**Note(s):** OTM: out-of-the-money. SVI: the SSVI model of Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Short/Long: maturity in calendar days. MPE: $\frac{1}{n}\sum(P^{mod}-P^{mkt})/P^{mkt}$. MAE: $\frac{1}{n}\sum\left|P^{mod}-P^{mkt}\right|/P^{mkt}$. RMSE: $\sqrt{\frac{1}{n}\sum(P^{mod}-P^{mkt})^2}$. $P^{mkt}$: market option price. $P^{mod}$: Black-Scholes-Merton option price (modified for known cash dividends) with the volatility from the SVI or spline model. $N$: number of options priced by the SVI or spline model
**Source(s):** Authors' own work

**Table 10.**
P&L's of volatility arbitrage strategy

| Maturity | Model | MPL | $\boldsymbol{E}[\boldsymbol{e}_1/\boldsymbol{v}_0]$ (%) | A | G (%) |
|---|---|---|---|---|---|
| *A. China 50 ETF options* | | | | | |
| 10–90 days | SVI | −0.0019 | −12.98 | 434 | 46.88 |
| | Spline | 0.0008 | 1.94 | 439 | 57.86 |
| >90 days | SVI | 0.0006 | 0.33 | 139 | 53.24 |
| | Spline | 0.0007 | 0.43 | 164 | 51.83 |
| All | SVI | −0.0013 | −9.76 | 573 | 48.43 |
| | Spline | 0.0008 | 1.53 | 603 | 56.22 |
| *B. CSI 300 index options* | | | | | |
| 10–90 days | SVI | −0.1463 | −0.31 | 584 | 52.23 |
| | Spline | 0.9192 | 1.09 | 632 | 50.63 |
| >90 days | SVI | −0.6928 | −0.15 | 269 | 49.44 |
| | Spline | 0.5453 | 0.28 | 714 | 49.30 |
| All | SVI | −0.3186 | −0.26 | 853 | 51.35 |
| | Spline | 0.7209 | 0.66 | 1,346 | 49.93 |
| *C. Soybean meal futures options* | | | | | |
| 10–90 days | SVI | 2.0165 | 5.05 | 307 | 63.19 |
| | Spline | 2.5306 | 3.97 | 316 | 62.66 |
| 91–180 days | SVI | 5.1163 | 9.99 | 276 | 65.94 |
| | Spline | 0.7903 | 1.70 | 282 | 59.22 |
| >180 days | SVI | 5.0666 | 5.71 | 161 | 60.25 |
| | Spline | 3.7108 | 3.42 | 166 | 66.27 |
| All | SVI | 3.8265 | 7.03 | 744 | 63.58 |
| | Spline | 2.1447 | 3.02 | 764 | 62.17 |
| *D. Copper futures options* | | | | | |
| 10–90 days | SVI | 19.8712 | 1.99 | 306 | 53.59 |
| | Spline | 25.5436 | 3.05 | 371 | 59.84 |
| >90 days | SVI | 144.3640 | 16.23 | 204 | 65.69 |
| | Spline | 31.3453 | 4.42 | 222 | 51.80 |
| All | SVI | 69.6683 | 7.69 | 510 | 58.43 |
| | Spline | 27.7156 | 3.56 | 593 | 56.83 |

**Note(s):** SVI: the SSVI model in Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Vol-arb strategy: long/short the option with the market implied volatility most deviated from that of the SVI or spline model among the six options closest to at-the-money. Maturity: in calendar days. MPL: means of arbitrage P&L. $E[e_1/v_0]$: average of $e_1/v_0$, where $e_1$ is the day-1 arbitrage P&L and $v_0$ is the day-0 option price. A: number of arbitrage days. G (%): percentage of arbitrage days with gains
**Source(s):** Authors' own work

*4.1.2 Arbitrage P&L.* Because the prices of the four kinds of options vary widely, we follow Liu *et al.* (2022) and focus on Column 4 of Table 10, which are the average results of dividing the day-1 arbitrage P&L, $e_1$, by the day-0 option price, $v_0$ (see Equation 4). For the long-maturity group of 50ETF options (Panel A), neither SVI nor spline makes money because the absolute P&Ls are less than 1%. The spline model is marginally profitable for the short-maturity group, however. CSI 300 index options seem similar to 50ETF options (Panel B).

In contrast, the SVI models with soybean meal futures options for all three maturity groups are substantially profitable and more profitable than the spline model (Panels C). For cooper futures options, the SVI model for the long-maturity groups is the most profitable, with a 16.23% profit. These results are robust if we use eight options that are closest to at-the-money to arbitrage.

**Table 11.** Out-of-sample pricing errors of China 50 ETF options from January 4, 2021 to December 31, 2021

| | Short (10–90 days) | | | | Long (>90 days) | | | |
|---|---|---|---|---|---|---|---|---|
| | MPE (%) | MAE (%) | RMSE | *N* | MPE (%) | MAE (%) | RMSE | *N* |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| SVI | −2.73 | 22.37 | 0.0012 | 181 | 8.66 | 12.24 | 0.0036 | 135 |
| Spline | −12.21 | 19.94 | 0.0010 | 181 | −2.75 | 8.23 | 0.0023 | 140 |
| Linear | −10.94 | 19.24 | 0.0009 | 181 | −1.65 | 8.03 | 0.0023 | 140 |
| iCubic | −7.49 | 16.68 | 0.0008 | 181 | −1.08 | 7.66 | 0.0023 | 140 |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| SVI | 12.50 | 18.77 | 0.0051 | 2091 | −1.86 | 10.18 | 0.0142 | 714 |
| Spline | −1.56 | 11.45 | 0.0039 | 2,115 | −1.33 | 6.97 | 0.0105 | 839 |
| Linear | −0.90 | 10.48 | 0.0032 | 2,115 | −0.15 | 4.72 | 0.0073 | 839 |
| iCubic | −0.67 | 10.32 | 0.0032 | 2,115 | −0.12 | 4.69 | 0.0075 | 839 |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| SVI | 18.65 | 22.45 | 0.0059 | 2063 | 3.33 | 7.94 | 0.0109 | 685 |
| Spline | 0.59 | 12.10 | 0.0043 | 2087 | 2.75 | 7.34 | 0.0120 | 819 |
| Linear | −0.01 | 11.06 | 0.0040 | 2087 | 1.08 | 4.78 | 0.0085 | 819 |
| iCubic | 0.04 | 10.83 | 0.0040 | 2087 | 0.99 | 4.74 | 0.0086 | 819 |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| SVI | 3.05 | 25.20 | 0.0013 | 622 | −0.74 | 8.87 | 0.0031 | 273 |
| Spline | −10.50 | 17.71 | 0.0009 | 623 | −3.11 | 8.24 | 0.0033 | 365 |
| Linear | −7.48 | 15.46 | 0.0008 | 623 | −0.52 | 6.18 | 0.0025 | 365 |
| iCubic | −5.11 | 13.68 | 0.0007 | 623 | 0.27 | 5.88 | 0.0024 | 365 |

**Note(s):** OTM: out-of-the-money. SVI: the SSVI model of Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Linear: linear interpolation on implied variance. iCubic: interpolating cubic spline on implied volatility. Short/Long: maturity in calendar days. MPE: $\frac{1}{n}\sum(P^{mod}-P^{mkt})/P^{mkt}$. MAE: $\frac{1}{n}\sum\left|P^{mod}-P^{mkt}\right|/P^{mkt}$. RMSE: $\sqrt{\frac{1}{n}\sum(P^{mod}-P^{mkt})^2}$. $P^{mkt}$: market option price. $P^{mod}$: Black-Scholes-Merton option price with a volatility forecasted from the previous trading day. *N*: number of out-of-sample options

**Source(s):** Authors' own work

In summary, the SVI model is significantly profitable in four out of nine groups, which is roughly consistent with the results of in-sample pricing errors. In theory, though, if a model leads to significant arbitrage P&L, the model is highly suspicious because the arbitrage P&L may be purely an artifact of the model. To conclude, the 5th-order spline is a better model for options traded in China.

### *4.2 Volatility forecasting*

We use implied volatility (variance) curves to determine (i.e. forecast) the volatility (variance) for an option with the same expiry day that is traded on the immediate next trading day. Again, we work only with filtered options; i.e. we require that both the trading day for modeling and the trading day for forecasting are in our filtered dataset. As the data filtering shows, a high percentage of convex arbitrage exists in the price data. Arguably, we believe a model may be worse and more difficult to justify if it yields smaller errors for prices admitting arbitrage.

As was discussed in Section 2, we employ four approaches for forecasting volatility. For SVI, we can compute volatility via Equation (1) for any given log forward moneyness ($k$) of an option. For the 5th-order spline, we estimate volatility using Equation (3). For linear interpolation on implied variance and interpolating cubic spline on implied volatility, we

**Table 12.**
Out-of-sample pricing errors of CSI 300 index options from January 4, 2021 to December 31, 2021

| | Short (10–90 days) | | | | Long (>90 days) | | | |
|---|---|---|---|---|---|---|---|---|
| | MPE (%) | MAE (%) | RMSE | *N* | MPE (%) | MAE (%) | RMSE | *N* |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| SVI | 31.38 | 45.32 | 3.35 | 701 | 26.78 | 29.87 | 12.81 | 861 |
| Spline | 1.66 | 18.86 | 1.63 | 719 | 14.75 | 17.22 | 11.28 | 1,181 |
| Linear | 0.61 | 16.15 | 1.19 | 719 | 12.49 | 14.35 | 8.19 | 1,181 |
| iCubic | 2.56 | 14.99 | 1.17 | 719 | 12.81 | 14.44 | 8.21 | 1,181 |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| SVI | 10.56 | 21.72 | 12.82 | 6,805 | −0.66 | 15.60 | 29.55 | 1,800 |
| Spline | 4.82 | 14.29 | 8.30 | 7,319 | 4.08 | 12.26 | 27.14 | 4,714 |
| Linear | 5.30 | 12.51 | 7.63 | 7,319 | 8.13 | 9.75 | 22.48 | 4,769 |
| iCubic | 5.43 | 12.47 | 7.72 | 7,319 | 8.23 | 9.77 | 23.07 | 4,769 |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| SVI | 10.12 | 20.88 | 11.37 | 4,250 | −0.69 | 16.48 | 30.72 | 1,067 |
| Spline | −1.96 | 15.43 | 10.73 | 4,562 | 3.64 | 17.32 | 36.84 | 2,827 |
| Linear | −4.64 | 12.53 | 8.54 | 4,562 | −7.31 | 12.12 | 28.83 | 2,857 |
| iCubic | −4.67 | 12.39 | 8.60 | 4,562 | −7.81 | 12.18 | 29.52 | 2,857 |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| SVI | −7.87 | 27.19 | 2.30 | 797 | −13.61 | 20.40 | 11.10 | 465 |
| Spline | −15.87 | 23.36 | 2.57 | 834 | −19.90 | 22.56 | 13.35 | 1,156 |
| Linear | −10.70 | 18.45 | 2.04 | 834 | −11.40 | 13.78 | 8.64 | 1,156 |
| iCubic | −7.61 | 15.86 | 1.85 | 834 | −10.39 | 12.87 | 8.07 | 1,156 |

**Note(s):** OTM: out-of-the-money. SVI: the SSVI model of Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Linear: linear interpolation on implied variance. iCubic: interpolating cubic spline on implied volatility. Short/Long: maturity in calendar days. MPE: $\frac{1}{n}\sum(P^{mod}-P^{mkt})/P^{mkt}$. MAE: $\frac{1}{n}\sum\left|P^{mod}-P^{mkt}\right|/P^{mkt}$. RMSE: $\sqrt{\frac{1}{n}\sum(P^{mod}-P^{mkt})^2}$. $P^{mkt}$: market option price. $P^{mod}$: Black-Scholes-Merton option price (modified for known cash dividends) with a volatility forecasted from the previous trading day. *N*: number of out-of-sample options
**Source(s):** Authors' own work

interpolate volatility for any given moneyness of an option from those spline curves. Using such forecasted volatility, we can price options out-of-sample via the BSM formulas (for European) or binomial tree (for American).

Tables 11–14 summarize the out-of-sample errors using the same categories and three error measures as for in-sample pricing. Let's again start with 50ETF options (Table 11) and focus on MAE. As is expected, the out-of-sample errors are larger than the corresponding in-sample errors for both SVI and 5th-order spline. Even though it outperforms SVI in all eight categories, spline underperforms the two interpolating splines. Overall, interpolating cubic spline turns out the best, with the largest MAE at 16.68% (deep OTM put - Short).

For CSI 300 index options (Table 12), the 5th-order spline outperforms SVI in six out of eight categories, with larger errors than in-sample. Again, both interpolating splines outperform the 5th-order spline; interpolating cubic spline, with the largest MAE of 15.86% (deep OTM call - Short), is slightly better than linear interpolation.

Table 13 shows the out-of-sample pricing errors for soybean meal futures options. In nine out of twelve categories, the 5th-order spline outperforms SVI. The performance of the two interpolating splines is mixed, however. Linear interpolation outperforms interpolating cubic spline in seven out of twelve categories and does SVI in ten out of twelve categories.

**Table 13.** Out-of-sample pricing errors of soybean meal futures options from January 4, 2021 to December 31, 2021

| | Short (10–90 days) | | | | Medium (91–180 days) | | | | Long (>180 days) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | MPE (%) | MAE (%) | RMSE | *N* | MPE (%) | MAE (%) | RMSE | *N* | MPE (%) | MAE (%) | RMSE | *N* |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | | | | | |
| SVI | −6.18 | 17.24 | 1.46 | 215 | 0.52 | 14.27 | 2.36 | 627 | 3.90 | 9.91 | 4.83 | 373 |
| Spline | −9.50 | 15.03 | 1.31 | 136 | −4.33 | 12.89 | 1.50 | 505 | 4.11 | 16.76 | 3.67 | 158 |
| Linear | −8.76 | 13.78 | 1.16 | 136 | −4.32 | 12.87 | 1.54 | 505 | 3.35 | 16.57 | 3.68 | 158 |
| iCubic | −6.29 | 12.55 | 1.10 | 136 | 4.26 | 17.64 | 11.57 | 490 | 3.49 | 16.75 | 3.83 | 158 |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | | | | | |
| SVI | 7.45 | 11.97 | 3.91 | 2,205 | 6.04 | 7.97 | 5.05 | 1,864 | 2.51 | 6.16 | 6.80 | 744 |
| Spline | −0.74 | 10.35 | 3.51 | 2,051 | 0.94 | 5.07 | 3.08 | 1,791 | 0.23 | 6.35 | 6.77 | 683 |
| Linear | −0.37 | 9.77 | 3.33 | 2,051 | 1.48 | 5.29 | 3.72 | 1,791 | 1.54 | 5.64 | 5.99 | 683 |
| iCubic | −0.35 | 9.67 | 3.36 | 2,040 | 1.73 | 5.68 | 4.43 | 1,760 | 2.30 | 6.33 | 8.05 | 683 |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | | | | | |
| SVI | 6.36 | 11.37 | 4.21 | 2,305 | 3.59 | 5.89 | 5.24 | 1,794 | 1.63 | 5.61 | 7.86 | 438 |
| Spline | −0.14 | 9.45 | 4.38 | 2,402 | 0.18 | 4.23 | 4.78 | 1,869 | −0.71 | 3.86 | 5.47 | 688 |
| Linear | 0.38 | 9.77 | 4.22 | 2,402 | 0.46 | 4.23 | 5.06 | 1,869 | −0.32 | 3.78 | 5.25 | 688 |
| iCubic | 0.26 | 9.67 | 4.18 | 2,388 | 0.41 | 4.20 | 4.76 | 1,846 | −0.31 | 3.88 | 5.96 | 688 |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | | | | | |
| SVI | 3.80 | 16.02 | 2.48 | 341 | 0.63 | 8.06 | 3.31 | 546 | 1.13 | 5.37 | 4.16 | 109 |
| Spline | −7.35 | 14.03 | 1.88 | 393 | −1.41 | 6.56 | 2.27 | 719 | −2.47 | 6.06 | 4.64 | 158 |
| Linear | −7.82 | 14.24 | 1.88 | 393 | −1.11 | 6.70 | 2.26 | 719 | −2.31 | 6.08 | 4.86 | 158 |
| iCubic | −4.07 | 13.79 | 1.98 | 388 | −0.34 | 6.97 | 2.53 | 711 | −1.57 | 6.59 | 5.37 | 158 |

**Note(s):** OTM: out-of-the-money. SVI: the SSVI model of Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Linear: linear interpolation on implied variance. iCubic: interpolating cubic spline on implied volatility. Short/Long: maturity in calendar days. MPE: $\frac{1}{n}\sum(P^{mod}-P^{mkt})/P^{mkt}$. MAE: $\frac{1}{n}\sum\left|P^{mod}-P^{mkt}\right|/P^{mkt}$. RMSE: $\sqrt{\frac{1}{n}\sum(P^{mod}-P^{mkt})^2}$. $P^{mkt}$: market option price. $P^{mod}$: option price via binomial tree (with the Trigeorgis parametrization and 500 steps) with a volatility forecasted from the previous trading day. *N*: number of out-of-sample options

**Source(s):** Authors' own work

**Table 14.**
Out-of-sample pricing errors of copper futures options from November 2, 2020 to October 29, 2021

| | Short (10–90 days) | | | | Long (>90 days) | | | |
|---|---|---|---|---|---|---|---|---|
| | MPE (%) | MAE (%) | RMSE | *N* | MPE (%) | MAE (%) | RMSE | *N* |
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | | | |
| SVI | 21.34 | 57.23 | 38.54 | 541 | 9.42 | 30.89 | 64.78 | 549 |
| Spline | −18.77 | 31.68 | 14.68 | 1,260 | −5.06 | 20.02 | 39.19 | 603 |
| Linear | −12.90 | 31.73 | 14.23 | 1,260 | −1.50 | 21.92 | 41.67 | 603 |
| iCubic | −7.95 | 33.31 | 16.88 | 1,260 | −0.68 | 20.43 | 39.66 | 603 |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | | | |
| SVI | 39.79 | 42.33 | 100.88 | 1,922 | 19.58 | 22.97 | 174.31 | 213 |
| Spline | −0.01 | 14.37 | 60.83 | 2,443 | 0.28 | 12.67 | 191.20 | 250 |
| Linear | −1.56 | 13.26 | 60.20 | 2,443 | 7.26 | 12.68 | 147.17 | 250 |
| iCubic | −1.82 | 13.33 | 60.52 | 2,443 | 6.05 | 11.96 | 142.96 | 250 |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | | | |
| SVI | 17.56 | 22.31 | 92.44 | 1,685 | 10.17 | 15.14 | 269.48 | 171 |
| Spline | −0.91 | 13.42 | 83.69 | 1,945 | −2.54 | 9.87 | 219.32 | 202 |
| Linear | −0.05 | 13.03 | 82.15 | 1,945 | −1.01 | 9.53 | 202.20 | 202 |
| iCubic | 0.35 | 13.36 | 83.20 | 1,945 | −1.40 | 9.65 | 209.09 | 202 |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | | | |
| SVI | 10.62 | 31.79 | 34.03 | 135 | 32.44 | 38.22 | 392.26 | 13 |
| Spline | −5.87 | 23.01 | 25.91 | 175 | 0.37 | 8.53 | 58.30 | 11 |
| Linear | −4.82 | 23.39 | 26.04 | 175 | 0.36 | 8.72 | 59.09 | 11 |
| iCubic | 0.32 | 25.07 | 27.49 | 175 | 2.43 | 10.74 | 87.76 | 11 |

**Note(s):** OTM: out-of-the-money. SVI: the SSVI model of Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Linear: linear interpolation on implied variance. iCubic: interpolating cubic spline on implied volatility. Short/Long: maturity in calendar days. MPE: $\frac{1}{n}\sum(P^{mod}-P^{mkt})/P^{mkt}$. MAE: $\frac{1}{n}\sum\left|P^{mod}-P^{mkt}\right|/P^{mkt}$. RMSE: $\sqrt{\frac{1}{n}\sum(P^{mod}-P^{mkt})^2}$. $P^{mkt}$: market option price. $P^{mod}$: Black-Scholes-Merton option price with a volatility forecasted from the previous trading day. *N*: number of out-of-sample options
**Source(s):** Authors' own work

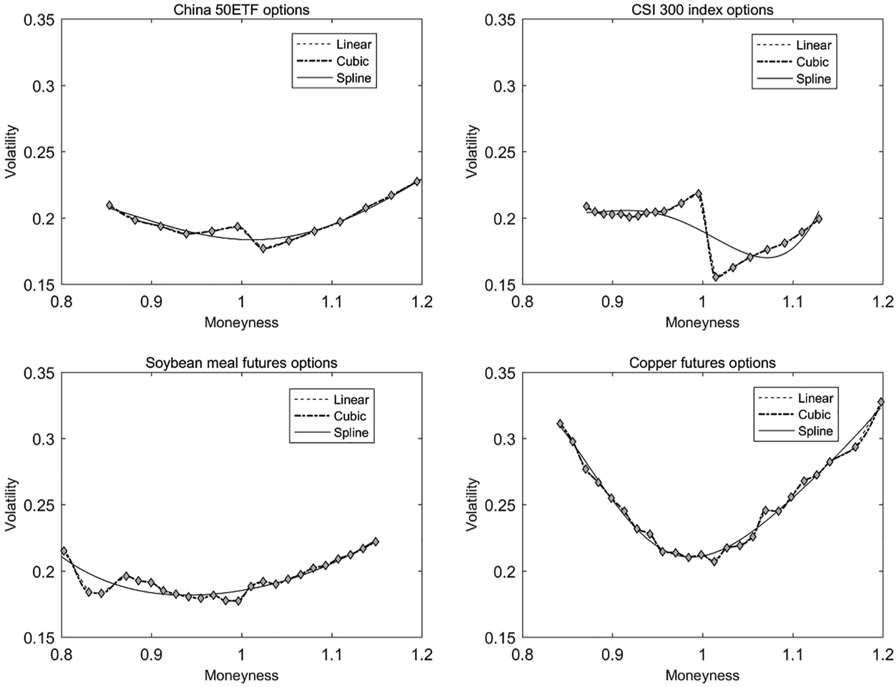
The largest MAE errors are 16.57% (deep OTM put - Long) and 17.64% (deep OTM put - medium) for linear interpolation and interpolating cubic spline, respectively.

Finally, errors for copper futures options are summarized in Table 14. Clearly, the 5th-order spline outperforms SVI significantly in every category. The performance of the 5th-order spline, linear interpolation, and interpolating cubic spline, however, are hardly distinguishable. Finally, the largest MAE errors are over 30% and seem unacceptable.

Graphs can give us certain visual insight (Figure 2). SVI can significantly deviate from the observed implied volatilities, as is evident from the diagrams for China's 50 ETF options, CSI 300 index options, and soybean meal futures options. Further, the three splines differ little for copper futures options.

In summary, we can draw two empirical conclusions. First, in-sample pricing appears to be an excellent indicator of out-of-sample pricing. Second, the three splines outperform SVI for all four kinds of options. This implies that even though SVI is quite popular on Wall Street, it was not employed by traders and market-makers in China.

The explanations for our conclusions are as follows [20]. First, compared to the 5th-order spline, SVI faces greater difficulty in parameter optimization, is less robust, and can be numerically unstable. As we write in Section 2.1, "to mitigate the trap of local minima, we generate 100 sets of random initial parameter values" in the calibration of SVI parameters.



**Note(s):** The trading date and expiry date are June 30, 2021 and September 22, 2021 (China 50 ETF options), June 30, 2021 and October 14, 2021 (CSI 300 index options), July 6, 2021 and December 7, 2021 (soybean meal futures options), and July 6, 2021 and July 26, 2021 (copper futures options), respectively. Data points are marked by diamonds

**Source(s):** Authors' own work

**Figure 2.**
Linear interpolation, interpolating cubic spline, and 5th-order spline for four kinds of options

Further, we report in Section 3.2.2 that "[For CSI 300 index options], only roughly one-third (37.1%) of the trading-expiry days' combinations in the long-maturity group yield optimal parameters." Worse, for all four kinds of options, SVI prices fewer options in-sample (Tables 6–9) and out-of-sample (Tables 11–14) and hedges fewer options (Table 10). In extreme cases, only 1,067 (2,827) OTM puts with long maturity are priced by SVI (spline) (Table 12). Second, spline is more straightforward to understand and simpler to implement. In fact, Wind provides cubic-spline interpolated implied volatilities so that traders can use them directly. Consequently, it is reasonable to argue that if traders and market-makers had employed spline for pricing, trading, and hedging, the markets would have been more closely aligned with spline, resulting in more favorable empirical evidence in historical data in China for spline.

## 5. Conclusions

This paper initiates studies of the empirical performance of implied volatility modeling and forecasting. As two representative approaches, we choose the Wall Street parametric SVI model and propose utilizing the nonparametric 5th-order spline model for comparison. The empirical research includes four representative options from four exchanges in China: China

50 ETF European options, CSI 300 index European options, soybean meal futures American options, and copper futures European options.

We include the most recent whole-year daily data in 2021 and use only out-of-money options. With lower pricing errors and smaller volatility arbitrage P&Ls, the 5th-order spline is found to be a better model for the four options traded in China. In addition to the 5th-order spline, we employ linear interpolation on implied variance and interpolating cubic spline on implied volatility to forecast volatility. Empirically, we find that the three splines outperform SVI for all four options.

Further research can be carried out in at least two aspects in the future. The current research focuses on the SVI and spline curve without considering the time dimension or the volatility surface. Studying the performance of the whole volatility surface in China can be worthwhile. In addition, it could be an exciting and challenging project to extend the spline model to the surface.

## Notes

1. In addition to SVI, parametric models include polynomial (Dumas *et al.*, 1998) and Vanna-Volga models (Castagna and Mercurio, 2007).
2. Nonparametric models include interpolation (Kahale, 2004; Lee, 2004), kernel density smoothing (Aït-Sahalia and Lo, 1998; Rosenberg, 2000; Cont and Fonseca, 2002), spline smoothing (Fengler, 2009), and local polynomial smoothing method (Benko *et al.*, 2007).
3. Of course, the US options market is interesting to study. But unfortunately, we do not have access to US options data.
4. We thank Reviewer No. 2 for making the exposition of our contributions better.
5. Note that this definition of "at-the-money" is slightly different from moneyness of one later.
6. The square of BSM implied volatility will be termed implied variance.
7. We drop the subscript for convenience.
8. The initial values of $\{\rho,\eta,\gamma\}$ are uniformly drawing from $\{[-0.999,0.999],[0.001,100.0],[0.001,0.999]\}$.
9. Figlewski (2010) and Jiao *et al.* (2021) use the term "4th-order spline" unconventionally. They actually mean 5th-order spline, which is a more commonly used term.
10. We define moneyness as the ratio of the strike price to the underlying price.
11. We utilize the Matlab function spap2 with k (spline order) being equal to 5.
12. Arguably, it makes more sense to interpolate linearly on variance, because variance is additive.
13. Wind database utilizes linear and cubic spline interpolations for implied volatility modeling.
14. Ye Du suggested this arbitrage as a practical trading strategy.
15. With trading volumes of 629 (China 50 ETF option), 67 (CSI 300 option), and 38 (soybean meal futures option) million contracts within the sample periods, three chosen options out of four are most actively traded.
16. For the two futures options, our unreported results show that using the futures settlement prices in the empirical analyses does not change the conclusions.
17. The minimum ticks are 0.0001 (50ETF options), 0.2 (CSI 300 index options), 0.5 (soybean meal futures options), and 1 (copper futures options), respectively.
18. Following Jiao *et al.* (2021), arbitrage means that $xv(K_1) + (1-x)v(K_3) > v(K_2) + 10t_m$, where $v(K_j)$ is the market price of an option with strike $K_j$, $t_m$ is the minimum tick of the option, and $K_2 = xK_1 + (1-x)K_3$.

19. Using 50ETF as an example, we show in the Appendix that Spline outperforms SVI in the dynamic (delta) hedge of options.
20. We thank Reviewer No. 1 for the suggestion of explaining our results.
21. Twenty-one, Twenty-six, Twenty-five, and Twenty-eight component stocks are replaced on June 15, 2020, December 14, 2020, June 15, 2021, and December 13, 2021, respectively.

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(The Appendix follows overleaf)

## Appendix

### Contracts information for four options

*China 50 ETF options* European options. The minimum tick is 0.0001. The contract size is 10,000 units of 50 ETFs. The contract trades for the current month, the next month, and the following two march-cycle months. The contract expires on the fourth Wednesday of the contract month. The trading hours are between 9:30 a.m. and 11:30 a.m. and between 13:00 and 15:00 (Beijing time). The opening call auction is from 9:15 to 9:25 a.m., and the closing call auction is from 14:57 to 15:00.

*CSI 300 index options* European options. The minimum tick is 0.2 points, with a multiplier of 100 Chinese yuan. The contract trades for the current month, the next two months, and the following three march-cycle months. The contract expires on the third Friday of the contract month. The trading hours and call auctions are the same as those of China 50 ETF options.

*Soybean meal futures options* American options. The minimum tick is 0.5 Chinese yuan per ton. The contract size is 10 tons. Contract months are January, March, May, July, August, September, November, and December. The contract expires on the 5th trading day of the month before the delivery month of the futures contract. The trading hours are from 9:30 a.m. to 11:30 a.m. and from 13:00 to 15:00 (Beijing time).

*Copper futures options* European options before November 16, 2021. The minimum tick was one yuan per ton. The contract size was 5 tons. The contract was traded for all twelve months. The contract expired on the 5th last trading day of the month prior to the delivery month of the futures contract. The trading hours were the same as those of soybean meal futures options.

### Dividends of the CSI 300 stock index

Chinese firms predominately pay one dividend per year between May and August (Table A1). Consequently, it makes more sense to treat the CSI 300 index as an investment asset with “known dollar dividends” in pricing derivatives on the CSI 300 index (Liu and Qiao, 2017).

In practice, traders “know” the cash dividend for a firm by using the prior-year dividend payment and ex-dividend date. We follow the same logic here. For any trading day, we obtain the prior-year dividend payment and ex-dividend date for a component stock of the current CSI 300 index from WIND. For pricing index options in 2021, we have to include all component stocks of the CSI 300 index in 2020 and 2021. Note that the CSI 300 index is adjusted twice a year [21].

Following Bos and Vandermark (2002) and Guo and Liu (2019), we partition a dividend payment $D$ into two parts, $D_t$ and $D_T$. Similar to Liu and Qiao (2017), we estimate the two parts as follows:

**Table A1.** The monthly numbers of dividend payments by CSI 300 index component firms in 2020 and 2021

| Month | Year 2020 | Year 2021 |
|---|---|---|
| January | 0 | 0 |
| February | 0 | 0 |
| March | 0 | 3 |
| April | 7 | 11 |
| May | 45 | 45 |
| June | 95 | 99 |
| July | 79 | 84 |
| August | 26 | 25 |
| September | 8 | 6 |
| October | 4 | 6 |
| November | 5 | 5 |
| December | 4 | 2 |
| Total | 273 | 286 |

**Source(s):** Authors’ own work

$$D_t = \frac{P_t}{V_t}\sum_{k=1}^{300} E_{k,t}\left[\sum_i D_k^i I_{\{t\le s_k^i<T\}} \times \frac{T-s_k^i}{T-t} \times \exp\left(-r_t\left[s_k^i - t\right]\right)\right] \tag{A1}$$

$$V_t = \sum_{k=1}^{300} S_{k,t} \times E_{k,t}$$

$$D_T = \frac{P_t}{V_t}\sum_{k=1}^{300} E_{k,t}\left[\sum_i D_k^i I_{\{t\le s_k^i<T\}} \times \frac{s_k^i - t}{T-t} \times \exp\left(r_t\left[T - s_k^i\right]\right)\right] \tag{A2}$$

where $t$ is the valuation date, $P_t$ is the day-$t$ closing point of the CSI 300 index, $S_{k,t}$ is the closing price of the $k$th stock in the CSI 300 index, $E_{k,t}$ is the number of shares used for computing the index for stock $k$ (available from Wind), $D_k^i$ is the $i$th prior-year cash dividend for stock $k$ with the ex-dividend date $s_k^i$, $T$ is the expiry date of an option, $r_t$ is the day-$t$ term-$T$ risk-free rate, and $I$ is the indicator function.

With $D_t$ and $D_T$, we use Formulas (2-3) of Guo and Liu (2019), which are the BSM formulas in which stock price $S_t$ (strike price $K$) is replaced by $S_t - D_t$ ($K + D_T$), to price CSI 300 index options. Note that for 2021, $D_t$ ($D_T$) is between 0.0 and 67.2 index points (between 0.0 and 67.3 index points).

## The hedge of 50ETF options

It is complicated to test hedge empirically. We follow the procedure of Liu *et al.* (2022) as follows. First, we explicitly identify an option series by maturity and strike, which is a time series of trading days for a specific option. In order to ensure the totality of hedging, we include options that expire on or before December 22, 2021 (the last expiration date in 2021). The hedge is rebalanced weekly (i.e. seven calendar days). When a chosen hedge rebalancing day does not fall on a trading day, we decide to use the next, next second, or next third trading day instead.

We obtain the implied volatility for an option from either the SVI or spline models and compute the delta needed for hedging for each hedge-rebalancing day. On days when implied-volatility models are unavailable, we utilize the implied volatility for the immediate previous trading day. As a result, the number of options hedged is the same for both models.

To empirically assess the hedge effectiveness of SVI and spline, we compute the single-option hedging error for options according to Equation (15) in Liu *et al.* (2022), and report the mean hedging error (MHE) and mean absolute hedging error (MAHE).

Table A2 presents the hedging errors for China 50 ETF put and call options. For put options, the MHEs (MAHEs) exhibit significant variations across all eight categories, ranging from 0.2246 (0.5627) to 1.5247 (1.5279) for SVI and from 0.2015 (0.5513) to 1.4713 (1.4772) for spline model. The hedging errors for DOTM puts are much greater than those for OTM puts, and the errors for long-term puts are greater than those for short-term puts, which are consistent with the simulated results under BSM. Notably, The errors of the spline model are smaller than that of the SVI model in all four categories.

For calls, the MHEs (MAHEs) range from 0.0274 (0.1409) to 0.5530 (0.7301) for SVI and from 0.0488 (0.1422) to 0.5938 (0.7794) for the spline model. Except for OTM-long calls and DOTM-short calls, the errors of the spline model are smaller than those of the SVI model in the other two call categories.

In summary, the proposed spline outperforms in delta hedging of 50ETF OTM options in six out of eight categories. Economically speaking, this is expected and logically consistent with the superior performance of the spline model in pricing (both in-sample and out-of-sample), as extensively discussed in our paper. Therefore, our findings provide additional evidence that the spline model not only surpasses the SVI model in option pricing but also excels in option hedging for 50ETF options, which is highly encouraging.

**Table A2.**
Hedging errors of China 50 ETF options from January 4, 2021 to December 22, 2021

| Model | Short (10–90 days) MHE | Short (10–90 days) MAHE | Short (10–90 days) $N$ | Long (>90 days) MHE | Long (>90 days) MAHE | Long (>90 days) $N$ |
|---|---|---|---|---|---|---|
| *A. Deep OTM put (Moneyness<0.85)* | | | | | | |
| SVI | 1.0936 | 1.1904 | 182 | 1.5247 | 1.5279 | 145 |
| Spline | 1.0778 | 1.1333 | | 1.4713 | 1.4772 | |
| *B. OTM put (0.85<Moneyness<1)* | | | | | | |
| SVI | 0.2246 | 0.9961 | 1974 | 0.5439 | 0.5627 | 775 |
| Spline | 0.2015 | 0.9698 | | 0.5314 | 0.5513 | |
| *C. OTM call (1<Moneyness<1.15)* | | | | | | |
| SVI | 0.0545 | 0.6291 | 1951 | 0.0274 | 0.1409 | 765 |
| Spline | 0.0488 | 0.5991 | | 0.0690 | 0.1422 | |
| *D. Deep OTM call (Moneyness>1.15)* | | | | | | |
| SVI | 0.5530 | 0.7301 | 605 | 0.1875 | 0.2535 | 361 |
| Spline | 0.5938 | 0.7794 | | 0.1799 | 0.2485 | |

**Note(s):** SVI: the SSVI model in Gatheral and Jacquier (2014). Spline: 5th-order spline interpolation with one knot at moneyness one. Hedging is rebalanced with intervals of seven calendar days. The delta is calculated using the BSM formulas with the model volatility. Short/Long: maturity in calendar days. MHE: mean hedging error defined in Liu *et al.* (2022). MAHE: mean absolute hedging error defined in Liu *et al.* (2022). $N$: the number of options with valid hedging series in each category
**Source(s):** Authors' own work

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