Appendix A The Asymmetric Student-*t* and Related Distribution

A.1 Bivariate Asymmetric Student-t Model

The bivariate asymmetric Student-t is a four-parameter family, where, besides the parameter related to the correlation, the remaining parameters are ν, α_1 and α_2. The ν parameter is related to the thickness of the tails of the marginal distributions, whereas α_i is the asymmetry parameter for the i-th marginal, $i = 1, 2$.

In what follows, we use the notation

$$Q(\rho, u, v) = \frac{u^2 - 2\rho uv + v^2}{1 - \rho^2}. \tag{4}$$

Furthermore, we denote by $T(y; \nu)$ and $t(y; \nu)$ the cumulative distribution and density function of the Student-t random variable with ν degrees of freedom, respectively.

Following Azzalini (2005), the density of the asymmetric Student-t distribution is given by

$$\begin{aligned} f(\eta_{1,t}, \eta_{2,t}) =& \frac{b_1 b_2}{\pi\sqrt{1-\rho_t^2}} \left[1 + \frac{Q(\rho_t, x_{1,t}, x_{2,t})}{\nu}\right]^{-\frac{\nu+2}{2}} \\ & \times T\left((\alpha_1 x_{1,t} + \alpha_2 x_{2,t})\sqrt{\frac{\nu+2}{\nu + Q(\rho_t, x_{1,t}, x_{2,t})}}; \nu + 2\right) \end{aligned} \tag{5}$$

with

$$\begin{aligned} x_{i,t} &= a_i \eta_{i,t} + b_i, \quad i = 1, 2 \\ b_\nu &= \sqrt{\frac{\nu}{\pi}} \frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \\ a_i &= \delta_i b_\nu \\ b_i &= \sqrt{\frac{\nu}{\nu-2} - a_i^2} \\ \alpha_*^2 &= \alpha_1^2 + \alpha_2^2 + 2\rho_t \alpha_1 \alpha_2 \\ \delta_1 &= \frac{\alpha_1 + \rho_t \alpha_2}{\sqrt{1+\alpha_*^2}} \\ \delta_2 &= \frac{\rho_t \alpha_1 + \alpha_2}{\sqrt{1+\alpha_*^2}}. \end{aligned}$$

Note that the above density implies that $\mathbb{E}_{t-1}[\boldsymbol{\eta}_t] = 0$, $\mathbb{E}_{t-1}\left[\eta_{1,t}^2\right] = \mathbb{E}_{t-1}\left[\eta_{2,t}^2\right] = 1$. Note also that ρ_t does not coincide with the correlation at time t as

$$\text{corr}_{t-1}(\eta_1, \eta_2) = \frac{\frac{\nu}{\nu-2}\rho_t - b_\nu^2 \delta_1 \delta_2}{b_1 b_2}.$$

The following result gives the score to be used in eq. (2)

Result 1. *The derivative of the log-likelihood with respect to ρ_t is*

$$\begin{aligned}\frac{\partial \ell_t}{\partial \rho_t} =& \frac{\rho_t}{1-\rho_t^2} + \frac{b_1'}{b_1} + \frac{b_2'}{b_2} - \frac{\nu+2}{2\nu} Q' \left[1 + \frac{Q}{\nu}\right]^{-1} \\ &+ \sqrt{\frac{\nu+2}{\nu+Q}} \left[\alpha_1 x_{1,t}' + \alpha_2 x_{2,t}' - \frac{1}{2}(\alpha_1 x_{1,t} + \alpha_2 x_{2,t}) \frac{Q'}{\nu+Q}\right] \\ &\frac{t\left((\alpha_1 x_{1,t} + \alpha_2 x_{2,t})\sqrt{\frac{\nu+2}{\nu+Q}}; \nu+2\right)}{T\left((\alpha_1 x_{1,t} + \alpha_2 x_{2,t})\sqrt{\frac{\nu+2}{\nu+Q}}; \nu+2\right)}\end{aligned} \tag{6}$$

where

$$\begin{aligned}\delta_1' &= \frac{\alpha_2}{\sqrt{1+\alpha_*^2}} - \frac{\alpha_1\alpha_2\delta_1}{1+\alpha_*^2} \\ \delta_2' &= \frac{\alpha_1}{\sqrt{1+\alpha_*^2}} - \frac{\alpha_1\alpha_2\delta_2}{1+\alpha_*^2} \\ a_i' &= \delta_i' b_\nu \\ b_i' &= -a_i'\frac{a_i}{b_i} \\ x_{i,t}' &= a_i'\eta_{i,t} + b_i' \quad \textit{for } i = 1,2 \\ Q &= Q(\rho_t, x_{1,t}, x_{2,t}) \\ Q' &= 2\frac{x_{1,t}'x_{1,t} + x_{2,t}'x_{2,t} - x_{1,t}x_{2,t} - \rho_t(x_{1,t}'x_{2,t} + x_{1,t}x_{2,t}') + \rho_t Q}{1-\rho_t^2}\end{aligned}$$

A.2 Bivariate Student-t Model

We consider the Student-t density, derived from (5) with the parameters $\alpha_1 = \alpha_2 = 0$, and expressed as

$$f(\eta_{1,t}, \eta_{2,t}) = \frac{\nu}{2\pi(\nu-2)\sqrt{1-\rho_t^2}} \left(1 + \frac{1}{\nu-2} Q(\rho_t, \eta_{1,t}, \eta_{2,t})\right)^{-\frac{\nu+2}{2}}. \tag{7}$$

Result 2 presents the score for this model.

Result 2. *The derivative of the log-likelihood with respect to* ρ_t *is*

$$\frac{\partial \ell_t}{\partial \rho_t} = \frac{\rho_t}{1-\rho_t^2} - \frac{\frac{\nu+2}{(\nu-2)(1-\rho_t^2)^2}\left[\rho_t(\eta_{1,t}^2+\eta_{2,t}^2)-(1+\rho_t^2)\eta_{1,t}\eta_{2,t}\right]}{1+\frac{1}{(\nu-2)\left(1-\rho_t^2\right)}\left[\eta_{1,t}^2-2\rho_t\eta_{1,t}\eta_{2,t}+\eta_{2,t}^2\right]} \tag{8}$$

Furthermore

$$\mathbb{E}_{t-1}\left[\left(\frac{\partial \ell_t}{\partial \rho_t}\right)^2\right] = -\frac{\rho_t^2}{(1-\rho_t^2)^2} + \mathbb{E}_{t-1}[g(\boldsymbol{\eta}_t)^2] = -\frac{\rho_t^2}{(1-\rho_t^2)^2} + \frac{\nu+2}{\nu+4}\frac{1+2\rho_t^2}{(1-\rho_t^2)^2} \tag{9}$$

where

$$g(\boldsymbol{\eta}_t) = \frac{\nu+2}{(\nu-2)(1-\rho_t^2)^2}\frac{\left[\rho_t(\eta_{1,t}^2+\eta_{2,t}^2)-(1+\rho_t^2)\eta_{1,t}\eta_{2,t}\right]}{1+\frac{1}{(\nu-2)\left(1-\rho_t^2\right)}\left[\eta_{1,t}^2-2\rho_t\eta_{1,t}\eta_{2,t}+\eta_{2,t}^2\right]}$$

$$\mathbb{E}_{t-1}[g(\boldsymbol{\eta}_t)] = \frac{\rho_t}{1-\rho_t^2}$$

$$\mathbb{E}_{t-1}[g(\boldsymbol{\eta}_t)^2] = \frac{\nu+2}{\nu+4}\frac{1+2\rho_t^2}{(1-\rho_t^2)^2}$$

Proof.

$$\begin{aligned}\mathbb{E}_{t-1}[g(\boldsymbol{\eta}_t)^2] &= C(\nu,\rho_t)\int\int_{\mathbb{R}^2}\left[\rho_t(\eta_{1,t}^2+\eta_{2,t}^2)-(1+\rho_t^2)\eta_{1,t}\eta_{2,t}\right]^2 \\ &\quad \times\left(1+\frac{1}{(\nu-2)\left(1-\rho_t^2\right)}\left[\eta_{1,t}^2-2\rho_t\eta_{1,t}\eta_{2,t}+\eta_{2,t}^2\right]\right)^{-\frac{\nu+6}{2}}\mathrm{d}\eta_{1,t}\mathrm{d}\eta_{2,t}\end{aligned}$$

$$C(\nu,\rho_t) = \frac{\nu}{2\pi(\nu-2)\sqrt{1-\rho_t^2}}\left(\frac{\nu+2}{(\nu-2)(1-\rho_t^2)^2}\right)^2$$

The results needed for the calculation of $\mathbb{E}_{t-1}[g(\boldsymbol{\eta}_t)^2]$ are

1.

$$\left[1+\frac{x_1^2-2\rho x_1x_2+x_2^2}{(1-\rho^2)\nu}\right]^{-\frac{\nu+6}{2}} = \left[1+\frac{x_2^2}{\nu}\right]^{-\frac{\nu+6}{2}}\left[1+\frac{(x_1-\rho x_2)^2}{(1-\rho^2)(\nu+x_2^2)}\right]^{-\frac{\nu+6}{2}}$$

2.

$$\begin{aligned} I_0(x_2,\nu,\rho) &= \int_{\mathbb{R}} x_1^4 \left[1+\frac{(x_1-\rho x_2)^2}{(1-\rho^2)(\nu+x_2^2)}\right]^{-\frac{\nu+6}{2}} dx_1 \\ &= (1-\rho^2)^{5/2}(\nu+x_2^2)^{5/2} B\left(\frac{\nu+1}{2},\frac{5}{2}\right) \\ &\quad +6\rho^2 x_2^2(1-\rho^2)^{3/2}(\nu+x_2^2)^{3/2} B\left(\frac{\nu+3}{2},\frac{3}{2}\right) \\ &\quad +\rho^4 x_2^4(1-\rho^2)^{1/2}(\nu+x_2^2)^{1/2} B\left(\frac{\nu+5}{2},\frac{1}{2}\right) \end{aligned}$$

3.

$$\int_{\mathbb{R}} \left[1+\frac{x_2^2}{\nu}\right]^{-\frac{\nu+6}{2}} I_0(x_2,\nu,\rho)\mathrm{d}x_2 = \frac{6\pi\nu^2}{(\nu+4)(\nu+2)}\sqrt{1-\rho^2}$$

4.

$$\begin{aligned} & C(\nu,\rho)\int\int_{\mathbb{R}^2} \rho^2\eta_1^4\left(1+\frac{1}{(\nu-2)(1-\rho^2)}\left[\eta_1^2-2\rho\eta_1\eta_2+\eta_2^2\right]\right)^{-\frac{\nu+6}{2}}\mathrm{d}\eta_1\mathrm{d}\eta_2 \\ &= C(\nu,\rho)\rho^2\left(\frac{\nu-2}{\nu}\right)^3\int\int_{\mathbb{R}^2} x_1^4\left(1+\frac{1}{\nu(1-\rho^2)}\left[x_1^2-2\rho x_1x_2+x_2^2\right]\right)^{-\frac{\nu+6}{2}}\mathrm{d}x_1\mathrm{d}x_2 \\ &= C(\nu,\rho)\rho^2\left(\frac{\nu-2}{\nu}\right)^3\frac{6\pi\nu^2}{(\nu+4)(\nu+2)}\sqrt{1-\rho^2} \\ &= 3\frac{\nu+2}{\nu+4}\frac{\rho^2}{(1-\rho^2)^4} \end{aligned}$$

5.

$$\begin{aligned} I_1(x_2,\nu,\rho) &= \int_{\mathbb{R}} x_1^2 \left[1+\frac{(x_1-\rho x_2)^2}{(1-\rho^2)(\nu+x_2^2)}\right]^{-\frac{\nu+6}{2}} \mathrm{d}x_1 \\ &= (1-\rho^2)^{3/2}(\nu+x_2^2)^{3/2} B\left(\frac{\nu+3}{2},\frac{3}{2}\right) \\ &\quad +\rho^2 x_2^2(1-\rho^2)^{1/2}(\nu+x_2^2)^{1/2} B\left(\frac{\nu+5}{2},\frac{1}{2}\right) \end{aligned}$$

6.

$$\int_{\mathbb{R}} x_2^2\left[1+\frac{x_2^2}{\nu}\right]^{-\frac{\nu+6}{2}} I_1(x_2,\nu,\rho)\mathrm{d}x_2 = \frac{2\pi\nu^2}{(\nu+4)(\nu+2)}\sqrt{1-\rho^2}(1+2\rho^2)$$

7.

$$
\begin{aligned}
& C(\nu,\rho)\int\int_{\mathbb{R}^2}[(1+\rho^2)^2+2\rho^2]\eta_1^2\eta_2^2 \\
& \qquad \left(1+\frac{1}{(\nu-2)\,(1-\rho^2)}\left[\eta_1^2-2\rho\eta_1\eta_2+\eta_2^2\right]\right)^{-\frac{\nu+6}{2}}\mathrm{d}\eta_1\mathrm{d}\eta_2 \\
= \; & C(\nu,\rho)[(1+\rho^2)^2+2\rho^2]\left(\frac{\nu-2}{\nu}\right)^3 \\
& \int\int_{\mathbb{R}^2}x_2^2x_1^2\left(1+\frac{1}{\nu\,(1-\rho^2)}\left[x_1^2-2\rho x_1x_2+x_2^2\right]\right)^{-\frac{\nu+6}{2}}\mathrm{d}x_1\mathrm{d}x_2 \\
= \; & C(\nu,\rho)[(1+\rho^2)^2+2\rho^2]\frac{2\pi\nu^2}{(\nu+4)(\nu+2)}\sqrt{1-\rho^2}(1+2\rho^2) \\
= \; & \frac{\nu+2}{\nu+4}\frac{(1+\rho^2)^2+2\rho^2}{(1-\rho^2)^4}(1+2\rho^2)
\end{aligned}
$$

8.

$$
\begin{aligned}
I_2(x_2,\nu,\rho) &= \int_{\mathbb{R}}x_1^3\left[1+\frac{(x_1-\rho x_2)^2}{(1-\rho^2)(\nu+x_2^2)}\right]^{-\frac{\nu+6}{2}}\mathrm{d}x_1 \\
&= 3\rho x_2(1-\rho^2)^{3/2}(\nu+x_2^2)^{3/2}B\left(\frac{\nu+3}{2},\frac{3}{2}\right) \\
&\quad +\rho^3x_2^3(1-\rho^2)^{1/2}(\nu+x_2^2)^{1/2}B\left(\frac{\nu+5}{2},\frac{1}{2}\right)
\end{aligned}
$$

9.

$$
\int_{\mathbb{R}}x_2\left[1+\frac{x_2^2}{\nu}\right]^{-\frac{\nu+6}{2}}I_1(x_2,\nu,\rho)\mathrm{d}x_2=\frac{6\pi\nu^2}{(\nu+4)(\nu+2)}\sqrt{1-\rho^2}\rho
$$

10.

$$
\begin{aligned}
& -2C(\nu,\rho)\int\int_{\mathbb{R}^2}\rho(1+\rho^2)\eta_1^3\eta_2 \\
& \qquad \left(1+\frac{1}{(\nu-2)(1-\rho^2)}\left[\eta_1^2-2\rho\eta_1\eta_2+\eta_2^2\right]\right)^{-\frac{\nu+6}{2}}\mathrm{d}\eta_1\mathrm{d}\eta_2 \\
= & \quad -2C(\nu,\rho)\rho(1+\rho^2)\left(\frac{\nu-2}{\nu}\right)^3 \\
& \quad \int\int_{\mathbb{R}^2} x_2x_1^3\left(1+\frac{1}{\nu(1-\rho^2)}\left[x_1^2-2\rho x_1x_2+x_2^2\right]\right)^{-\frac{\nu+6}{2}}\mathrm{d}x_1\mathrm{d}x_2 \\
= & \quad -2C(\nu,\rho)\rho(1+\rho^2)\frac{6\pi\nu^2}{(\nu+4)(\nu+2)}\sqrt{1-\rho^2}\rho \\
= & \quad -6\frac{\nu+2}{\nu+4}\frac{\rho^2(1+\rho^2)}{(1-\rho^2)^4}
\end{aligned}
$$

Points 4, 7, and 10 above are based on the change of variable $x_1 = \sqrt{\frac{\nu}{\nu-2}}\eta_1$, $x_2 = \sqrt{\frac{\nu}{\nu-2}}\eta_2$, with the Jacobian $|J| = \frac{\nu-2}{\nu}$.

Putting all together

$$
\begin{aligned}
\mathbb{E}[g(\boldsymbol{\eta})^2] &= \frac{\nu+2}{\nu+4}\frac{6\rho^2+(1+2\rho^2)(1+\rho^2)^2+2\rho^2(1+\rho^2)^2-12\rho^2(1+\rho^2)}{(1-\rho^2)^4} \\
&= \frac{\nu+2}{\nu+4}\frac{(1+2\rho^2)(1-\rho^2)^2}{(1-\rho^2)^4} \\
&= \frac{\nu+2}{\nu+4}\frac{1+2\rho^2}{(1-\rho^2)^2}
\end{aligned}
$$

□

As expected, eq. (8) can also be obtained by setting $\alpha_1 = \alpha_2 = 0$ in (6).

A.3 Bivariate Asymmetric Normal

The density we use for the bivariate asymmetric normal distribution (Azzalini, 2005) is

$$
f(\eta_{1,t},\eta_{2,t}) = \frac{b_1b_2}{\pi\sqrt{1-\rho_t^2}}\exp\left(-\frac{1}{2}Q(\rho_t,x_{1,t},x_{2,t})\right)\Phi\left(\alpha_1x_{1,t}+\alpha_2x_{2,t}\right). \tag{10}
$$

with

$$
\begin{aligned}
x_{i,t} &= a_1 \eta_{i,t} + b_i \\
a_i &= \delta_i \sqrt{\frac{2}{\pi}} \\
b_i &= \sqrt{1 - a_i^2} \\
\alpha_*^2 &= \alpha_1^2 + \alpha_2^2 + 2\rho_t \alpha_1 \alpha_2 \\
\delta_1 &= \frac{\alpha_1 + \rho_t \alpha_2}{\sqrt{1 + \alpha_*^2}} \\
\delta_2 &= \frac{\rho_t \alpha_1 + \alpha_2}{\sqrt{1 + \alpha_*^2}},
\end{aligned}
$$

and with $\Phi(\cdot)$ being the standard normal cumulative distribution function. The above density can be obtained by letting $\nu \to \infty$ in (5). Note that it implies that $\mathbb{E}_{t-1}[\boldsymbol{\eta}_t] = 0$, $\mathbb{E}_{t-1}\left[\eta_{1,t}^2\right] = \mathbb{E}_{t-1}\left[\eta_{2,t}^2\right] = 1$. Note also that ρ_t does not coincide with the correlation at time t as

$$
\text{corr}_{t-1}(\eta_1, \eta_2) = \frac{\rho_t - \frac{2}{\pi}\delta_1 \delta_2}{b_1 b_2}.
$$

The score is derived in Result 3.

Result 3. *The derivative of the log-likelihood with respect to ρ_t is*

$$
\frac{\partial \ell_t}{\partial \rho_t} = \frac{\rho_t}{1 - \rho_t^2} + \frac{b_1'}{b_1} + \frac{b_2'}{b_2} - \frac{1}{2}Q' + (\alpha_1 x_{1,t}' + \alpha_2 x_{2,t}') \frac{\phi(\alpha_1 x_{1,t} + \alpha_2 x_{2,t})}{\Phi(\alpha_1 x_{1,t} + \alpha_2 x_{2,t})}
$$

where

$$
\delta_1' = \frac{\alpha_2}{\sqrt{1 + \alpha_*^2}} - \frac{\alpha_1 \alpha_2 \delta_1}{1 + \alpha_*^2}
$$

$$
\delta_2' = \frac{\alpha_1}{\sqrt{1 + \alpha_*^2}} - \frac{\alpha_1 \alpha_2 \delta_2}{1 + \alpha_*^2}
$$

$$
a_i' = \delta_i' \sqrt{\frac{2}{\pi}}
$$

$$
b_i' = -a_i' \frac{a_i}{b_i}
$$

$$
x_{i,t}' = a_i' \eta_{i,t} + b_i' \quad i = 1, 2
$$

$$
Q' = 2\frac{x_{1,t}' x_{1,t} + x_{2,t}' x_{2,t} - x_{1,t} x_{2,t} - \rho_t (x_{1,t}' x_{2,t} + x_{1,t} x_{2,t}') + \rho_t Q(\rho_t, x_{1,t}, x_{2,t})}{1 - \rho_t^2}.
$$

A.4 Bivariate normal

We derive the symmetric normal distribution by setting $\alpha_1 = \alpha_2 = 0$ in (10):

$$f(\eta_{1,t}, \eta_{2,t}) = \frac{1}{2\pi\sqrt{1-\rho_t^2}} \exp\left(-\frac{1}{2}Q(\rho_t, \eta_{1,t}, \eta_{2,t})\right). \tag{11}$$

The score is derived in Result 4 along with the Fisher information.

Result 4. *The derivative of the log-likelihood with respect to ρ_t is*

$$\frac{\partial \ell_t}{\partial \rho_t} = \frac{\rho_t}{1-\rho_t^2} - \frac{1}{(1-\rho_t^2)^2}\left[\rho_t(\eta_{1,t}^2 + \eta_{2,t}^2) - (1+\rho_t^2)\eta_{1,t}\eta_{2,t}\right] \tag{12}$$

Furthermore

$$\mathbb{E}_{t-1}\left[\left(\frac{\partial \ell_t}{\partial \rho_t}\right)^2\right] = \frac{1+\rho_t^2}{(1-\rho_t^2)^2}. \tag{13}$$

Proof. The proof of eq.(13) of the paper is based on the following results:

$$\begin{aligned}
\mathbb{E}_{t-1}\left[\eta_{1,t}\right] &= \mathbb{E}_{t-1}\left[\eta_{2,t}\right] = \mathbb{E}_{t-1}\left[\eta_{1,t}^3\right] = \mathbb{E}_{t-1}\left[\eta_{2,t}^2\right] = 0 \\
\mathbb{E}_{t-1}\left[\eta_{1,t}^2\right] &= \mathbb{E}_{t-1}\left[\eta_{2,t}^2\right] = 1 \\
\mathbb{E}_{t-1}\left[\eta_{1,t}^4\right] &= \mathbb{E}_{t-1}\left[\eta_{2,t}^4\right] = 3 \\
\mathbb{E}_{t-1}\left[\eta_{1,t}\eta_{2,t}\right] &= \rho_t \\
\mathbb{E}_{t-1}\left[\eta_{1,t}^2\eta_{2,t}^2\right] &= 1 + 2\rho_t^2 \\
\mathbb{E}_{t-1}\left[\eta_{1,t}^3\eta_{2,t}\right] &= \mathbb{E}_{t-1}\left[\eta_{1,t}\eta_{2,t}^3\right] = 3\rho_t
\end{aligned}$$

□

Appendix B Simulation Study

In this section, we conduct a comprehensive simulation study to assess the performance of our statistical model under varying parameter configurations. The model's key parameters are systematically explored to understand their impact on estimation accuracy. The parameter choices considered, based on values fitted to real data from diverse sources (equity, debt, crypto, utilities), are presented in Table 9.

Table 9: Parameter values for the simulation study.

Parameter	**Values**
ω	$[-0.12, 0, 0.12]$
a	$[-0.03, 0, 0.03]$
b	0.95
α_1	$[-0.3, 0, 0.3]$
α_2	$[-0.8, 0.2, 1.2]$
ν	4

Source: Authors own work

We examine a total of 81 parameter combinations, creating a robust exploration of the model's behaviour. We simulate $T = 250$, 500, and $1,000$ observations for each combination. The primary objective of these simulations is to assess the convergence of the estimated parameters to their true values.

Figures 5, 6, and 7 show the results of the experiment. The outcomes demonstrate that the mean of the estimated values from simulated samples matches the true values across all scenarios. Estimates obtained when T is low exhibit greater variability, particularly for ν.

Figure 8 illustrates the Root Mean Squared Errors (RMSE) across parameters for varying time periods T. The pattern is generally stable, with a notable decrease in RMSE for ν. The result of Figure 8 confirms the robustness of our estimation framework, even in datasets characterised by small sample sizes.

Appendix C Model Estimation for Ethereum (ETH) and Ripple (XRP)

To strengthen the robustness of our analytical framework for Bitcoin, we extend the analysis to Ethereum (ETH) and Ripple (XRP) to assess their dynamic correlations with traditional financial markets. ETH is distinguished

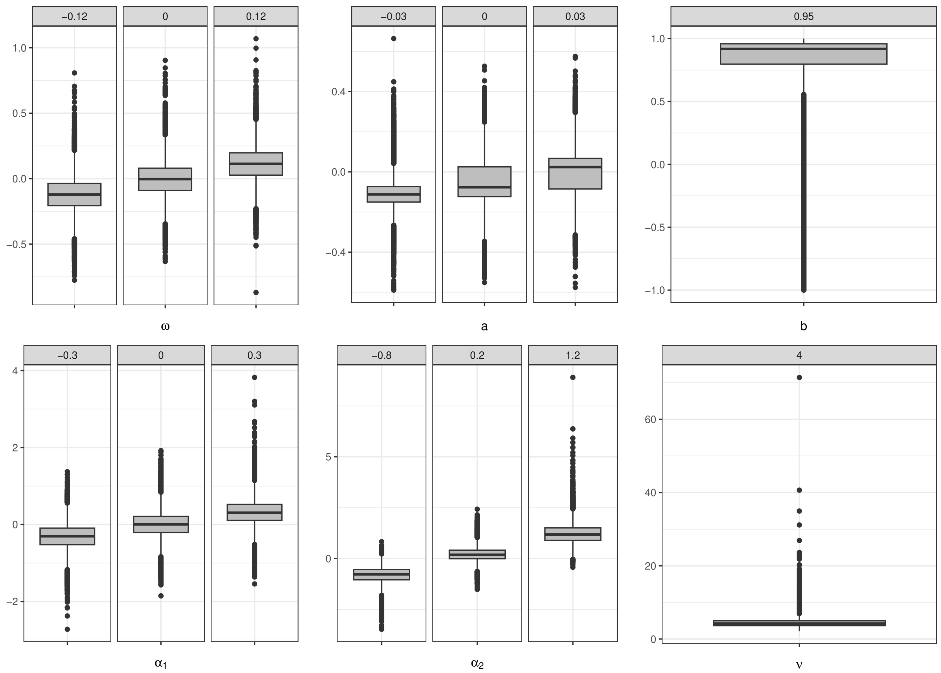

Figure 5: Boxplots illustrating simulation study results for $T = 250$, and across diverse parameter values. Source: Authors own work.

by its technological foundation in smart contracts and decentralised applications, offering utility beyond that of BTC. XRP, by contrast, is designed primarily for cross-border payments and institutional settlement—positioning it closer to a foreign-exchange utility token. Incorporating cryptocurrencies with diverse functional roles enables us to evaluate whether the observed patterns in Bitcoin are asset-specific or generalizable across broader segments of the crypto market.

Across all ETH and XRP pairs, the model selection criteria—Log-Likelihood, AIC, and BIC—consistently point to the Asymmetric Student-t distribution as the optimal specification, in line with results obtained for BTC. This model's superior performance, reflected in the lowest AIC and BIC values across all tables, indicates that heavy tails and asymmetry are intrinsic features of cryptocurrency data. The statistical significance of both the degrees of freedom (ν) and skewness (α_2) parameters at the 1% level further confirms that these assets exhibit pronounced kurtosis and asymmetry, which the selected model captures effectively. Estimation results are presented in Tables 10 and 12.

Table 10: Parameter estimates for the bivariate models fitted on the pairs (ETH-S&P500, ETH-NASDAQ, ETH-VIX, ETH-WTI), comparing Normal, Student-t, Asymmetric Normal, and Asymmetric Student-t distributions. The table includes estimates for the degrees of freedom (ν), asymmetry parameters (α_1, α_2), and other model parameters (ω, a, b), along with log-likelihood (LogLik), AIC, and BIC values.

(a) ETH-S&P500

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	5.0667 * * *	-	5.2639 * * *
	-	(0.2970)	-	(0.3440)
α_1	-	-	-0.3262 * *	0.1161
	-	-	(0.1371)	(0.1146)
α_2	-	-	-1.6290 * * *	-0.6055 * * *
	-	-	(0.1523)	(0.1405)
ω	0.0852	0.1611 * *	0.2745 *	0.1440 *
	(0.1259)	(0.0731)	(0.1405)	(0.0872)
a	0.0133 * *	0.0234 * *	0.0181 * *	0.0285 * *
	(0.0060)	(0.0111)	(0.0086)	(0.0136)
b	0.9963 * * *	0.9911 * * *	0.9967 * * *	0.9906 * * *
	(0.0047)	(0.0086)	(0.0042)	(0.0092)
LogLik	-5778.3509	-5541.6602	-5732.6098	-5531.0146
AIC	11562.7018	11091.3204	11475.2197	11074.0293
BIC	11579.5786	11113.8228	11503.3476	11107.7828

(b) ETH-NASDAQ

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	5.2534 * * *	-	5.4671 * * *
	-	(0.3202)	-	(0.3689)
α_1	-	-	-0.3399 * * *	0.1440
	-	-	(0.1211)	(0.1096)
α_2	-	-	-1.7084 * * *	-0.8430 * * *
	-	-	(0.1467)	(0.1365)
ω	0.0031	0.1447	0.1947	0.1156
	(0.1790)	(0.1041)	(0.2115)	(0.1256)
a	0.0127 * * *	0.0168 * *	0.0180 * * *	0.0222 * *
	(0.0046)	(0.0066)	(0.0068)	(0.0094)
b	0.9983 * * *	0.9963 * * *	0.9983 * * *	0.9962 * * *
	(0.0025)	(0.0038)	(0.0027)	(0.0042)
LogLik	-5768.3041	-5553.3843	-5720.9875	-5533.5225
AIC	11542.6081	11114.7686	11451.9750	11079.0450
BIC	11559.4849	11137.2710	11480.1029	11112.7986

(c) ETH-VIX

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	4.2505 * * *	-	4.4923 * * *
	-	(0.1945)	-	(0.2237)
α_1	-	-	-0.0239	0.0322
	-	-	(0.1217)	(0.0797)
α_2	-	-	2.5425 * * *	1.3626 * * *
	-	-	(0.1534)	(0.1489)
ω	0.0649	-0.1516	0.0720	-0.1779
	(0.1406)	(0.1092)	(0.1907)	(0.1343)
a	0.0078	0.0170 * * *	0.0148	0.0277 * *
	(0.0052)	(0.0064)	(0.0113)	(0.0112)
b	0.9999 * * *	0.9967 * * *	1.0000 * * *	0.9961 * * *
	(0.0013)	(0.0035)	(0.0014)	(0.0038)
LogLik	-5777.7622	-5430.3072	-5655.7800	-5374.8198
AIC	11561.5243	10868.6145	11321.5599	10761.6396
BIC	11578.4011	10891.1169	11349.6879	10795.3932

(d) ETH-WTI

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	5.1258 * * *	-	5.1745 * * *
	-	(0.3062)	-	(0.3066)
α_1	-	-	-0.3407 *	0.0097
	-	-	(0.1938)	(0.1010)
α_2	-	-	-1.4139 * * *	-0.7994 * * *
	-	-	(0.1626)	(0.1382)
ω	0.0671 * * *	0.0498	0.2428 * * *	0.0519
	(0.0229)	(0.0305)	(0.0771)	(0.0512)
a	0.0241 *	0.0082	0.0414	0.0117
	(0.0125)	(0.0081)	(0.0258)	(0.0107)
b	0.6552 * *	0.9765 * * *	0.6533 * * *	0.9775 * * *
	(0.2580)	(0.0263)	(0.2134)	(0.0245)
LogLik	-5836.5489	-5599.6867	-5806.7368	-5581.3513
AIC	11679.0979	11207.3734	11623.4735	11174.7026
BIC	11695.9747	11229.8758	11651.6015	11208.4562

Note: * 10%, ** 5%, and *** 1% significance levels. Source: Authors own work.

Table 11: Parameter estimates for the bivariate models with explanatory variables fitted on the pairs (ETH-S&P500, ETH-NASDAQ, ETH-VIX, ETH-WTI), comparing Normal, Student-t, Asymmetric Normal, and Asymmetric Student-t distributions. The table includes estimates for the degrees of freedom (ν), asymmetry parameters (α_1, α_2), and other model parameters (ω, a, b), along with log-likelihood (LogLik), AIC, and BIC values.

(a) ETH-S&P500

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	5.0756 * * *	-	5.2731 * * *
	-	(0.2978)	-	(0.3366)
α_1	-	-	-0.3777	0.1210
	-	-	(2.0068)	(0.1129)
α_2	-	-	-1.7804 * * *	-0.6077 * * *
	-	-	(0.3579)	(0.1392)
ω	0.0900	0.1385 * *	0.2059	0.1182 *
	(0.0778)	(0.0557)	(0.4498)	(0.0670)
a	0.0159 *	0.0289 *	-0.0067	0.0349 * *
	(0.0094)	(0.0149)	(0.1296)	(0.0157)
b	0.9901 * * *	0.9774 * * *	1.0000 * * *	0.9769 * * *
	(0.0126)	(0.0219)	(0.0036)	(0.0190)
ρ_{Gold}	1.5043	1.2252	0.6915	1.3818
	(1.0398)	(1.1282)	(4.0851)	(1.2772)
ρ_{DGS10}	0.2377	0.3732	0.1226	0.4065
	(0.2280)	(0.3474)	(1.6952)	(0.3560)
ρ_{T10Y3M}	0.0295	-0.0521	0.0584	-0.0609
	(0.0988)	(0.2036)	(2.7126)	(0.2014)
ρ_{T10YIE}	-0.1662	-0.1769	-0.0235	-0.1795
	(0.1860)	(0.3271)	(3.4497)	(0.4083)
LogLik	-5776.3130	-5540.2819	-5725.1952	-5529.6127
AIC	11566.6259	11096.5637	11468.3904	11079.2254
BIC	11606.0051	11141.5685	11519.0207	11135.4813

(b) ETH-NASDAQ

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	5.2631 * * *	-	5.4829 * * *
	-	(0.3217)	-	(0.4361)
α_1	-	-	-0.3114 * *	0.1376
	-	-	(0.1347)	(0.1247)
α_2	-	-	-1.7249 * * *	-0.8461 * * *
	-	-	(0.1606)	(0.2706)
ω	-0.0466	0.0495	0.1094	0.0073
	(0.1318)	(0.3670)	(0.1652)	(1.2989)
a	0.0099	0.0099	0.0139	0.0124
	(0.0061)	(0.0357)	(0.0088)	(0.0895)
b	0.9994 * * *	1.0000 * * *	0.9996 * * *	1.0000 * * *
	(0.0021)	(0.0131)	(0.0019)	(0.0223)
ρ_{Gold}	0.6286	0.4486	0.7464	0.5126
	(0.7336)	(1.2567)	(0.8614)	(0.5512)
ρ_{DGS10}	0.0307	0.0448	0.0335	0.0498
	(0.1099)	(0.2459)	(0.1265)	(0.3010)
ρ_{T10Y3M}	0.0964	0.1347	0.1150	0.1508
	(0.0603)	(0.2331)	(0.0716)	(0.6093)
ρ_{T10YIE}	-0.0883	-0.1420	-0.1027	-0.1469
	(0.1130)	(0.2370)	(0.1420)	(0.2432)
LogLik	-5766.2336	-5552.3319	-5718.9173	-5532.4888
AIC	11546.4673	11120.6638	11455.8347	11084.9777
BIC	11585.8464	11165.6686	11506.4650	11141.2336

(c) ETH-VIX

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	4.2610 * * *	-	4.5015 * * *
	-	(0.2006)	-	(0.2441)
α_1	-	-	-0.0343	0.0378
	-	-	(0.1101)	(0.0797)
α_2	-	-	2.5396 * * *	1.3637 * * *
	-	-	(0.1585)	(0.1617)
ω	0.0122	-0.1044	0.0132	-0.1195
	(0.1157)	(0.0670)	(0.1918)	(0.0848)
a	0.0102 *	0.0218 * *	0.0175	0.0348 * *
	(0.0053)	(0.0094)	(0.0111)	(0.0162)
b	0.9961 * * *	0.9868 * * *	0.9971 * * *	0.9855 * * *
	(0.0038)	(0.0140)	(0.0037)	(0.0121)
ρ_{Gold}	-2.1587 * * *	-1.9590 *	-2.8065 * *	-2.4902 *
	(0.8294)	(1.1747)	(1.1510)	(1.3575)
ρ_{DGS10}	-0.3081 * *	-0.4278	-0.3741 *	-0.5500
	(0.1502)	(0.3412)	(0.2044)	(0.3480)
ρ_{T10Y3M}	-0.0550	0.0467	-0.0930	0.0669
	(0.0683)	(0.2256)	(0.0929)	(0.1789)
ρ_{T10YIE}	0.2907 * *	0.2150	0.3706 *	0.2780
	(0.1466)	(0.2809)	(0.2236)	(0.3673)
LogLik	-5771.5686	-5427.6387	-5649.7045	-5371.9805
AIC	11557.1372	10871.2775	11317.4090	10763.9610
BIC	11596.5164	10916.2823	11368.0394	10820.2170

(d) ETH-WTI

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	5.1362 * * *	-	5.1854 * * *
	-	(0.3150)	-	(0.2879)
α_1	-	-	-0.3688 *	0.0104
	-	-	(0.1895)	(0.0959)
α_2	-	-	-1.4052 * * *	-0.8013 * * *
	-	-	(0.1592)	(0.1377)
ω	0.0604 * *	0.0305	0.2560 * * *	0.0291
	(0.0268)	(0.0295)	(0.0746)	(0.0500)
a	0.0107	0.0081	0.0313	0.0118
	(0.0115)	(0.0098)	(0.0247)	(0.0118)
b	0.9164 * * *	0.9677 * * *	0.7000 * * *	0.9691 * * *
	(0.1054)	(0.0295)	(0.2379)	(0.0261)
ρ_{Gold}	0.4930	0.8583	-0.6733	0.9932
	(1.4644)	(1.1638)	(1.8216)	(1.4260)
ρ_{DGS10}	0.3932 *	0.2332	0.4862	0.2585
	(0.2306)	(0.2405)	(0.4561)	(0.3097)
ρ_{T10Y3M}	-0.1227	-0.1304	-0.1483	-0.1569
	(0.2153)	(0.1816)	(0.3983)	(0.1624)
ρ_{T10YIE}	-0.1370	-0.0692	0.2908	-0.0674
	(0.2541)	(0.2941)	(0.6903)	(0.3280)
LogLik	-5835.2281	-5598.4522	-5805.2603	-5580.0639
AIC	11684.4562	11212.9043	11628.5206	11180.1277
BIC	11723.8354	11257.9091	11679.1510	11236.3837

Note: The significance of each explanatory variable (Gold, DGS10, T10Y3M and T10YIE) coefficient is indicated with stars. * 10%, ** 5%, and *** 1% levels. Source: Authors own work.

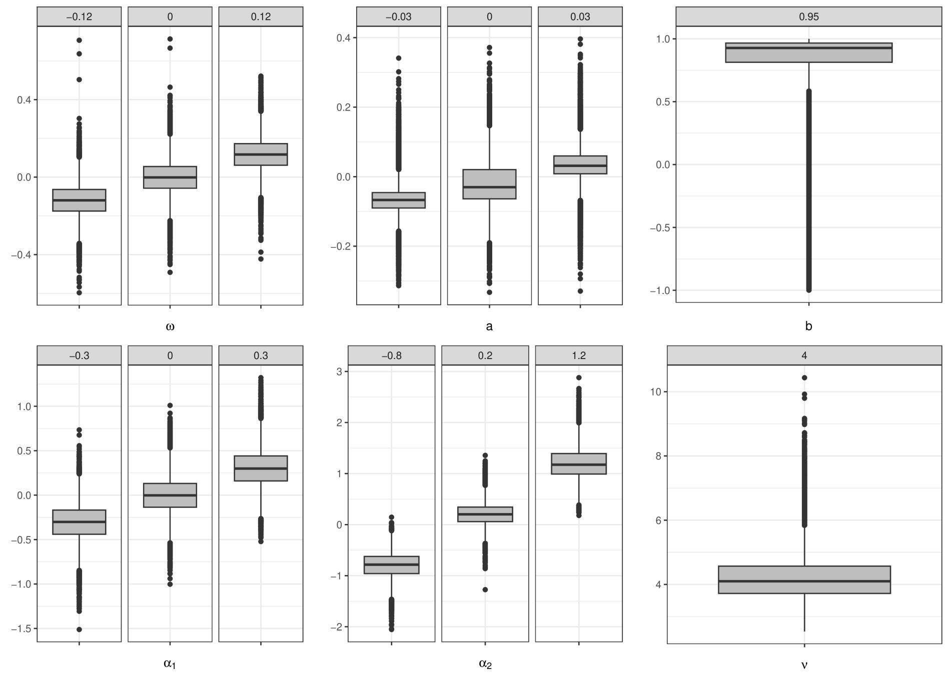

Figure 6: Boxplots illustrating simulation study results for $T = 500$, and across diverse parameter values. Source: Authors own work.

C.1 Impact of Explanatory Variables on Correlations for ETH and XRP

Tables 11 and 13 report how macroeconomic factors influence correlation dynamics, using interest rates (DGS10), the yield curve (T10Y3M), expected inflation (T10YIE), and Gold prices—consistent with the BTC analysis. The results reveal differentiated patterns across ETH and XRP.

For Ethereum, the inclusion of macroeconomic variables yields a mixed but informative picture. The most notable effects arise in the ETH–VIX pair. Gold and DGS10 display negative and significant coefficients in several models, particularly the Normal and Asymmetric Normal, suggesting that during periods of market stress (high VIX), increases in Gold prices and long-term interest rates weaken the positive correlation between ETH and the VIX. Expected inflation (T10YIE), however, exhibits a positive and significant relationship, indicating that higher inflation expectations tend to strengthen the ETH–VIX correlation. In contrast, the explanatory variables are largely insignificant for pairs involving equity indices (S&P 500, NASDAQ) and WTI. This suggests that ETH's co-movements with these assets may be shaped more by crypto-specific factors—such as internal market sen-

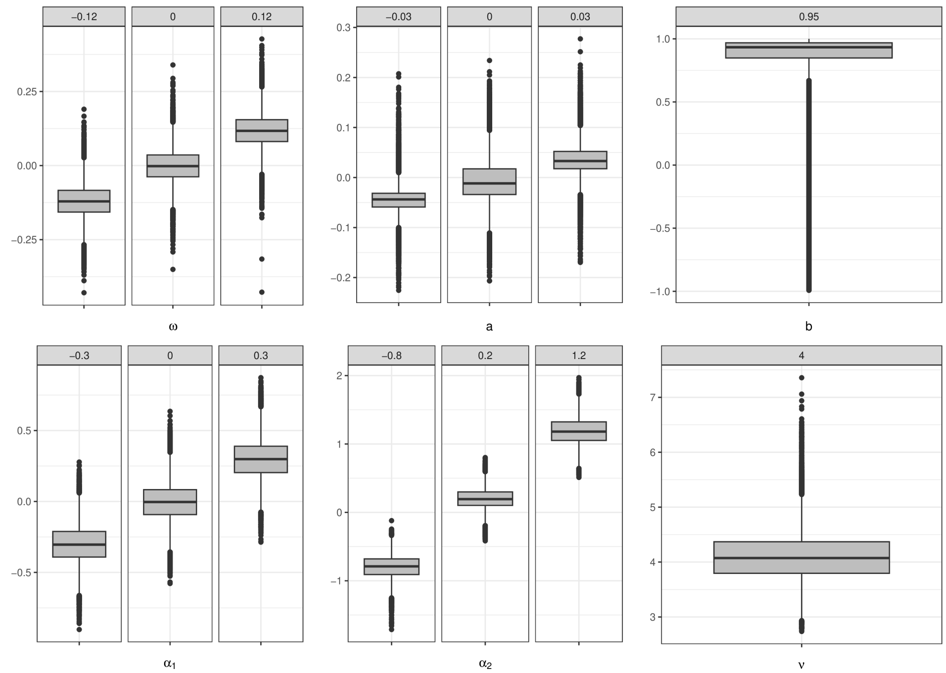

Figure 7: Boxplots illustrating simulation study results for $T = 1,000$, and across diverse parameter values. Source: Authors own work.

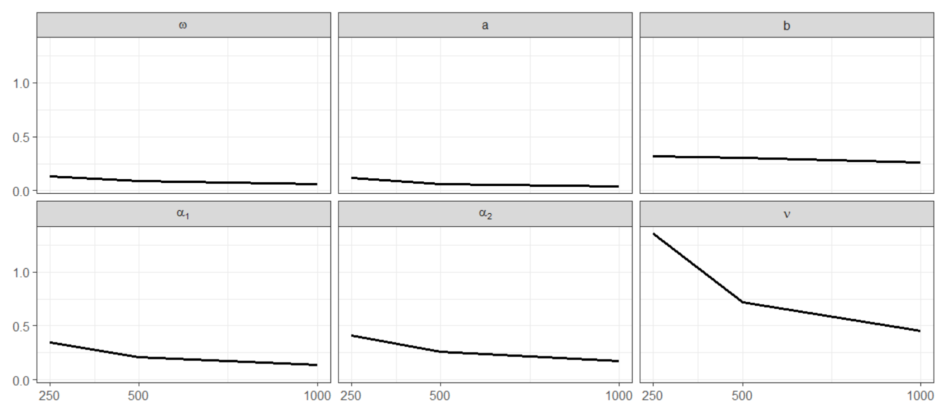

Figure 8: Root mean squared errors (RMSE) across different parameters and time periods $T = 250$, 500, $1,000$. Source: Authors own work.

timent or technological developments within the Ethereum ecosystem—than by the macroeconomic channels considered here.

For XRP, the influence of macroeconomic variables is stronger and more consistent, particularly for the XRP–WTI pair. Gold shows a robustly negative and statistically significant coefficient across all models, indicating that increases in Gold prices systematically reduce XRP–WTI correlations. This pattern is indicative of risk-off behaviour, in which capital shifts away from both cryptocurrencies and oil toward safe-haven assets, causing their price movements to decouple. DGS10 and T10YIE also display significant effects for the XRP–WTI pair in some models, highlighting the heightened sensitivity of this correlation to macroeconomic conditions. For equity index pairs (XRP–S&P500 and XRP–NASDAQ), Gold is again positive and consistently significant, suggesting a modest but systematic influence of Gold market dynamics on XRP's comovement with equity markets.

In short, although the Asymmetric Student-t distribution provides the best statistical fit across all assets, the macroeconomic drivers of correlation vary substantially by cryptocurrency. ETH's correlations are most sensitive to macroeconomic variables when paired with the VIX, highlighting its responsiveness during periods of financial stress. XRP exhibits broader and more consistent macro sensitivity, particularly with respect to WTI and Gold. These results confirm that while major cryptocurrencies share common statistical properties—such as heavy tails and asymmetry—their linkages to traditional financial markets are heterogeneous and multifaceted.

Table 12: Parameter estimates for the bivariate models fitted on the pairs (XRP-S&P500, XRP-NASDAQ, XRP-VIX, XRP-WTI), comparing Normal, Student-t, Asymmetric Normal, and Asymmetric Student-t distributions. The table includes estimates for the degrees of freedom (ν), asymmetry parameters (α_1, α_2), and other model parameters (ω, a, b), along with log-likelihood (LogLik), AIC, and BIC values.

(a) XRP-S&P500

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	3.9375 ***	-	3.9906 ***
	-	(0.1479)	-	(0.1544)
α_1	-	-	1.1639 ***	0.0536
	-	-	(0.0818)	(0.1022)
α_2	-	-	-0.1598 *	-0.2685 ***
	-	-	(0.0949)	(0.1018)
ω	0.1232 **	0.1389 **	0.0873	0.1345 **
	(0.0524)	(0.0628)	(0.0769)	(0.0661)
a	0.0150 ***	0.0144 *	0.0242 ***	0.0152 *
	(0.0058)	(0.0076)	(0.0093)	(0.0078)
b	0.9884 ***	0.9931 ***	0.9885 ***	0.9931 ***
	(0.0066)	(0.0074)	(0.0064)	(0.0074)
LogLik	-5774.5301	-5374.1334	-5739.3562	-5370.1770
AIC	11555.0602	10756.2668	11488.7124	10752.3539
BIC	11571.9370	10778.7691	11516.8403	10786.1075

(b) XRP-NASDAQ

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	4.0616 ***	-	4.1324 ***
	-	(0.1636)	-	(0.1867)
α_1	-	-	1.1738 ***	0.0979
	-	-	(0.0836)	(0.0981)
α_2	-	-	-0.1949 *	-0.5083 ***
	-	-	(0.1059)	(0.1085)
ω	0.1427 ***	0.1486 **	0.1003	0.1354 *
	(0.0539)	(0.0660)	(0.0782)	(0.0709)
a	0.0156 ***	0.0131 *	0.0254 ***	0.0154 *
	(0.0056)	(0.0069)	(0.0098)	(0.0080)
b	0.9889 ***	0.9941 ***	0.9890 ***	0.9939 ***
	(0.0062)	(0.0061)	(0.0071)	(0.0063)
LogLik	-5770.9193	-5396.2515	-5735.2131	-5383.3826
AIC	11547.8385	10800.5029	11480.4263	10778.7652
BIC	11564.7153	10823.0053	11508.5542	10812.5187

(c) XRP-VIX

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	4.0413 ***	-	4.0483 ***
	-	(0.1550)	-	(0.1577)
α_1	-	-	1.1398 ***	0.1025
	-	-	(0.0808)	(0.0967)
α_2	-	-	-0.0436	-0.5467 ***
	-	-	(0.0881)	(0.1067)
ω	0.0432 ***	0.0309	0.0339	0.0085
	(0.0167)	(0.0249)	(0.0452)	(0.0341)
a	-0.0253 *	-0.0239	-0.0391 *	-0.0262
	(0.0146)	(0.0222)	(0.0215)	(0.0288)
b	0.8760 ***	0.6590 ***	0.8774 ***	0.6471 *
	(0.0616)	(0.2427)	(0.0609)	(0.3450)
LogLik	-5812.8406	-5418.3112	-5779.1994	-5402.9669
AIC	11631.6812	10844.6224	11568.3988	10817.9338
BIC	11648.5579	10867.1248	11596.5268	10851.6874

(d) XRP-WTI

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	3.5480 ***	-	3.6650 ***
	-	(0.1078)	-	(0.1207)
α_1	-	-	-0.1060	0.0501
	-	-	(0.0721)	(0.0753)
α_2	-	-	2.5372 ***	0.9579 ***
	-	-	(0.1607)	(0.1246)
ω	-0.1254 **	-0.1457 *	-0.2289 ***	-0.1567
	(0.0502)	(0.0831)	(0.0760)	(0.1013)
a	0.0139 **	0.0132 *	0.0286 **	0.0180 **
	(0.0063)	(0.0068)	(0.0129)	(0.0088)
b	0.9872 ***	0.9962 ***	0.9869 ***	0.9961 ***
	(0.0074)	(0.0044)	(0.0076)	(0.0044)
LogLik	-5780.4252	-5250.7161	-5657.1225	-5206.6176
AIC	11566.8504	10509.4322	11324.2449	10425.2352
BIC	11583.7272	10531.9346	11352.3729	10458.9887

Note: * 10%, ** 5%, and *** 1% significance levels. Source: Authors own work.

Table 13: Parameter estimates for the bivariate models with explanatory variables fitted on the pairs (XRP-S&P500, XRP-NASDAQ, XRP-VIX, XRP-WTI), comparing Normal, Student-t, Asymmetric Normal, and Asymmetric Student-t distributions. The table includes estimates for the degrees of freedom (ν), asymmetry parameters (α_1, α_2), and other model parameters (ω, a, b), along with log-likelihood (LogLik), AIC, and BIC values.

(a) XRP-S&P500

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	3.9457 ***	-	3.9993 ***
	-	(0.1645)	-	(0.1552)
α_1	-	-	1.1613 ***	0.0550
	-	-	(0.0832)	(0.1022)
α_2	-	-	-0.1462	-0.2710 ***
	-	-	(0.0953)	(0.0969)
ω	0.0715	0.0968 *	0.0278	0.0910 *
	(0.0539)	(0.0496)	(0.0798)	(0.0507)
a	0.0155 **	0.0146 *	0.0246 **	0.0157 *
	(0.0067)	(0.0084)	(0.0101)	(0.0087)
b	0.9830 ***	0.9828 ***	0.9831 ***	0.9827 ***
	(0.0088)	(0.0133)	(0.0075)	(0.0124)
ρ_{Gold}	2.7660 ***	2.2929 *	3.3897 ***	2.3608 **
	(0.9957)	(1.2298)	(1.2587)	(1.1225)
ρ_{DGS10}	0.3251 *	0.3485	0.4051 *	0.3610
	(0.1662)	(0.2498)	(0.2300)	(0.2229)
ρ_{T10Y3M}	-0.0186	-0.0412	-0.0358	-0.0426
	(0.1096)	(0.1336)	(0.1491)	(0.1265)
ρ_{T10YIE}	-0.2757	-0.3084	-0.3350	-0.3241
	(0.2083)	(0.3021)	(0.2440)	(0.2415)
LogLik	-5769.7005	-5370.8153	-5734.7300	-5366.8216
AIC	11553.4010	10757.6305	11487.4600	10753.6431
BIC	11592.7802	10802.6353	11538.0904	10809.8991

(b) XRP-NASDAQ

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	4.0651 ***	-	4.1355 ***
	-	(0.1683)	-	(0.1722)
α_1	-	-	1.1707 ***	0.0987
	-	-	(0.0839)	(0.1019)
α_2	-	-	-0.1791	-0.5104 ***
	-	-	(0.1117)	(0.1079)
ω	0.0844	0.1069 **	0.0317	0.0905
	(0.0576)	(0.0523)	(0.0845)	(0.0606)
a	0.0147 **	0.0131	0.0240 **	0.0155 *
	(0.0062)	(0.0090)	(0.0098)	(0.0084)
b	0.9865 ***	0.9859 ***	0.9863 ***	0.9855 ***
	(0.0088)	(0.0114)	(0.0068)	(0.0128)
ρ_{Gold}	2.4224 **	2.1043 **	2.9805 **	2.2713 **
	(1.0166)	(1.0354)	(1.1828)	(1.1038)
ρ_{DGS10}	0.2824	0.3194	0.3591 *	0.3485
	(0.1861)	(0.2042)	(0.2098)	(0.2423)
ρ_{T10Y3M}	0.0087	0.0027	-0.0067	-0.0006
	(0.1392)	(0.1255)	(0.1443)	(0.1520)
ρ_{T10YIE}	-0.2812	-0.3866	-0.3420	-0.4228
	(0.1828)	(0.2764)	(0.2395)	(0.2656)
LogLik	-5766.5960	-5392.9277	-5731.1201	-5380.0065
AIC	11547.1919	10801.8554	11480.2401	10780.0130
BIC	11586.5711	10846.8601	11530.8705	10836.2689

(c) XRP-VIX

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	4.0397 * * *	-	4.0469 * * *
	-	(0.1560)	-	(0.1561)
α_1	-	-	1.1395 * * *	0.1052
	-	-	(0.0800)	(0.0885)
α_2	-	-	-0.0408	-0.5458 * * *
	-	-	(0.0901)	(0.1118)
ω	0.0267	0.0255	0.0149	0.0024
	(0.0235)	(0.0249)	(0.0469)	(0.0360)
a	-0.0209	-0.0235	-0.0317	-0.0263
	(0.0145)	(0.0225)	(0.0234)	(0.0317)
b	0.9192 * * *	0.7008 * *	0.9210 * * *	0.6912 * *
	(0.0586)	(0.3214)	(0.0589)	(0.3474)
rho:Gold	2.0366 *	1.9300	2.5373 *	1.9822
	(1.1788)	(2.6423)	(1.5365)	(2.6303)
rho:DGS10	0.3247 *	0.3191	0.3994	0.3327
	(0.1953)	(0.3366)	(0.2500)	(0.4283)
rho:T10Y3M	-0.0682	0.1109	-0.0888	0.1343
	(0.1363)	(0.3654)	(0.1656)	(0.4438)
rho:T10YIE	-0.4345 *	-0.5552	-0.5295 *	-0.6097
	(0.2619)	(0.4582)	(0.3146)	(0.4537)
LogLik	-5810.6921	-5417.3202	-5777.0781	-5401.9663
AIC	11635.3843	10850.6403	11572.1561	10823.9326
BIC	11674.7634	10895.6451	11622.7865	10880.1885

(d) XRP-WTI

	Normal	Student-t	Asymmetric Normal	Asymmetric Student-t
ν	-	3.5552 ***	-	3.6732 ***
	-	(0.1108)	-	(0.1219)
α_1	-	-	-0.1075	0.0537
	-	-	(0.0793)	(0.0746)
α_2	-	-	2.5458 ***	0.9626 ***
	-	-	(0.1619)	(0.1381)
ω	-0.1703 ***	-0.0825	-0.2916 ***	-0.0847
	(0.0298)	(0.0583)	(0.0483)	(0.0708)
a	0.0517 ***	0.0136 *	0.1144 ***	0.0193 *
	(0.0098)	(0.0071)	(0.0268)	(0.0101)
b	0.8321 ***	0.9866 ***	0.8208 ***	0.9856 ***
	(0.0605)	(0.0132)	(0.0567)	(0.0145)
rho:Gold	-4.5435 ***	-2.5325 **	-6.9377 ***	-2.9712 **
	(1.5352)	(1.2208)	(2.1651)	(1.4937)
rho:DGS10	-1.7973 ***	-0.4054	-2.7988 ***	-0.5034
	(0.6023)	(0.3267)	(0.8664)	(0.3774)
rho:T10Y3M	0.3079	0.0520	0.7551	0.0696
	(0.4727)	(0.1385)	(0.7311)	(0.1641)
rho:T10YIE	1.0650 ***	0.2657	1.6265 ***	0.3520
	(0.3494)	(0.3088)	(0.4848)	(0.3546)
LogLik	-5772.9531	-5246.3971	-5648.8003	-5202.2376
AIC	11559.9062	10508.7941	11315.6005	10424.4753
BIC	11599.2854	10553.7989	11366.2309	10480.7312

Note: The significance of each explanatory variable (Gold, DGS10, T10Y3M and T10YIE) coefficient is indicated with stars. * 10%, ** 5%, and *** 1% levels. Source: Authors own work.