

On Gustav Robert Kirchhoff's contributions to electrodynamics

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Abstract

Purpose – This work is dedicated to the two-hundredth anniversary of the birth of German physicist Gustav Robert Kirchhoff. The purpose of the article is to point out that Kirchhoff's research includes an impressive number of fundamental publications that laid the foundation for new disciplines in physics and even electrical and mechanical engineering.

Design/methodology/approach – While many biographies mainly discuss Kirchhoff's research in physics, this paper focuses on Kirchhoff's research in electrodynamics and its significance for electrical engineering, particularly the theory of electrical circuits.

Findings – Kirchhoff's importance is evident in his fundamental contributions to physics, where, with his distinctive research style, he was among the founders of theoretical physics. However, it is shown that Kirchhoff's contributions to the mathematical and physical understanding of electrical circuits are often not presented in full detail, which motivated us to fill this gap in this paper.

Originality/value – Based on this historical analysis of Kirchhoff's contributions to electromagnetic theory, it becomes clear that modern circuit theory as a physical model of concentrated electromagnetic phenomena without radiation is essentially based on Kirchhoff's research.

Keywords History of electrical circuit theory, Circuit topology, Motion of electricity, Telegraph equation

Paper type General review

1. Introduction

Gustav Robert Kirchhoff (see [Plate 1](#)) was one of the most important German physicists and therefore known in his physics community primarily for his fundamental physical results. On the other hand, Kirchhoff also developed essential physical principles of electrical engineering, in particular the foundations for the theory of electrical circuits, which became a new technical discipline in the second half of the 19th century. Interestingly, many physicists do not know why some of Kirchhoff's results are so important for electrical engineering and, conversely, electrical engineers do not know which of Kirchhoff's results are so essential for physics ([Inan, 2010](#)). Regarding these statements, we should emphasize that Kirchhoff himself never asked what the benefit of his research was, which Robert von Helmholtz (1862–1889) pointed out in 1887 and continued that most of Kirchhoff's works began with the same sentence: "I believe I have found this and that and allow myself to give the proof below" ([Helmholtz, 1887/88](#)).



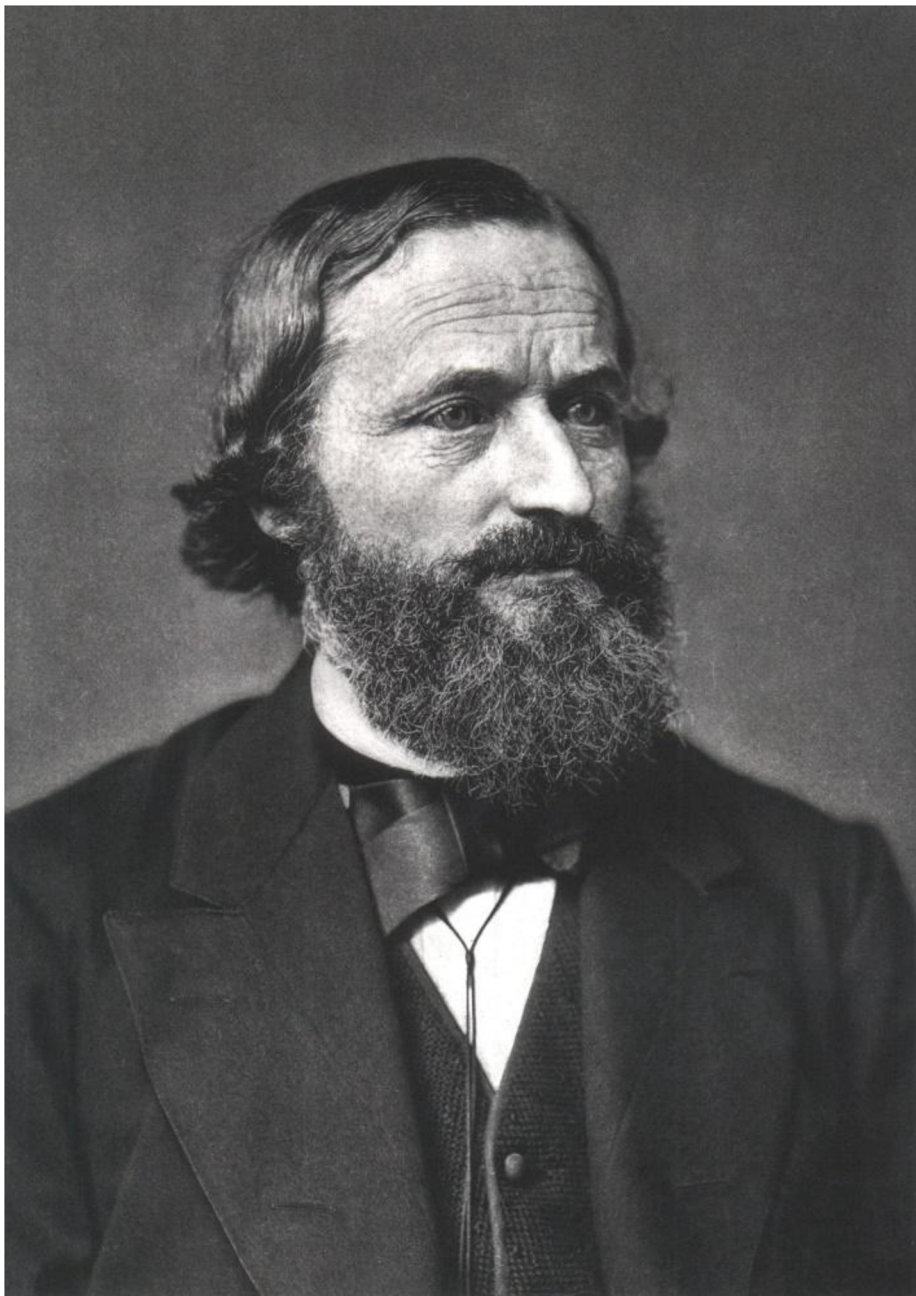


Plate 1. Gustav Kirchhoff in his time in Berlin (around 1880)

Source: https://commons.wikimedia.org/wiki/File:Gustav_Robert_Kirchhoff.jpg

When looking at Kirchhoff's collected works, one might be surprised to find that they contain less than 70 articles from his 40 years of research. However, there are some articles that opened up entirely new areas of scientific research. For physicists, Kirchhoff's most important achievements include the discovery of spectral analysis ([Hentschel, 1997](#)), developed together with Robert Wilhelm Bunsen (1811–1899) in 1857, the laws of thermal radiation, and his approximate theory of diffraction for scalar waves ([Menzel et al., 1973](#)). Mechanical engineers are particularly interested in Kirchhoff's plate theory based on the Bernoulli beam theory and its application to Cladni figures. However, in the majority of his articles, Kirchhoff dealt with the problems of electrodynamics. In this paper, we will particularly address the question of why some of Kirchhoff's results ultimately served as the physical basis of electrical engineering and, above all, circuit theory.

2. A biographical sketch

Kirchhoff is one of the most famous German physicists of the 19th century. As already mentioned in the introduction, Kirchhoff is known among scientists for very different reasons. Therefore, there are numerous biographies and biographical articles that examine Kirchhoff's scientific significance from different perspectives: [Voigt \(1888\)](#), R.v. Helmholtz (1887/88), [American Academy \(1887\)](#), [Boltzmann \(1888\)](#), [Pockels \(1903\)](#), [Warburg \(1925\)](#) and [Kant \(1988\)](#). Excellent biographies and further references can be found in [Hentschel \(1997\)](#) and [Hübner \(2010\)](#). For this reason, only a brief biographical outline of Kirchhoff's scientific life is given here, based on the biographical sketch in [Mathis \(2023\)](#).

Gustav Robert Kirchhoff was born on March 12, 1824 in Königsberg, Prussia (now Kaliningrad, Russia) as the son of Friedrich Kirchhoff, a lawyer, and Johanna Henriette Wittke. After he left the Kneiphöfische Gymnasium at age 18, Kirchhoff studied mathematics and physics at the Königliche Albertus University of Königsberg (now Kaliningrad in Russia). Under the scientific direction of Franz Ernst Neumann (1798–1895), he worked since 1843 in the mathematical-physical seminar, which was founded by Carl Gustav Jacob Jacobi (1804–1851). In 1847, Kirchhoff was honored for solving a philosophical faculty prize task, using the revised manuscript as a dissertation. In the same year he moved to the Friedrich-Wilhelms-Universität Berlin, where he received his habilitation (*venia legendi*) and became a private lecturer (*Privatdozent*). In 1850, at the age of 25, Kirchhoff moved to the University of Breslau (now Wrocław, Poland), but as early as 1854 he followed his friend Bunsen to the University of Heidelberg, where he spent his most fruitful years from an academic point of view. In 1857, Kirchhoff married Clara Richelot, the daughter of his former Mathematics Professor Friedrich Julius Richelot (1808–1875). Finally, in 1875, Kirchhoff received an exceptional and honorable vocation to the Prussian Academy of Sciences and at the same time to the University of Berlin. Kirchhoff died in Berlin in 1887 at the age of 63.

3. Kirchhoff's laws

It was probably a decisive step that Kirchhoff became a member of the famous mathematical-physical seminar in the winter semester of 1842 / 43, which was founded in 1834 at the University of Königsberg. As the seminar leader at the time, Neumann described Kirchhoff's potential in his report on the years 1843–1845, published in 1845, and pointed out that “a true pedagogical talent could be recognized” ([Hübner, 2010](#), p. 38). In addition, Neumann also mentioned a forthcoming paper by Kirchhoff, which was published in 1845 in *Poggendorff's Annalen der Physik* and signed “*Studiosus Kirchhoff – Member of the Physics Seminar in Königsberg*” ([Kirchhoff, 1845](#)). Together with some of Kirchhoff's later works, it contained the mathematical basis of a new discipline that was later called circuit theory and became an essential foundation of electrical engineering.

The main topic of Kirchhoff's first paper was the distribution of electric current in a circular plane, where his mathematical results were also verified by measurement results. Regarding his calculations of current distribution, Kirchhoff pointed out in the introduction to this paper that "The type of distribution can be determined theoretically according to the principles established by Ohm." While the main part of the 18-page work dealt with solid mathematical physics, the 2½-page appendix attracted considerable attention among Berlin physicists and especially among Johann C. Poggendorff (1796–1877) and Wilhelm Weber (1804–1891) (Hübner, 2010, p. 40). The purpose of this appendix was an analysis of the measurement arrangement which goes back to Charles Wheatstone (1802–1875) in 1843 (Wheatstone (1843) where Kirchhoff's variant is well-known today as Wheatstone bridge (Hübner, 2010, p. 40). To analyze the bridge circuit, Kirchhoff provided some of the results of Georg Simon Ohm's 1827 monograph (Ohm, 1827) in a more general and particularly mathematical form.

Kirchhoff considered systems of wires that are connected in any way and in which electric currents flow. Then, we have for the currents I_μ at each node of the system:

$$I_1 + I_2 + \dots + I_\mu = 0.$$

If the wires form a closed figure, we have:

$$I_1 \cdot \omega_1 + I_2 \cdot \omega_2 + \dots + I_v \cdot \omega_v = K_1 + K_2 + \dots + K_v,$$

where ω_v are the resistances of wires and K_v the electromotive forces in the closed figure. These two types of equations were later called Kirchhoff's laws and formed the basis of the theory of electrical circuits.

4. Reception of Kirchhoff's laws

As already mentioned, Kirchhoff's method for analyzing resistance circuits was immediately discussed in the physics community. Already in January 1846, Poggendorff gave a lecture at the Academy in Berlin in which he also discussed Weber's earlier calculations on this topic (Poggendorff, 1846). He introduced his paper "On a problem with linear branching of electric currents" with "Until recently, with regard to the branching of electric currents, only those cases were considered in which the branching originates from two points" and then continued "The matter is more complicated if the branching starts from more than two points." After Poggendorff explained Weber's analysis approach he proceeded: "Mr Kirchhoff has suggested a method by which the problems arising from linear branching of electrical currents can be solved in the most general way." Poggendorff closed his paper with a final remark:

An essential advantage of Kirchhoff's method is that, in the case of several electromotive forces contained in the different branches of the combination, it makes the calculation of the partial currents of each of these forces and their superposition superfluous, and allows the effective currents to be found at once, thus greatly simplifying the task.

Poggendorff showed also that Weber was not able to calculate the currents in the bridge in a completely general way (Hübner, 2010), pp. 48–50). The young Kirchhoff's first publication attracted great attention from the renowned physicists of his time.

Some other authors, such as the Dutch physicist Willem Smaasen (1820–1850) (Smaasen, 1846) and Hermann von Helmholtz (1821–1894) (v. Helmholtz, 1853), took Kirchhoff's (1845) work into account, but did not emphasize the importance of the appendix. It took more than ten years before another paper on Kirchhoff's laws was published. In 1854 in a letter to the editor of Poggendorff's *Annalen* (Bosscha, 1855), the Dutch physicist Johannes

Bosscha (1831–1911) presented a method to determine the electromotive force. Three years later, Bosscha explained his method in more detail and introduced his work as follows (Bosscha, 1858):

The general equations given by Mr. Kirchhoff for determining the intensities of currents in a system of any linear conductors containing any electromotive forces not only lead in a safe and easy way to the solution of often very complicated problems, but also enable us to discover some laws of linear branching of galvanic currents, which in many cases considerably simplify the calculation.

Note that in Bosscha's (1858) paper one finds for the first time a representation of Kirchhoff's laws with summation signs as short notations:

$$\sum(i) = 0, \quad \sum(ir - k) = 0.$$

In 1863, Gustav Wiedemann (1826–1899) published a monograph "The theory of galvanism and electro-magnetism" where the first volume includes a chapter "Current Branches" that discusses Kirchhoff's laws and illustrates its application (Wiedemann, 1863). Heinrich Schellen (1818–1884), a teacher and well-known author on physical and technical topics, also devoted a detailed chapter to "Kirchhoff's Laws" in his monograph "The electromagnetic telegraph in the main stages of its development and in its present form and application" in the fourth edition of 1867 (Schellen, 1867). A first edition was published in 1851. In the French technical literature, Kirchhoff's laws were apparently first mentioned by Jules Raynaud (1843–1888), as Andrew J. Butrica pointed out in his dissertation (Butrica, 1986). According to Butrica, Raynaud became aware of Kirchhoff's work through the work of the Swiss physicist Auguste De la Rive (1801–1873). In 1863, De la Rive published a paper (De la Rive, 1863) on the number of independent Kirchhoff equations, based on Kirchhoff's (1847) work (Kirchhoff, 1847). It is interesting that James Clerk Maxwell (1831–1879) in his first edition of 1873 (Maxwell, 1873) mentioned Kirchhoff's laws only in a verbal manner in a short remark.

A relationship between Kirchhoff's laws and analysis situs (now: algebraic topology) was presented by the mathematician Oswald Veblen (1880–1960) at the Cambridge Colloquium in 1916 and published in 1922 (Veblen, 1922) which also included the definition of linear graphs. Hermann Weyl (1885–1955) applied the framework of analysis situs to the "current distribution in a conducting network" in 1918, but this work was not published in Spanish until 1923 (Weyl, 1923). Although the German mathematician and electrical engineer Wilhelm Cauer (1900–1945) discussed Weyl's concept in his monograph (Cauer, 1941, p. 81), their importance was not fully recognized by circuit theorists until after the Second World War (Ghenzi, 1953), especially in Japan (Kondo, 1963) and later in the USA (Roth, 1955; Branin, 1967). Recently, Albrecht Reibiger presented the mathematical framework of Kirchhoff networks in a more abstract context (Reibiger, 2011). It should be emphasized that the reception by the latter researchers was particularly related to the properties of Kirchhoff's laws hidden in the connection structure.

5. Kirchhoff's laws and network topology

The problem of calculating the current distribution in electrical networks with Ohmic resistances and electrical energy sources was solved in principle by the formulation of Kirchhoff's laws. Kirchhoff, however, found that their concrete application to more complex circuits leads to complicated calculations if Gabriel Cramer's (1704–1752) method (Cramer, 1750), which was already available at the time, is used to solve the resulting linear equation

systems. With this method, the currents i_k of an electrical network can be calculated as quotients of determinants:

$$i_k = \frac{\det(C_k)}{\det(C)},$$

With matrices C_k and C , although Kirchhoff did not know this notation. There are obviously two difficulties associated with Cramer's method:

- (1) Independent linear equations are required for its application.
- (2) The calculation of the determinants is very laborious, since many terms arise that cancel each other out.

In accordance with Robert von Helmholtz's characterization, Kirchhoff presented an approach to circumvent these difficulties, but refrained from providing a justification and merely provided a proof of his results. So, in 1847 Kirchhoff published a paper "On the Solution of the Equations Obtained from the Investigation of the Linear Distribution of Galvanic Currents" (Kirchhoff, 1847); (Kirchhoff, 1958) which included his approach.

A central idea of his work was that Kirchhoff not only used the physical laws he had formulated two years earlier, but also certain properties of the structure of the interconnected resistors and electrical sources. It would be very interesting to know whether Kirchhoff knew the 1847 paper "Preliminary Studies in Topology" by Johann Benedict Listing (1808–1882), whose mentor was Johann Carl Friedrich Gauss (1777–1855).

In his paper, Kirchhoff defined the structure of a wire system (electrical circuit), consisting of wires, each containing a resistance and a voltage source (electromotive force), nodes and closed figures (today: meshes), and related it to the algebraic equations of Kirchhoff's laws. In the first two theorems, Kirchhoff uses this structure to determine n independent linear equations for the network of n wires and m nodes. To do this, an equation of "voltage" sums is assigned to certain "closed figures," resulting in a total number of $\mu = n - m + 1$ independent equations. Furthermore, $m - 1$ equations of "current" sums are added.

In the next step, the common denominator of all currents is determined. For this purpose, all products of the resistances of μ wires are formed, so that after extracting these wires from the circuit, no closed figures are left in the circuit. The denominator consists of the sum of these products. In a somewhat more complicated way, the numerators of the corresponding currents are determined as products of resistances, again involving certain closed figures. Obviously, no determinants appear explicitly in the calculation process, so Kirchhoff's approach was interpreted as an alternative to the determinant calculation of the Cramer method.

6. Reception of Kirchhoff's topological approach

Although Kirchhoff's method for calculating the current distribution in resistor networks is very elegant because it circumvents the difficulties of the Cramer method and avoids the explicit calculation of determinants (Feussner, 1902), it has been very slow to be accepted by the electrical engineering community. Thus, the results of Kirchhoff's work from 1847 were not mentioned in the monographs by Wiedemann and Schellen. The Swiss physicist De la Rive was probably the first to discuss at least one aspect of Kirchhoff's results from 1847. In his paper published in French, De la Rive was interested in "On the number of independent equations in the solution of a system of linear currents" (De la Rive, 1863).

While Kirchhoff's laws had already found their way into the electrical engineering literature, it took almost another 30 years until the mathematician Wilhelm Ahrens (1872–1927) reconsidered Kirchhoff's work from 1847. Ahrens, who later became interested in mathematical games, was well trained in topology and was also familiar with Listing's study mentioned above. Therefore, Ahrens reformulated Kirchhoff's results in the context of topology and proved Kirchhoff's theorems on this basis. Although the conceptual basis of Kirchhoff's results was mathematically clarified by Ahrens and related to earlier results of Leonhard Euler (1707–1783), this did not represent any progress for practical calculations. Maybe this was also the reason because Anton Wassmuth (1844–1927) did not mention Ahrens's paper. In his work "On linear current branches" (Wassmuth, 1896) from 1896, Wassmuth illustrated Kirchhoff's approach using an example and compared it with an approximation method by Maxwell (Fleming, 1885).

In his 1902 paper "On the branching of current in network-shaped conductors" (Feussner, 1902), Wilhelm Feussner (1843–1928) discussed Kirchhoff approach in much more detail than Wassmuth, discussed a connection with determinants and compared it with Maxwell's method. In his 1904 paper "On the Calculation of Current in Network-Type Conductors" (Feussner, 1904), Feussner significantly expanded Kirchhoff's approach. Finally, Kirchhoff's approach of 1847 appeared for the first time in the form presented by Feussner in a textbook, namely, in the 1905 book "The Calculation of Electrical Networks in Theory and Practice" (Herzog and Feldmann, 1905) published by the Austrian engineer Josef Herzog (1859–1915) and Dutch electrician Clarence Feldmann (1867–1941), who was a early assistant of Erasmus Kittler (1852–1929) at the TH Darmstadt.

This seemed to represent a breakthrough in this important approach of Kirchhoff's, but unfortunately this was not the case. Only occasionally did some researchers recall Kirchhoff's approach, such as Philip Franklin (Franklin, 1925), who used Veblen's graph definition (Veblen, 1922), p. 2), the Chinese electrical engineer Yu Hsiu Ku (1902–2002) (Ku, 1932) and the German mathematician and electrical engineer Cauer (Cauer, 1938), but this initially had no consequences.

In 1932, Ku was a professor of electrical engineering at Tsinghua University, Beijing, China. However, he was educated from 1923 to 1928 at MIT, Boston, USA, where he received his doctorate under the famous Vannevar Bush (1890–1974). He then returned to China and worked at various universities before leaving the country in 1949 after the communists took power there. At MIT, Ku wrote his review article "Résumé of Maxwell's and Kirchhoff's Rules for Network Analysis" in 1952 (Ku, 1952), which was probably a starting point of a whole wave of work in the 1950s dealing with Kirchhoff's approach and the related approach of Maxwell. For an overview of the literature of the 1950s and 1960s, see Ku's 1965 review (Ku and Bedrosian, 1965) on circuit theory and synthesis, especially Louis Weinberg's (1919–2012) paper "Kirchhoff's 'Third and Fourth Laws'" (Weinberg, 1958), and Hans Edelmann (1918–1982) (Edelmann, 1959) on the analysis of power grids.

Although Kirchhoff's approach to circuit analysis was discussed and partially realized with the advent of digital computers in the early 1960s, it gradually disappeared from the literature and textbooks on circuit theory and is rarely cited today (Kirby *et al.*, 2017). Therefore, as before 1952, Kirchhoff's approach is unknown to most electrical engineering students and engineers.

7. Physical justification of Ohm's law

In Kirchhoff's first two works, a mathematical framework was developed based on Ohm's principles for linear resistance networks ("galvanic chains"), which could be used to efficiently analyze such networks. However, these were not Kirchhoff's only achievements

in this field of research, as he also succeeded in solving a conceptual difficulty of Ohm's theory that had been known from the beginning in 1826 / 27. Kirchhoff informed his former supervisor Neumann in a letter from October 13, 1848 that Ohm's theory is not compatible with the well-known electrostatics. Kirchhoff added that he had "thought of a way of deriving these formulas [Ohm's law] which seems to me to be preferable to Ohm's" (Hübner, 2010), pp. 68–69).

In fact, the details of Ohm's physical explanation of current flow in an electrical resistance network are quite complicated because Ohm changed the denotation for what is now known as "voltage" several times. The reader is left to detailed studies of Thomas Archibald (Archibald, 1988) and more recently the dissertation of Charlotte Elizabeth Connelly (Connelly, 2022). In short, Ohm developed his theory of electric currents in resistance networks using some elements of theory of heat of Joseph Fourier (1768–1830), although Ohm did not mention Fourier until his 1827 monograph "The Galvanic Circuit investigated Mathematically" (Ohm, 1827; Connelly, 2022), p. 122). Ohm proposed that the cause of electrical current was what he called "exciting force" – later on he denoted it "electroscopic force." Archibald pointed out that "Ohm saw electric current [...] as a changing spatial distribution of electroscopic force produced by the attempt of the system of conductors to reach static equilibrium" and proceeded "Ohm, however, interpreted his electroscopic force as though it were what we now term a charge density" (Archibald, 1988), pp. 146–147). This leads to a contradiction, which Kirchhoff made clear in the introduction to his 1849 paper (Kirchhoff, 1849). According to Kirchhoff, this contradiction consists in the fact that Ohm, in contrast to electrostatics, assumes "that the electricity in a conductor is at rest when it fills the volume of the conductor with uniform density." Then, Kirchhoff proceeded: "If electricity is imparted to a conductor, it will be in equilibrium when the forces exerted by the free electricity on a particle of electricity located at any point inside the conductor cancel each other out [...] The calculation shows that this can only be the case if the free electricity has arranged itself in a certain way on the surface of the conductor." Kirchhoff now introduced the potential of the total free electricity with respect to any point on the conductor, assumed stationarity and arrived "at the same equations for the potential of free electricity that result from Ohm's concept of the electroscopic force, i.e. the density of electricity." However, he came to completely different conclusions regarding the distribution of free electricity: "According to Ohm, the value of u [here: electroscopic force] at every point in the system directly indicates the density of electricity, not so with the view developed here; from this it follows that even in the closed chain [closed circuit], free electricity can only be found on the surface of the conductor." The last statement follows from the fact that u in the sense of an electric potential must satisfy Laplace's differential equations. Finally, Kirchhoff pointed out that he used the law of electrostatics for his derivation but he mentioned that Weber had found a more general law. Therefore, he formulated the task "to derive the laws of electrical current in the closed circuit from Weber's laws."

In summary, it can be stated with Hübner (Hübner, 2010), p. 69) that Ohm's law in its current interpretation basically goes back to Kirchhoff. Unfortunately, this essential fact was forgotten for a long time until Archibald presented a detailed study on the history of Ohm's law in 1988 (Archibald, 1988). Most recently, in her 2022 dissertation, Connelly presented further insights into the historical development of Ohm's Law (Connelly, 2022).

8. Motion of electricity

Kirchhoff pointed out in his paper as early as 1849 that it would be interesting to study the laws of electric current on the basis of Weber's law of action at a distance. In 1857, he published two papers in which he investigated the movement of electricity in wires and

conductors (Kirchhoff, 1857a; Kirchhoff, 1857b). While Kirchhoff's first article was published in English in the Philosophical Magazine (Kirchhoff, 1857c), in the same year, the second article did not appear in English translation until 1994 (Graneau and Assis, 1994).

Based on experiments on the motion of electricity by Weber and Rudolf Kohlrausch (1809–1858) (Wiederkehr, 2004), the father of the famous physicists Friedrich Wilhelm Kohlrausch (1840–1910) and Wilhelm Kohlrausch (1855–1936), Weber postulated a new force law that generalized Coulomb's law:

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{\vec{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r}{c^2} \frac{\ddot{r}}{r} \right),$$

where c is the light velocity. This law contains both electromagnetic force laws, i.e. the Coulomb force, and Ampère's law, and Weber was also able to derive the law of induction (Weber, 1846; Härtel, 2020). Furthermore, Weber's force law can be derived from Maxwell's equations (Kinzer and Fukai, 1996).

As Wiederkehr (Wiederkehr, 2004) emphasized, Kirchhoff's articles on the motion of electricity are among the most important articles using Weber's electrodynamics, which is based on Weber's force law. Four years before Maxwell (1865) paper published the first version of his concept of electrodynamics, Kirchhoff showed that the speed of propagation of wire waves in very thin wires is almost identical to the speed of light. This contradicts the general assumption that the propagation of electrical signals can only be explained by free energy moving through space, as Graneau and Assis pointed out, who continued by stating: "Moreover, this velocity would be independent of the nature of conductors, of the cross-section of the wire and of the density of free electricity" (Graneau and Assis, 1994).

To get an idea of how Kirchhoff came up with an equation describing the movement of electricity, let us briefly comment on his work on the movement of electricity in conductors, following (Graneau and Assis, 1994; Assis, 1994). First, Kirchhoff generalized Ohm's law by taking a first part resulting from the electrostatic potential and adding a second part based on Weber's law of induction. Then, Kirchhoff derived two partial differential equations which can be used to analyze the current in closed conductors. Today these equations correspond the telegraph equations (Wiederkehr, 2004). When solving these equations for a special geometry of the conductor, he showed in his first paper (Kirchhoff, 1857a) that by reducing the resistance of the conductor "the electricity in the wire propagates in a similar way to a wave in a taut string, namely with the speed that light has in empty space." In his second work (Kirchhoff, 1857b), he investigated the case where the resistance increases and obtained an equation which is "of the same form as that which determines the propagation of the conducted heat. In the case under consideration, the electricity propagates in a similar way to the conducted heat." Finally, Kirchhoff discussed a paper by William Thomson, i.e. Lord Kelvin (1824–1907), from 1855 (Thomson, 1855). In it, Thomson discussed the propagation of electricity in an underwater telegraph wire using the heat equation, i.e. neglecting inductive effects. Using parameters of telegraph wires, Kirchhoff confirmed that his case of large resistance is comparable to Thomson's results.

It should be noted that in his second paper, Kirchhoff also mentioned the mechanical effects of the discharge current of a Leyden jar. In a paper of 1864 (Kirchhoff, 1864), Kirchhoff showed, based on Weber's electrodynamics, that the oscillation of the discharge can be calculated using Thomson's famous oscillation formula (Wiederkehr, 2004). In addition, Kirchhoff derived the results of Thomson's (1855) paper in 1877 (Kirchhoff, 1877) based on more general principles.

In 1865, Maxwell presented the final version of his field-theoretical electrodynamics based on the principle of locality including “Maxwell’s equations,” and Weber’s electrodynamics, which was based on the action-at-the-distance principle, gradually lost its supporters. For the same reason, Kirchhoff’s results, which were based on Weber’s theory, are unknown to the vast majority of physicists and electrical engineers, with the exception of a few rare publications (Assis, 1994; Graneau and Assis, 1994).

9. Conclusions

Gustav Robert Kirchhoff was one of the most important German physicists of the 19th century and, due to his approach to scientific research, also became one of the founders of theoretical physics. Kirchhoff’s importance can also be seen in the fact that his scientific work contains a considerable number of fundamental contributions to physics. Although Kirchhoff was mainly interested in contributions to physical topics, some of his results also achieved enormous importance in technology. It follows that most physicists emphasize the discovery of spectral analysis (together with Robert Wilhelm Bunsen), which became extremely important in many eras of physics, as well as Kirchhoff’s law of thermal radiation, which later became the starting point of quantum theory. On the other hand, electrical engineers emphasize that Kirchhoff developed the mathematical foundations of circuit theory, which became an essential basis of the electronic age in the 20th century, and in particular of circuit simulation and circuit design. However, Kirchhoff’s contributions to the mathematical and physical understanding of electrical circuits are often not presented in full detail, which motivated us to close this gap in this article.

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Further reading

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