

Improved equivalent circuit representation of air-core transformer for higher-frequency operation

Milena Kurzawa, Rafał M. Wojciechowski and Paweł Idziak
*Institute of Electrical Engineering and Electronics,
Poznan University of Technology, Poznan, Poland*

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Abstract

Purpose – The purpose of this work is to develop an improved equivalent circuit model for air-core transformers operating at higher frequencies. By refining the classical Cauer-based representation, this study aims to achieve more accurate impedance and current behavior over a wide frequency range, with particular emphasis on improving low-frequency performance.

Design/methodology/approach – This research uses a full field-circuit modeling approach. Initially, a finite element method (FEM) of coupled coils is formulated to capture electromagnetic interactions. These complex matrix equations are then reduced using the Padé via Lanczos (PvL) method, which accurately approximates the frequency response while significantly lowering the computational order. Based on the reduced model, impedance characteristics are determined for both magnetizing and horizontal branches. A modified circuit structure is then proposed and rigorously validated by comparing its performance – specifically impedance characteristics and current waveforms – against field-model simulations under various load conditions.

Findings – The analysis demonstrates that the classical equivalent circuit fails to accurately reproduce the impedance behavior of the magnetizing branch at lower frequencies. The proposed modified structure rectifies this, achieving significantly better agreement with results of field-model. Simulation results confirm that the proposed equivalent circuit in the more accurately replicates both the amplitude and phase of load currents, eliminating the discrepancies found in the classical model.

Originality/value – This study identifies a limitation of the classical Cauer-based representation of the magnetizing branch in air-core transformers, related to its improper low-frequency behavior. It is shown that the traditional structure does not satisfy the zero-pulsation boundary condition, leading to a non-zero remainder in the PvL approximation and reduced modeling accuracy at low frequencies. To overcome this issue, a physically consistent modification is proposed by introducing an additional resistance R_0 into the magnetizing branch. By combining field-based parameter extraction with PvL reduction, the method preserves computational efficiency while significantly improving low-frequency accuracy, providing a reliable tool for wide-frequency analysis of higher-frequency coupled systems.

Keywords Equivalent model, Cauer circuits, Equivalent circuits, Air-core coil, Magnetic systems

Paper type Research paper

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1. Introduction

The modern analysis and design of magnetic circuits increasingly demand highly accurate mathematical representations that effectively capture the physical properties of real-world systems. While maintaining a high level of modeling fidelity is essential, the evaluation of these systems’ operational states must also remain computationally efficient. Consequently, there is a persistent search for analysis and synthesis methods that ensure rapid convergence and minimal computational overhead. Simultaneously, the ongoing advancement of electronic technologies has enabled the development of systems operating with greater efficiency at progressively higher frequencies. This trend necessitates modeling approaches capable of capturing circuit behavior across wide frequency ranges.

Magnetic circuits incorporating air core coils – commonly referred to as air-core transformers [Figure 1(a)] are widely used in various higher frequency applications, such as wireless power transfer (WPT) systems (Kurzawa *et al.*, 2020; Mulders *et al.*, 2022). In practical engineering, the precise design and optimization of such resonant systems heavily rely on accurate wideband modeling. Even minor phase discrepancies in the equivalent circuit can lead to significant detuning of the resonant converters, resulting in reduced energy transfer efficiency and increased thermal losses. Due to the complex electromagnetic phenomena that govern the operation of such systems, their analysis is ideally conducted using full field-based models (Demenko *et al.*, 2014; Kurzawa, 2023). However, given the substantial computational cost associated with these models, designers often opt for simplified circuit-based or circuit-field models.

In this work, authors propose an analysis of a magnetically coupled coil system using a circuit-field model. In this approach, values of the lumped parameters of circuit are calculated using the field model and subsequently implemented in dedicated software. In the authors’ software, the Pade via Lanczos (PvL) method was used (Kurzawa, 2022), enabling the synthesis of an equivalent circuit based on Cauer circuits. It is worth noting that, in the most basic modeling approach, magnetically coupled coil systems may be represented using a classical equivalent circuit of an air-core transformer, as illustrated in Figure 1.

In many circuit analyses, it is sufficient to use a classic equivalent model. However, when circuit calculations are performed for different values of the supply frequency, it is recommended to use Foster or Cauer circuits. The theoretical foundation for the Cauer Ladder Network (CLN) synthesis traces back to classical network theory (Cauer, 1958), and its modern implementations have been proven highly effective for electromagnetic field modeling (e.g. Kameari *et al.*, 2018; Shindo *et al.*, 2020). The application of such equivalent circuits enables the analysis of the system without the need for multiple calculations of lumped parameters for different frequency values, which speeds up the analysis process of the investigated system. An example of an equivalent circuit for a magnetically coupled coil system using Cauer circuits is shown in Figure 2.

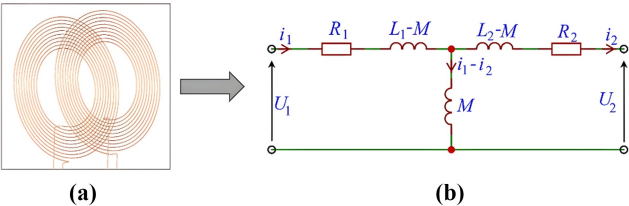


Figure 1. Air-core transformer (a) view and (b) classical equivalent circuit

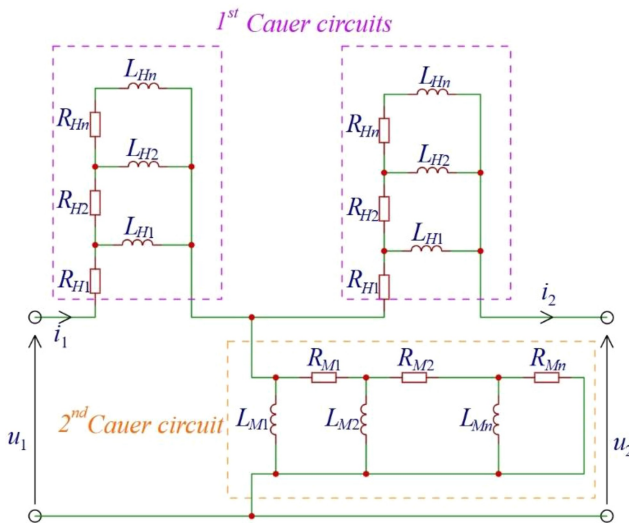


Figure 2. Equivalent circuit of an air-core transformer using Cauer circuits of the first- and second order

This approach can be found in many literature sources, for example [Otomo *et al.* \(2018\)](#) and [Kurzawa *et al.* \(2020\)](#). It allows for the analysis of the system over a wide range of frequency variations. In constructing the equivalent circuit of an air-core transformer, many authors use the classical approach (see [Figure 2](#)).

The parameters of Cauer circuits can be determined using several techniques, including fitting methods ([Belahcel *et al.*, 2015](#); [Shimotani *et al.*, 2016a](#); [Wojciechowski *et al.*, 2020](#); [Kurzawa *et al.*, 2025](#)) as well as Model Order Reduction (MOR) approaches ([Shimotani *et al.*, 2016b](#); [Sato *et al.*, 2017](#); [Kurzawa, 2023](#)). MOR methods reduce the original full-order system to a simplified model with fewer state variables while preserving the essential dynamic characteristics of the system. This substantially decreases computational complexity and improves the efficiency of frequency-response analysis. MOR techniques commonly used for determining Cauer network parameters include PvL method or Proper Orthogonal Decomposition (POD) method. In practice, classical fitting methods are also widely applied, which enable accurate reconstruction of frequency characteristics ([Shimotani *et al.*, 2016a](#)).

In this work, the PvL method was used due to its high accuracy and efficiency in modeling systems with nonlinear frequency characteristics.

2. Methodology for determining parameters of Cauer circuits

The procedure for determining the equivalent parameters of Cauer circuits using the Padé via Lanczos method is presented in [Figure 3](#). The first step involves formulating the finite element method (FEM) equations that describe the analyzed system. In the work, the system under consideration is an air-core transformer consisting of two magnetically coupled coils. Next, by applying the PvL algorithm, the transmittance of the system and the parameters of the Cauer circuits are obtained.

To determine the equivalent parameters of the Cauer circuits for the analyzed transformer, the PvL method was used. This technique enables the reduction of the order of the matrix equations obtained from the field model, allowing for an accurate representation of the

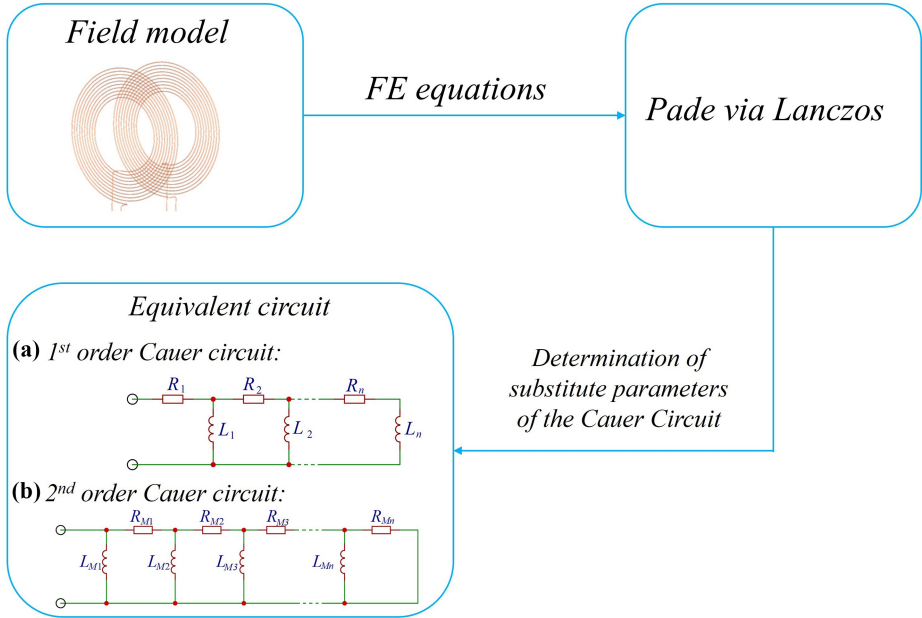


Figure 3. Procedure for formulating Cauer equivalent circuits using the PvL method

system's frequency response. The parameter identification process begins with formulating the FEM equations that describe the electromagnetic coupling phenomena occurring within the coils of the air-core transformer. The field equations were obtained using in-house software (Kurzawa and Wojciechowski, 2022; Kurzawa, 2023; Tuck-Lee *et al.*, 2008; Feldmann and Freund, 1995; Sato and Igarashi, 2013). For the analyzed system, the field equations can be expressed in the general matrix form:

$$(\mathbf{A} + j\omega\mathbf{B})\underline{\mathbf{x}} = \mathbf{b}\underline{\mathbf{U}}, \quad (1a)$$

$$\underline{\mathbf{Y}} = \mathbf{l}^T \underline{\mathbf{x}}, \quad (1b)$$

where: $\mathbf{l}$ and $\mathbf{b}$ are unit vectors, $\underline{\mathbf{U}}$ is complex vector representing the input and output voltages of the analyzed system, $\mathbf{A}$ represents the stiffness matrix of FEM equations, whereas $\mathbf{B}$ is the damping matrix accounting for eddy currents and conductivity. The complex vector $\underline{\mathbf{x}}$ represents the sought magnetic fluxes and currents, and vector $\underline{\mathbf{Y}}$ represents the system response (Kurzawa, 2023).

Equation (1) are transformed into its operator form using the Laplace transform. Then, the expansion point s_0 is defined as $s_0 = 2\pi f_{max}$, where f_{max} denotes the maximum frequency considered in the analysis. The choice of the maximum frequency as the expansion point was strictly deliberate, as it ensures the highest numerical stability and accuracy in the upper frequency band, where skin and proximity effects are most pronounced. By defining the parameter σ (where $\sigma = s - s_0$ representing the neighborhood around the point), the system transmittance $H(s)$ is obtained in the following form:

$$H(s_0 + \sigma) = \mathbf{l}^T (\mathbb{I} + \sigma \mathbf{\Lambda})^{-1} \mathbf{r}, \quad (2)$$

where: $\mathbf{\Lambda} = \mathbf{B}/(\mathbf{A} + s_0 \mathbf{B})^{-1}$, $\mathbf{r} = (\mathbf{A} + s_0 \mathbf{B})^{-1} \mathbf{b}$ and $\mathbb{I}$ is the identity matrix.

Since the direct diagonalization of the full matrix $\mathbf{\Lambda}$ is computationally expensive for large number of variables of FEM equations, the PvL approach is used. This powerful MOR technique, originally introduced for linear circuit analysis by [Feldmann and Freund \(1995\)](#), combines Padé approximation with the Lanczos process, enabling the replacement of the large $N \times N$ matrix $\mathbf{\Lambda}$ with a tridiagonal Lanczos matrix $\mathbf{T}_q$ of significantly lower order qxq , where $q \ll N$. In this study, a modified Lanczos algorithm proposed in ([Sato and Igarashi, 2016](#)) was implemented by the authors for computing the matrix $\mathbf{T}_q$. After determining the Lanczos matrix according to $\mathbf{T}_q = \mathbf{S}_q \mathbf{\Lambda}_q \mathbf{S}_q^T$ the system transmittance can be expressed as the following function:

$$H_q(s_0 + \sigma) = \underbrace{\mathbf{l}^T \mathbf{e}^T \mathbf{S}_q}_{\boldsymbol{\mu}^T} (\mathbb{I} + \sigma \mathbf{\Lambda}_q)^{-1} \underbrace{\mathbf{S}_q^{-1} \mathbf{e} \cdot \mathbf{r}}_{\mathbf{v}} = \sum_{j=1}^q \frac{\mu_j \nu_j}{1 + \sigma \lambda_j}, \quad (3)$$

where: $\mathbf{e} = [1, 0, 0, \dots, 0]^T \in \mathbb{R}^q$; $\mathbf{\Lambda}_q = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_q\}$ contains the q eigenvalues of the matrix $\mathbf{T}_q$, while the values μ_j and ν_j are components of the vectors $\boldsymbol{\mu}$ and $\mathbf{v}$; the matrix $\mathbf{S}_q$ represents the eigenvectors of the Lanczos matrix $\mathbf{T}_q$. The function $H_q(s_0 + \sigma)$ obtained above is a function of the variable s in the Laplace domain.

To convert the operator-form expression (3) into its frequency-domain representation, the inverse substitution $s = (s_0 + \sigma) = j\omega$, is applied, yielding:

$$\underline{H}_q(\omega) = \sum_{i=1}^q \frac{1}{\frac{1}{\mu_i \nu_i} + j\omega \frac{\lambda_i}{\mu_i \nu_i}}. \quad (4)$$

Although the tridiagonal Lanczos matrix $\mathbf{T}_q$ can be directly expanded into a continued fraction to directly yield Caue network parameters, our numerical workflow uses the pole-residue (partial fraction) representation derived from the eigen-decomposition of $\mathbf{T}_q$. This form allows us to efficiently evaluate the system's broadband frequency response in the form of impedance $\underline{Z}(\omega)$ which is subsequently used to synthesize the multi-branch Caue parameters via our network-fitting routine. The parameters of the Caue circuit are determined based on this known impedance of the system, expressed by the following relationship:

$$\underline{H}(\omega) = \frac{1}{\underline{H}_q(\omega)} = \frac{1}{\sum_{i=1}^q \frac{1}{\frac{1}{\mu_i \nu_i} + j\omega \frac{\lambda_i}{\mu_i \nu_i}}}. \quad (5)$$

In the process of estimating the lumped parameters, the authors adopted a classical approach based on the analysis of two operating states of the air-core transformer, namely: (a) the short-circuit state and (b) the no-load state.

For the no-load condition, the input impedance $\underline{Z}_0(\omega)$ was determined using [equation \(5\)](#), under the constraint of zero current in the secondary winding, i.e. $i_2 = 0$ (see [Figure 2](#)):

$$Z_0(\omega) = \frac{1}{\underline{H}_q(\omega)} \Big|_{i_2=0}. \quad (6)$$

In an analogous manner, the short-circuit impedance was determined by applying the condition $u_2 = 0$, i.e.:

$$Z_z(\omega) = \frac{1}{\underline{H}_q(\omega)} \Big|_{u_2=0}. \quad (7)$$

The obtained functions $Z_0(\omega)$ and $Z_z(\omega)$ were generated by running the FEM-PvL reduction procedure twice under these two distinct boundary conditions. Subsequently, they were used to determine the parameters of the individual branches of the equivalent circuit. Assuming a perfectly symmetric air-core transformer, the short-circuit impedance is equally divided between the primary and secondary horizontal branches. Therefore, the impedance of the horizontal branches, denoted as $\underline{Z}_H$, was determined using the following relationship:

$$\underline{Z}_H(\omega) = 0.5 \cdot \underline{Z}_z(\omega). \quad (8)$$

While the parameters of the magnetizing branch, denoted as $\underline{Z}_M$, were determined from:

$$\underline{Z}_M(\omega) = \{ \underline{Z}_0(\omega) - 0.5 \cdot \underline{Z}_z(\omega) \}. \quad (9)$$

3. Improved equivalent circuits

The classical approach described in (Otomo *et al.*, 2018; Kurzawa and Wojciechowski, 2022) and shown in Figure 2 allows the system to be analyzed over a wide range of frequency variations. In this method, the vertical branch includes the magnetizing inductance, noted as L_{M1} , which plays the key role. The remaining parameters of this branch represent the dynamic behavior of the system.

However, the authors noted that the system shown in Figure 2 exhibits a specific drawback. Upon examining the applied Cauer circuit representing the magnetizing branch, it should be noted that the impedance (ω) in the pulsation domain does not satisfy the following condition, where:

$$\lim_{\omega \rightarrow 0} Z_M(\omega) = \text{const} > 0, \quad (10)$$

which is revealed when representing systems operating at lower pulsations. Therefore, authors have proposed a modification of the equivalent circuit from Figure 2.

The paper considers a system consists of two magnetically coupled air core coils (Kurzawa, 2023) [Figure 1(a)]. To determine the parameters of the equivalent circuit, a classical approach was used, in which the parameters were determined from the open-circuit test and test of short-circuit state. The parameters of the horizontal branch were defined based on the short-circuit test, while those of the magnetizing branch were solved using both the open-circuit and short-circuit tests – that is, the impedance values $Z_0(\omega)$ for the open-circuit condition (no-load state) and $Z_z(\omega)$ for the short-circuit condition. Finally, the parameters of the horizontal branch were calculated using the relations:

$$R(\omega) = \text{Re} \{ 0.5 \cdot \underline{Z}_z(\omega) \}, \quad (11)$$

$$L(\omega) = \frac{\text{Im}\{0.5 \cdot Z_z(\omega)\}}{\omega}, \quad (12)$$

whereas the parameters of the magnetizing branch were determined from:

$$R_M(\omega) = \text{Re}\{Z_M(\omega)\}, \quad (13)$$

$$L_M(\omega) = \frac{\text{Im}\{Z_M(\omega)\}}{\omega}, \quad (14)$$

where $Z_M(\omega) = \{Z_0(\omega) - 0.5 \cdot Z_z(\omega)\}$ and its graphical form can be presented in the following figure.

Analyzing the function (ω) from Figure 4 across the entire pulsation spectrum, the authors observed that the use of the circuit shown in Figure 2 leads to discrepancies. It should be noted that when a second-order Cauer circuit is used to represent the magnetizing branch, the resulting circuit does not satisfy condition (10), as the expression describing Z_M for this circuit is equal to zero, whereas it should equal a constant value. From a theoretical circuit-modeling standpoint, this non-zero limit at $\omega \rightarrow 0$ represents the static (DC) resistive properties of the system. In the fundamental derivation of the T-equivalent circuit, the magnetizing branch impedance is formulated as $Z_M(\omega) = Z_0(\omega) - 0.5 \cdot Z_z(\omega)$. Consequently, at extremely low frequencies, the static DC resistance of the physical windings is algebraically distributed among both the horizontal branches and the magnetizing branch. The classical Cauer topology for the magnetizing branch (Figure 2) begins with a parallel inductor L_{M1} . At DC conditions, this inductance acts as a short circuit, artificially forcing the equivalent DC resistance of this branch to zero and thus violating the fundamental physical boundary condition of the static

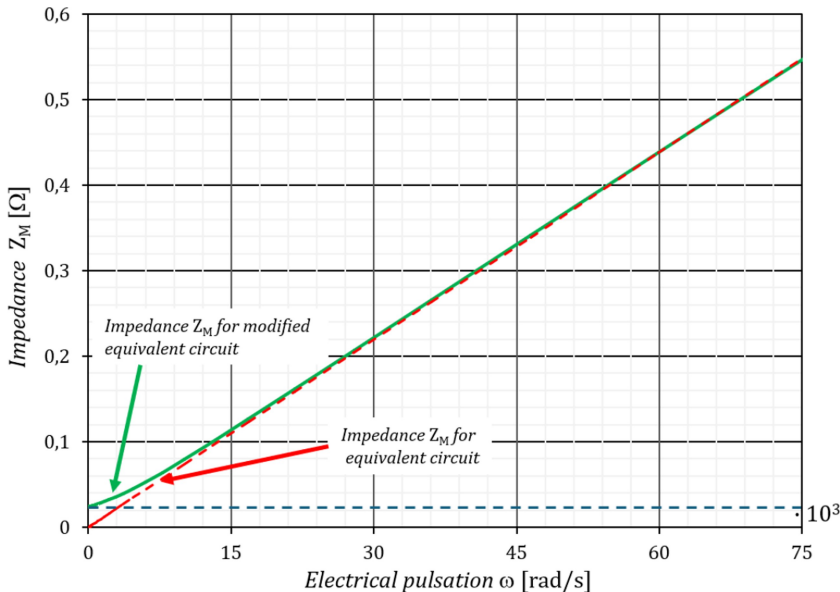


Figure 4. Dependencies of the magnetizing branch impedance as a function of pulsation ω for the considered example

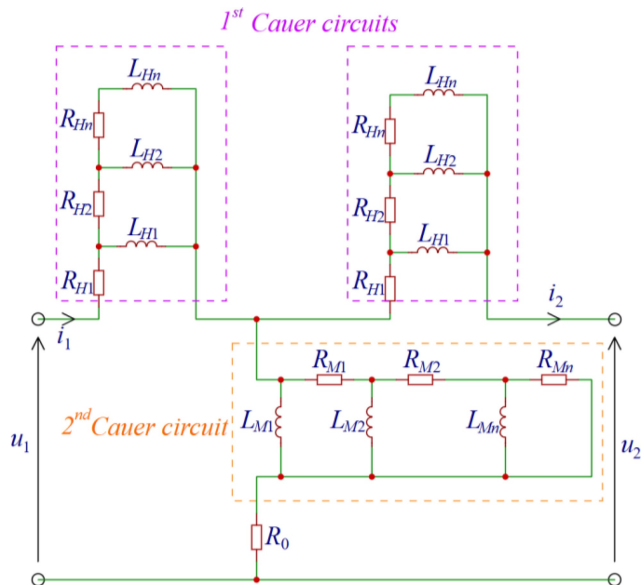


Figure 5. Modified equivalent circuit of an air transformer using Cauer circuits with additional resistance R_0

resistance distribution in the T-model. This theoretical limitation is strictly identified by the PvL algorithm, which yields a nonzero remainder (*a constant value*) from the division of the polynomials describing the function (ω). Therefore, this remainder is not merely a numerical artifact of the approximation process, but a direct physical reflection of the static resistive component. Based on this solid theoretical observation, the authors propose a physically consistent modification to the circuit by explicitly introducing this boundary parameter as an additional resistance R_0 , resulting in the improved equivalent circuit shown in [Figure 5](#).

4. Results

This section presents selected simulation results for the system of magnetically coupled coils analyzed in this work. [Table 1](#) summarizes the resistance and inductance values obtained from the proposed circuit–field model using the PvL synthesis procedure described in Section 2. The listed parameters correspond to both the horizontal branch and

Table 1. Synthesized resistance and inductance values for the horizontal (RHn, LHn) and magnetizing (RMn, LMn) branches for consecutive approximation orders n , including the proposed modification parameter R_0 (at $n = 0$)

n	$R_{Hn} [\Omega]$	$L_{Hn} [\mu H]$	$R_{Mn} [\Omega]$	$L_{Mn} [\mu H]$
1	0.025	9.711	0.039	10.137
2	0.036	4.934	29.666	26.243
3	4.651	15.691	197.649	203.179
4	24.707	34.415	207.628	580.461
5	101.658	69.194	175.147	938.043
6	321.003	171.006	13.193	116.625
0 (R_0)	–	–	0.025	–

the magnetizing branch of the equivalent transformer circuit, including the additional resistance R_0 (represented by row $n=0$). The parameter n denotes the order of approximation, which corresponds to the number of consecutive RL branches in the Cauer circuits. Each subsequent line ($n=1$ to 6) represents additional lumped parameters calculated to capture frequency-dependent phenomena, such as skin and proximity effects, with increasing precision.

As detailed in Table 1, the higher-order parameters ($n=1$ to 6) capture the high-frequency dynamic behaviors of the transformer, such as skin and proximity effects, which become increasingly dominant at higher pulsations. Conversely, the proposed modification parameter R_0 ($n=0$) strictly governs the static DC characteristics of the magnetizing branch. This clear separation of frequency-dependent phenomena confirms the capability of the PvL synthesis to accurately map complex electromagnetic interactions into distinct lumped elements.

Based on the equivalent parameters obtained for the analyzed transformer, the impedance fitting functions $Z_M(\omega)$ for the magnetizing branch and $Z_H(\omega)$ for the horizontal branch were determined as functions of the electrical pulsation ω , using equations (8) and (9). The impedance characteristics $Z_M(\omega)$ and $Z_H(\omega)$ are presented in Figures 6 and 7, respectively.

As can be observed from the presented characteristics, the impedance of both the horizontal and magnetizing branches remains non-zero even at low ripple values. This confirms that the condition defined by equation (10) is satisfied. Moreover, the monotonic increase in impedance with pulsation indicates consistent inductive behavior in both branches, validating the correctness of the proposed equivalent circuit representation and demonstrating good agreement between the fitted impedance functions and the physical characteristics of the transformer.

In this work, the current waveforms at the load of the system with $R_{load} = 10 \Omega$ were also analyzed for various values of the supply frequency. Simulations were performed for both the classical equivalent circuit using Cauer networks and the modified equivalent circuit proposed in this study, also based on Cauer networks (see Figure 8).

The peak values of the resulting current waveforms and the phase shift between them were compared. Figures 9–11 show a comparison of the load current waveforms for

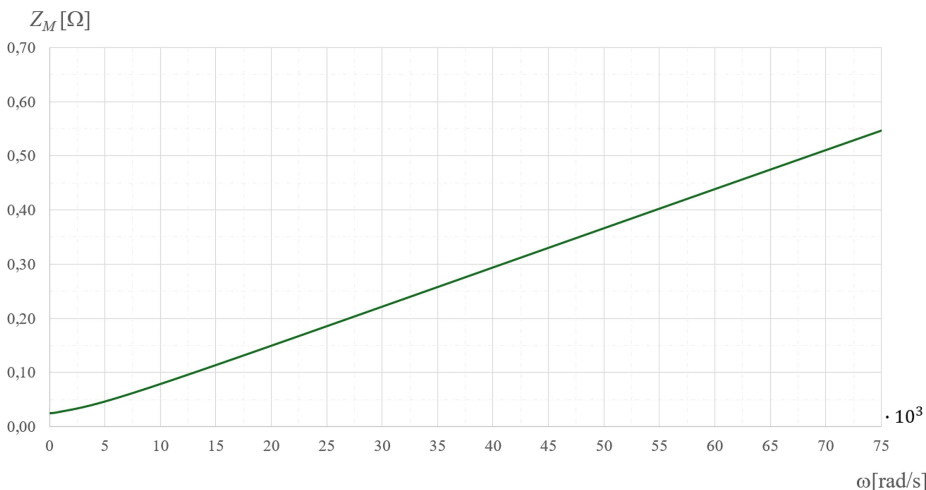
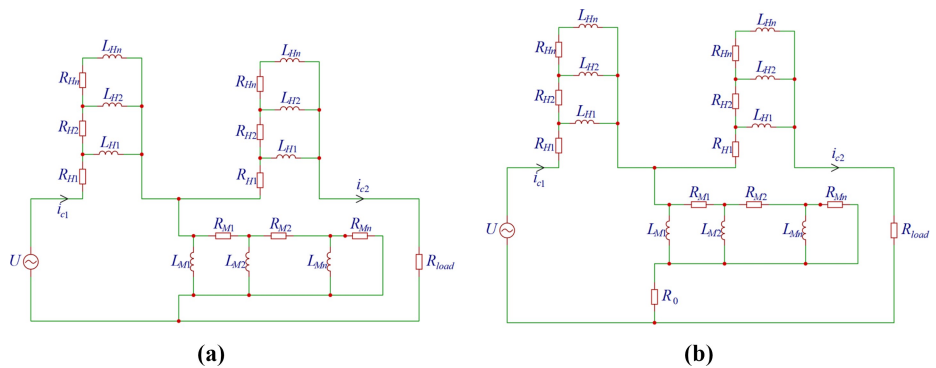
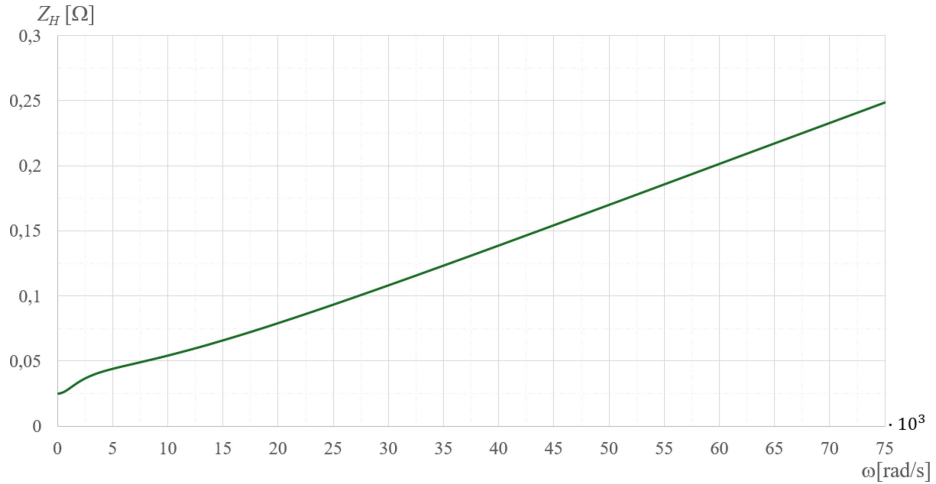


Figure 6. Impedance characteristics as a function of the electrical pulsation for the magnetizing branch



While [Figures 9–11](#) provide an intuitive, time-domain visualization of the phase-shift discrepancies at specific operating frequencies (typical for inverter-fed systems), a comprehensive wideband evaluation of the model is necessary to fully capture its dynamic behavior. To precisely determine the range of applicability and assess the numerical stability of the proposed model across the entire frequency spectrum, a comparative analysis of current amplitude and phase characteristics was performed. [Figure 12](#) presents the secondary current amplitudes, while [Figure 13](#) illustrates the phase shift φ discrepancy between the waveforms obtained using the classical approach and those resulting from the improved model. The analysis of [Figure 12](#) indicates significant discrepancies in current amplitude

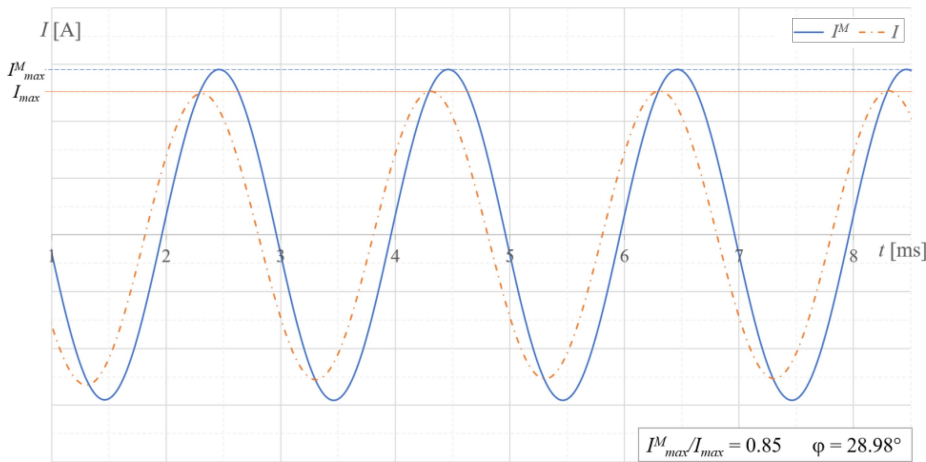


Figure 9. Comparison of current waveforms I for the classic circuit and the modified circuit at an electrical pulsation of 3.14 krad/s

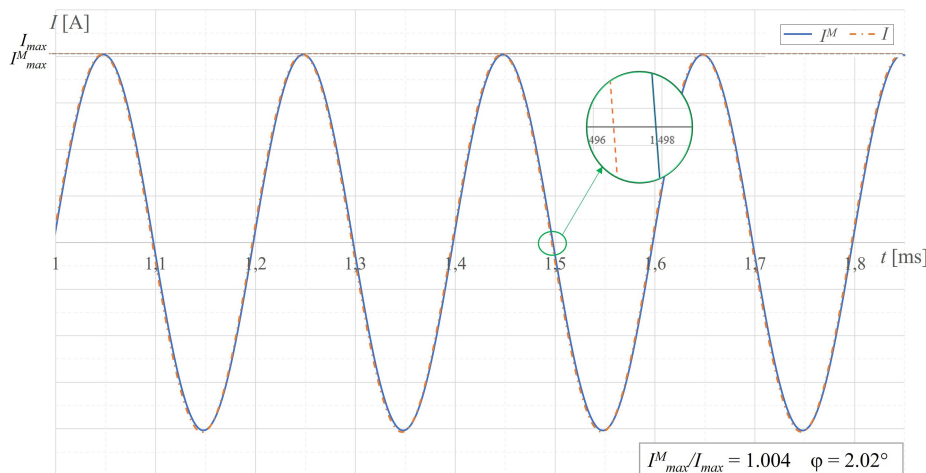


Figure 10. Comparison of current waveforms I for the classic circuit and the modified circuit at an electrical pulsation of 31.4 krad/s

determination within the low pulsation range, which is a direct consequence of the classical approach's limitations in representing the DC component at low frequencies. Convergence of the current amplitudes for both considered circuit structures occurs only at an angular frequency of $\omega = 35$ krad/s (corresponding to approximately 5.5 kHz). Above this value, both models exhibit acceptable agreement, confirming the validity of the network synthesis for higher operating electrical pulsation. A decisive argument for using the modified structure with an additional resistance R_0 is further provided by the phase discrepancy analysis presented in Figure 13. In wireless power transfer systems and impulse systems,

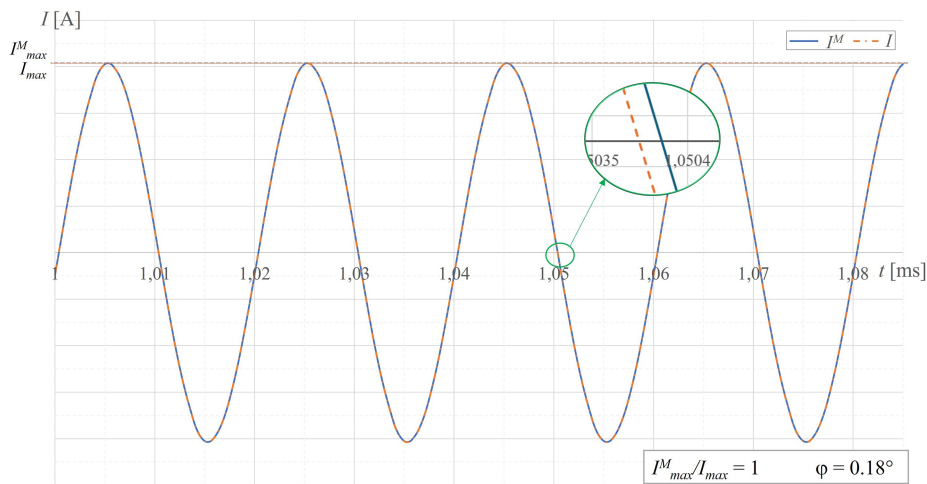


Figure 11. Comparison of current waveforms I for the classic circuit and the modified circuit at an electrical pulsation of 314 krad/s

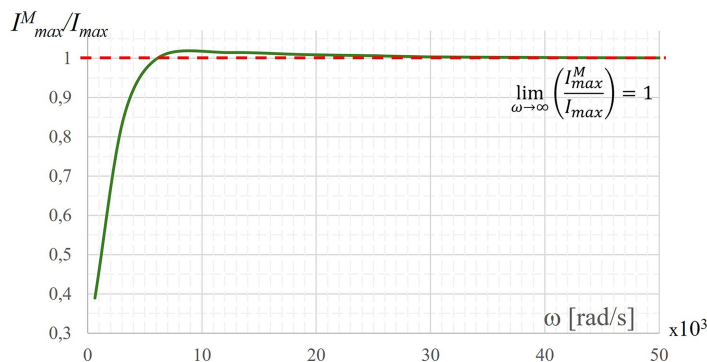


Figure 12. Comparison of secondary current amplitudes calculated using the classical and improved equivalent circuit models as a function of pulsation ω

correct representation of the phase shift is crucial for the precise determination of power losses and resonant parameters. As demonstrated, for a pulsation of $\omega = 50$ krad/s (approximately 10 kHz), the application of the model based on the classical approach generates a phase error of 0.83° , which disqualifies it for high-precision broadband analyses. A high level of phase representation accuracy (*discrepancy below 0.2°*) is achieved by the classical equivalent circuit model only in the high pulsation range exceeding 300 krad/s (approximately 50 kHz) – see Figure 11. These results clearly demonstrate that incorporating the R_0 parameter into the magnetizing branch of the transformer equivalent circuit is essential to ensure the physical consistency of the model in both steady-state and transient conditions at lower power supply pulsation. The proposed modification effectively

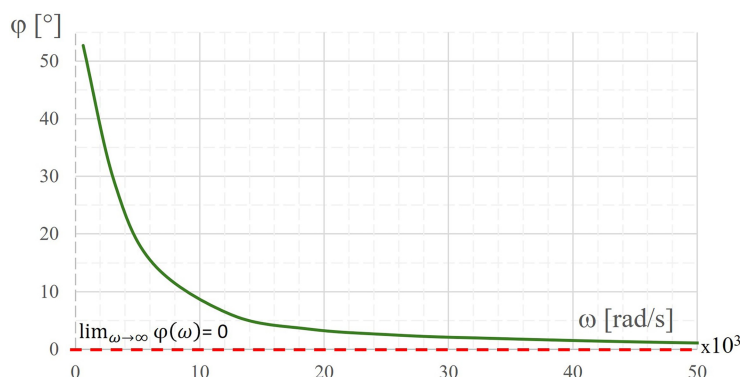


Figure 13. Phase shift ϕ discrepancy of between current waveforms obtained for the classical approach and the improved model as a function of pulsation ω

eliminates discrepancies resulting from the undefined behavior of the classical Cauer circuits as $\omega \rightarrow 0$, rendering the model a universal tool for the analysis of magnetically coupled systems operating in a wideband regime.

5. Conclusions

This study successfully proposes and validates an improved equivalent circuit for air-core transformers powered by higher frequency sources. Authors have identified a fundamental theoretical limitation in the classical Cauer-based representation of the T-equivalent model, specifically its failure to satisfy the DC boundary condition. Because the traditional structure completely short-circuits the magnetizing branch at zero frequency, it inaccurately distributes the static resistive properties of the windings, leading to significant low-frequency discrepancies. By using the PvL algorithm, authors have demonstrated that this missing static component manifests mathematically as a non-zero remainder in the model order reduction process. Authors have corrected this physical inconsistency by explicitly introducing an additional resistance R_0 into the magnetizing branch. The comprehensive wideband analysis confirms that the proposed modified structure achieves good agreement with the full field-based model. It eliminates the amplitude and phase differences present in the classical approach at lower frequencies, while seamlessly preserving computational efficiency and high-fidelity representation of eddy-current phenomena at high frequencies. Consequently, the improved model provides a highly reliable, physically consistent and universally applicable tool for the wideband analysis of magnetically coupled systems. From a practical standpoint, the proposed methodology bridges the gap between complex electromagnetic field theory and circuit engineering, enabling designers of modern resonant converters and WPT systems to significantly reduce computational overhead, accelerate the prototyping phase and ultimately develop more energy-efficient power electronics.

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Corresponding author

Milena Kurzawa can be contacted at: milena.kurzawa@put.poznan.pl

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