The evaluation of grey relative incidence

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Abstract

Purpose – With the use of the grey incidence analysis (GIA), indicators such as the absolute degree of grey incidence (ε_{ij}), relative degree of grey incidence (r_{ij}) or synthetic degree of grey incidence (ρ_{ij}) are calculated. However, it seems that some assumptions made to calculate them are arguable, which may also have a material impact on the reliability of test results. In this paper, the authors analyse one of the indicators of the GIA, namely the relative degree of grey incidence. The aim of the article was to verify the hypothesis: in determining the relative degree of grey incidence, the method of standardisation of elements in a series significantly affects the test results.

Design/methodology/approach – To achieve the purpose of the article, the authors used the numerical simulation method and the logical analysis method (in order to draw conclusions from our tests).

Findings – It turned out that the applied method of standardising elements in series when calculating the relative degree of grey incidence significantly affects the test results. Moreover, the manner of standardisation used in the original method (which involves dividing all elements by the first element) is not the best. Much more reliable results are obtained by a standardisation that involves dividing all elements by their arithmetic mean.

Research limitations/implications – Limitations of the conducted evaluation involve in particular the limited scope of inference. This is since the obtained results referred to only one of the indicators classified into the GIA.

Originality/value – In this article, the authors have evaluated the model of GIA in which the relative degree of grey incidence is determined. As a result of the research, the authors have proposed a recommendation regarding a change in the method of standardising variables, which will contribute to obtaining more reliable results in relational tests using the grey system theory.

Keywords Grey incidence analysis, Relative degree of grey incidence, Grey relation

Paper type Full length paper

1. Introduction

The Grey Incidence Analysis (GIA) is a group of models that make it possible to analyse the relation existing between two sets of variables (Liu *et al.*, 2017a, b, c). The essence of GIA models comes down to testing the geometric similarity between two data vectors. The more similar they are, the higher will be the values of the indicators covered by the GIA. The GIA is commonly used for solving problems in engineering (Kokocińska *et al.*, 2020), natural and social sciences (Nowak *et al.*, 2020). Example applications of GIA models in the recent years

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include: service quality analysis in healthcare (Javed and Liu, 2018), the Industry 4.0 research (Fahim *et al.*, 2021), green supplier selection (Quan *et al.*, 2018), project management (Javed and Liu, 2019), grain production (Zhang *et al.*, 2021), research on the effect of electrification on the level of economic development (Nowak *et al.*, 2021), research on the development of science and technology (Wei and Wu, 2016), research on sustainable development (Yi *et al.*, 2021; Kaswan and Rathi, 2021), research on social media (Weng *et al.*, 2021), research on customer satisfaction (Peng *et al.*, 2021; Xiang, 2022), business performance analysis (Škrinjarić and Šego, 2021; Ellibeş and Candan, 2021), corporate social responsibility analysis (Diaz and Nguyen, 2021), quality management (Valmohammadi *et al.*, 2021), or financial management (Ramezani, 2022). In general terms, within the relational analysis in the grey system theory, we distinguish distance-, surface-, and panel-based models (Liu *et al.*, 2017a, b, c). The most commonly used models in the GIA are the area-based models, especially those in which we calculate the absolute degree of grey incidence, relative degree of grey incidence and synthetic degree of grey incidence. The gap noticed by the authors concerns the method of calculating all the known indicators, but the scope of this article is limited to the method of calculating the relative degree of grey incidence. The procedure of calculating the relative degree of grey incidence seems to have flaws that consist in an incomprehensible arbitrariness of some assumptions. The following stages seem to be particularly arbitrary:

(1) The standardisation of time series at the first stage, which involves dividing all terms in the series by their initial values. Why do we divide them by the first term and not, for example, the last term in the series?

(2) The standardisation of time series at the second stage, which involves subtracting the series of the initial value from all terms. Why exactly the initial value?

It should also be pointed out that all the indicated stages of the procedure for determining the relative degree of grey incidence, due to their assumptions, increase the importance of the first terms in the series. If the grey incidence analysis (GIA) concerns time series, then it blatantly contradicts the axiom of the grey system theory, according to which the greatest significance should be attached to those data that are the most up-to-date (fresh).

The identified problem served as a starting point for formulating the hypothesis:

H. The method of standardising elements in a series when determining the relative degree of grey incidence significantly affects the test results.

The purpose of the article is the verification of the posed hypothesis. To achieve the purpose of the article, we used the numerical simulation method and the logical analysis method (in order to draw conclusions from our tests).

In the second section of the article, we present a literature review concerning GIA models. In the third section, we introduce the methodology used in the article. In the fourth section, we present the results of conducted studies on simulations of GIA models. In the fifth section, we present the most important conclusions and outline the area of further research to be conducted by the authors.

2. Literature review

The history of Grey Systems Theory dates back to the 1980s and originates in China. It was developed by Professor Deng Julong at Huazhong University of Science and Technology (Julong, 1982). This theory is distinct in its ability to analyse and model systems characterised by a lack of sufficient data or information. The term “grey” refers to the existence of a certain level of uncertainty or ambiguity in such systems that can be studied and analysed (Liu and Forrest, 2007). Within the framework of Grey Systems Theory, various models have been developed, with one of the most significant being GIA (Liu *et al.*, 2006). This model was created to aid in the analysis and resolution of problems in various fields where uncertainty and missing data need to be considered. GIA is used to assess relationships between different variables under such conditions. The development of this model is the result of extensive research and efforts in advancing Grey Systems Theory, solidifying its importance as a valuable tool for analysing systems with uncertain data (Liu *et al.*, 2017a, b, c).

One of the initial models of the GIA was the distance-based model proposed by D. Julong (1989). The aim of that model is to calculate the similarity between two vectors or sets of points (depending on whether the data have a temporal nature or not). Figure 1 shows the idea of the distance-based model.

Two objects represented by different colours of the points in diagram 1 will be the more similar to each other, the closer the multi-colour points will be to each other. The coefficient of similarity between two vectors (sets) of data is, therefore, calculated with the use of the selected distance metric. Deng (1989) proposed a metric in the form of a grey degree of similarity between two data vectors (1).

$$\gamma(g_{ki}, g_{ji}) = \frac{min_k min_i |g_{ki} - g_{ji}| + \xi max_k max_i |g_{ki} - g_{ji}|}{|g_{ki} - g_{ji}| + \xi max_k max_i |g_{ki} - g_{ji}|} \tag{1}$$

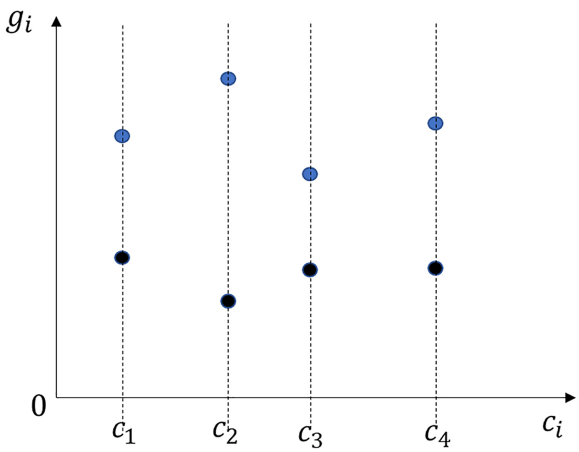
Where:

ξ – distinguishing coefficient with a value in the range of (0–1),

$\gamma(g_{ki}, g_{ji})$ – the indicator of the grey degree of similarity between two point sets g_{ki} i g_{ji},

g_{ki}, g_{ji} – two sets of points for which the similarity level is determined.

The most popular models of the grey relational analysis are the surface-based models. They allow us to calculate the coefficient of similarity between two vectors. The metric applied to determine the similarity between two vectors uses the surface formed between two vectors on a plane. The idea of surface models in the grey relational analysis is presented in Figure 2.



Source(s): Own work

Figure 1. The idea of the distance-based model

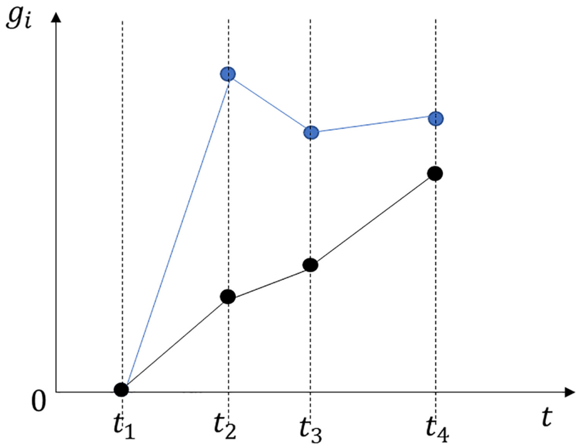


Figure 2.
The idea of the surface-based model

Source(s): Own work

The literature describes at least several surface-based models of the GIA. The most popular ones include the model using the absolute degree of grey incidence (ε_{k-ref}) (2), relative degree of grey incidence (r_{ij}) (3) and synthetic degree of grey incidence (ρ_{ij}) (4).

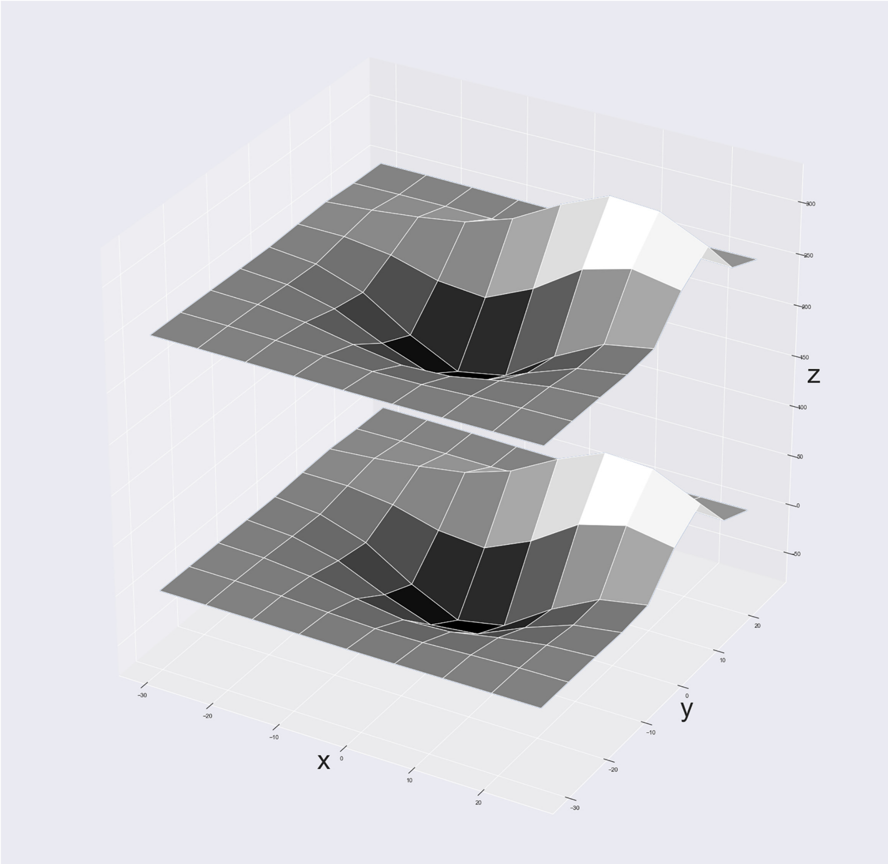
$$\varepsilon_{k-ref} = \frac{1 + \left|s_k^0\right| + \left|s_{ref}^0\right|}{1 + \left|s_k^0\right| + \left|s_{ref}^0\right| + \left|s_k^0 - s_{ref}^0\right|} \tag{2}$$

$$r_{ij} = \frac{1 + \left|s_i'\right| + \left|s_j'\right|}{1 + \left|s_i'\right| + \left|s_j'\right| + \left|s_i' - s_j'\right|} \tag{3}$$

$$\rho_{ij} = \theta\varepsilon_{k-ref} + (1-\theta)r_{ij}, \theta \in [0,1] \tag{4}$$

The last group of models classified into the grey relational analysis are the panel-based models. They are used to calculate the similarity between two three-dimensional spaces. These models can be used where we can create a three-dimensional data matrix, for example if we have m objects and n decision-making criteria that change in time t. The similarity between two planes is determined by calculating the three-dimensional absolute degree of grey similarity ε_{ab}. Figure 3 shows the idea of the panel-based models in a graphic form (Mierzwiak and Nowak, 2020).

GIA is the subject of research in many articles (Prakash *et al.*, 2023). This arises from the importance of relational analysis models both in theoretical and practical dimensions (Sun *et al.*, 2021). The literature on GIA has sparked a wide-ranging discussion about the evaluation of various mathematical models. For instance, Zhang and Liu (2010) not only explored relationships between curves but also extended their investigation to scrutinise associations among surfaces. This expansion led to the examination of relational analysis within three-dimensional spaces and even delved into the interrelations among hypersurfaces in n-dimensional spaces. The need for evaluating methods within GIA has been emphasised in Liu's recent article (2023). In this work, Liu introduces novel negative grey relational analysis models designed to effectively address the measurement of relationships in reverse sequences. These models are designed to satisfy the criteria of normalisation and reversibility. Wu and Qu have proposed a GIA model known as the Grey



Source(s): Mierzwiak, Nowak (2020)

Figure 3.
Idea of the panel-based model

Discrete Curvature Incidence model (GDCI). They utilise this model to ascertain relationships between panel data by representing it as discrete triangular surfaces. By employing the Mean and Gauss curvature of the discrete surface, they establish grey incidence models and explore their properties, including normality and symmetry. Numerical and practical examples demonstrate the effectiveness and rationality of the proposed model, highlighting its ability to reflect relationships between panel data (Wu and Xu). Zhang Qishan examined the favourable aspects of Deng's grey relational analysis model and introduced the concept of grey relational entropy to enhance the transmission model. Zhang also proposed a new technique for determining the degree of relation (Zhang, 1996; Zhang *et al.*, 1999). In another study related to GIA methods, Yang *et al.* developed a grey relational model that incorporates information diffusion to address the issue of rank reversal when faced with limited or changing decision information (2022). The researchers devised an ideal point diffusion method and, using a virtual-ideal sequence, constructed a grey relational model for sample classification. They also established an optimisation model aimed at minimising deviation.

The concept of calculating similarity indicators between sets of points or vectors is fundamentally significant from a theoretical perspective, mainly because it finds application in both relational, decision-making models (Javed *et al.*, 2020; Mal *et al.*, 2021), and predictive ones (Li *et al.*, 2022). Therefore, in the two most important journals for grey systems theory, namely Grey Systems: *Theory and Application and Journal of Grey System,* there is an advanced discussion about the verification and improvement of existing models as well as the creation of new GIA models. It appears that a practical demand plays a key role in this process. This is because GIA models have a range of practical applications. Among the most important of them are: to evaluate multilevel dispatching rules in wafer fabrication (Chia Yee *et al.*, 2021), optimisation of the investment portfolio (Škrinjarić, 2020), assessment of financial results of stock exchange companies (Javanmardi *et al.*, 2021), socio-economic policies related to sustainable development (Javanmardi *et al.*, 2020; Koçak, 2020), project management (Javed and Liu, 2019), selection of the best cities to live in selected countries around the world (Kose *et al.*, 2020), evaluation of provincial integration degree of "Internet + industry" (Yang and Xie, 2019), broadly understood health diagnostics (Zhang *et al.*, 2022).

It turns out, therefore, that the results of research conducted using GIA models have significant theoretical and practical importance. Evaluating these models may thus have a substantial impact on the development of grey systems theory as well as its practical application in problems of grey relational analysis.

3. Methodology

In this article, a simulation analysis will be conducted on a relational model using the relative degree of grey incidence. The procedure of determining that indicator can be presented in the following steps:

Step 1. Identifying the set of vectors subjected to the GIA with the use of the relative degree of grey incidence

A relational analysis requires determining the reference vectors, i.e. those to which other vectors will be compared. A reference vector can be denoted as follows:

$$X_k^{\mathrm{ref}} = \left[x_{k1}^{\mathrm{ref}}, x_{k2}^{\mathrm{ref}}, \ldots, x_{kj}^{\mathrm{ref}}, \ldots, x_{kl}^{\mathrm{ref}}\right] \tag{5}$$

where:

X_k^{ref}– reference vector for the kth object, where $k = 1, 2, \ldots, m$

j – the jth value in the reference series for the kth object, $j = 1, 2, \ldots, l$

The set of the other vectors is represented as follows:

$$X_k^{\mathrm{i}} = \left[x_{k1}^{\mathrm{i}}, x_{k2}^{\mathrm{i}}, \ldots, x_{kj}^{\mathrm{i}}, \ldots, x_{kl}^{\mathrm{i}}\right] \tag{6}$$

where:

X_k^i– the ith empirical vector for the kth object, where $i = 1, 2, \ldots, n$
j – the jth value in the empirical time series for the kth object, $j = 1, 2, \ldots, l$

Step 2. The first stage of time series unitarisation

At this stage of unitarisation, all elements in the series are divided by their initial values according to formulae (7) and (8).

$$Y_k^{\mathrm{ref}} = \left[\frac{x_{k1}^{\mathrm{ref}}}{x_{k1}^{\mathrm{ref}}}, \frac{x_{k2}^{\mathrm{ref}}}{x_{k1}^{\mathrm{ref}}}, \ldots, \frac{x_{kj}^{\mathrm{ref}}}{x_{k1}^{\mathrm{ref}}}, \ldots, \frac{x_{kl}^{\mathrm{ref}}}{x_{k1}^{\mathrm{ref}}}\right] = \left[y_{k1}^{\mathrm{ref}}, y_{k2}^{\mathrm{ref}}, \ldots, y_{kj}^{\mathrm{ref}}, \ldots, y_{kl}^{\mathrm{ref}}\right] \tag{7}$$

$$Y_k^{\mathrm{i}} = \left[\frac{x_{k1}^{\mathrm{i}}}{x_{k1}^{\mathrm{i}}}, \frac{x_{k2}^{\mathrm{i}}}{x_{k1}^{\mathrm{i}}}, \ldots, \frac{x_{kj}^{\mathrm{i}}}{x_{k1}^{\mathrm{i}}}, \ldots, \frac{x_{kl}^{\mathrm{i}}}{x_{k1}^{\mathrm{i}}}\right] = \left[y_{k1}^{\mathrm{i}}, y_{k2}^{\mathrm{i}}, \ldots, y_{kj}^{\mathrm{i}}, \ldots, y_{kl}^{\mathrm{i}}\right] \tag{8}$$

Step 3. The second stage of time series unitarisation

At this stage of unitarisation, initial values are subtracted from each element (this stage is also called the stage of zeroing against the first terms) (9) and (10).

$$Y'^{\mathrm{ref}}_k = \left[y_{k1}^{\mathrm{ref}} - y_{k1}^{\mathrm{ref}}, y_{k2}^{\mathrm{ref}} - y_{k1}^{\mathrm{ref}}, \ldots, y_{kj}^{\mathrm{ref}} - y_{k1}^{\mathrm{ref}}, \ldots, y_{kl}^{\mathrm{ref}} - y_{k1}^{\mathrm{ref}}\right] = \left[y'^{\mathrm{ref}}_{k1}, y'^{\mathrm{ref}}_{k2}, \ldots, y'^{\mathrm{ref}}_{kj}, \ldots, y'^{\mathrm{ref}}_{kl}\right] \tag{9}$$

$$Y'^{\mathrm{i}}_k = \left[y_{k1}^{\mathrm{i}} - y_{k1}^{\mathrm{i}}, y_{k2}^{\mathrm{i}} - y_{k1}^{\mathrm{i}}, \ldots, y_{kj}^{\mathrm{i}} - y_{k1}^{\mathrm{i}}, \ldots, y_{kl}^{\mathrm{i}} - y_{k1}^{\mathrm{i}}\right] = \left[y'^{\mathrm{i}}_{k1}, y'^{\mathrm{i}}_{k2}, \ldots, y'^{\mathrm{i}}_{kj}, \ldots, y'^{\mathrm{i}}_{kl}\right] \tag{10}$$

Step 4. Determining model parameters $\left|s'_{\mathrm{ref}}\right|$, $\left|s'_i\right|$ and $\left|s'_{\mathrm{ref}} - s'_i\right|$

Model parameters in the GIA, that is, $\left|s'_{\mathrm{ref}}\right|$, $\left|s'_i\right|$ and $\left|s'_{\mathrm{ref}} - s'_i\right|$ are determined according to formulae (11)–(13).

$$\left|s'_{\mathrm{ref}}\right| = \left|\sum_{j=2}^{l-1} y'^{\mathrm{ref}}_{kj} + 0.5 \cdot y'^{\mathrm{ref}}_{kl}\right| \tag{11}$$

$$\left|s'_{\mathrm{i}}\right| = \left|\sum_{j=2}^{l-1} y'^{\mathrm{i}}_{kj} + 0.5 \cdot y'^{\mathrm{i}}_{kl}\right| \tag{12}$$

$$\left|s'_{\mathrm{ref}} - s'_i\right| = \left|\sum_{j=2}^{l-1} \left(y'^{\mathrm{i}}_{kj} - y'^{\mathrm{ref}}_{kj}\right) + 0.5 \cdot \left(y'^{\mathrm{i}}_{kl} - y'^{\mathrm{ref}}_{kl}\right)\right| \tag{13}$$

Step 4. Calculating the relative degree of grey incidence $r_{\mathrm{ref}-i}$

The relative degree of grey incidence $r_{\mathrm{ref}-i}$ is determined with the use of formula (14).

$$r_{\mathrm{ref}-i} = \frac{1 + \left|s'_{\mathrm{ref}}\right| + \left|s'_{\mathrm{i}}\right|}{1 + \left|s'_{\mathrm{ref}}\right| + \left|s'_{\mathrm{i}}\right| + \left|s'_{\mathrm{ref}} - s'_i\right|} \tag{14}$$

The value of indicator $r_{\mathrm{ref}-i}$ is within the range of 0–1. The higher the value of that indicator, the higher the geometric similarity between two vectors. Conversely, the lower the value of that indicator, the lower the geometric similarity between the vectors.

In the simulation tests conducted on the models, in which the relative degree of grey incidence is determined, we will also use error statistics. For each of thousands of simulations, we will determine deviations from the expected values with the use of the error statistics presented in Table 1.

Statistics	Acronym	Formula
Mean error	ME	$ME = \frac{1}{n}\sum_{i=1}^{N} x - \bar{x}$
Mean percentage error	MPE	$MPE = \frac{1}{n}\sum_{i=1}^{N} \frac{x-\bar{x}}{\bar{x}}$
Mean absolute error	MAE	$MAE = \frac{1}{n}\sum_{i=1}^{N} \left\lvert x-\bar{x}\right\rvert$
Mean absolute percentage error	MAPE	$MAPE = \frac{1}{n}\sum_{i=1}^{n} \left\lvert \frac{x-\bar{x}}{\bar{x}}\right\rvert$
Mean squared error	MSE	$MSE = \frac{1}{n}\sum_{i=1}^{N} (x-\bar{x})^2$
Root mean squared error	RMSE	$RMSE = \sqrt{MSE}$

Source(s): Qi *et al.*, 2020; Zaman and Bulut, 2020; Eyo and Abbey, 2021

Table 1.
Error statistics used in simulation tests

4. Empirical research

The conducted simulation research can be presented in the form of a procedure consisting of three steps.

Step 1. Preparation of data for simulation

Simulations were conducted for three cases. In each of them, two sets were generated, consisting of 100 vectors each. In the first case, both sets contained natural numbers from the range 1–10. In the second case, both sets contained natural numbers from the range 1–100. In the third case, both sets had arbitrary numbers from the range 1–10. Considering the specified limitations, all values were random. The article's Appendix contains the Python code (along with the random seed) to replicate the simulation experiment. Table 2 presents an example of the first five randomly drawn values for each of the three cases (for the first and second vector sets).

Step 2. Determining the relative degree of grey incidence for all cases

For each of the three cases, we determined the values of the relative degree of grey incidence, taking consecutive vectors from the first set and determining the values of that indicator relative to all consecutive vectors from the second set for each of the three cases. In this way, we obtained $100 \cdot 100 \cdot 3 = 30000$ combinations for determining the relative degree of grey incidence. As part of the simulation studies, the relative degree of grey incidence was calculated for each of the 30,000 combinations three times, using three types of unitarisations of variables: by the first value, by the maximum value and by the average value in the vector. Table 3 presents sample simulation results for each case.

Step 3. Determination of error statistics for individual types of variable unitarisation.

For each of 10,000 combinations of variables, we calculated the values of the relative degree of grey incidence using standardisation by dividing by the first term (st_1), standardisation by dividing by the average value in the series (st_2), and standardisation by dividing by the maximum arithmetic value in the series (st_3). For each simulation, we used parameter 0.5 in the formula for $\left\lvert s'_{\mathrm{ref}}\right\rvert$, $\left\lvert s'_{\mathrm{i}}\right\rvert$ and $\left\lvert s'_{\mathrm{ref}} - s'_{\mathrm{i}}\right\rvert$.

For each combination, we calculated the following types of errors: ME, MPE, MAE, MAPE, MSE, RMSE. When calculating the errors, the expected value assumed was the arithmetic mean of the results obtained in three different standardisations. Table 4 shows sample results of the simulation.

	Set 1	Set 2
Case 1	[3, 7, 9, 6, 10, 4, 2, 5, 1, 8]	[4, 3, 2, 1, 8, 5, 6, 10, 9, 7]
	[1, 10, 6, 5, 4, 9, 2, 7, 3, 8]	[8, 1, 7, 5, 2, 9, 4, 6, 10, 3]
	[7, 10, 8, 1, 3, 5, 4, 6, 9, 2]	[6, 10, 5, 9, 2, 4, 1, 8, 3, 7]
	[1, 3, 2, 10, 7, 4, 8, 9, 5, 6]	[6, 4, 7, 10, 5, 1, 2, 9, 8, 3]
	[8, 3, 9, 6, 1, 4, 2, 7, 10, 5]	[3, 8, 9, 4, 5, 2, 6, 1, 10, 7]
	.	.
	.	.
	.	.
Case 2	[88, 85, 32, 35, 38, 8, 29, 74, 53, 79]	[44, 20, 74, 89, 65, 73, 64, 7, 86, 27]
	[78, 34, 52, 92, 46, 87, 100, 28, 44, 15]	[73, 42, 79, 27, 58, 94, 99, 21, 1, 24]
	[31, 17, 50, 73, 100, 40, 43, 1, 76, 85]	[23, 54, 63, 41, 95, 10, 79, 62, 4, 31]
	[50, 96, 18, 70, 47, 6, 94, 56, 71, 2]	[38, 59, 11, 92, 41, 10, 47, 56, 99, 13]
	[34, 37, 80, 19, 16, 64, 60, 9, 55, 41]	[19, 75, 2, 78, 36, 22, 40, 72, 73, 13]
	.	.
	.	.
	.	.
Case 3	[7.17, 9.54, 4.65, 3.9, 1.99, 4.15, 9.03, 8.77, 8.45, 7.59]	[3.95, 3.87, 8.76, 8.57, 6.65, 5.31, 9.39, 1.0, 5.31, 7.69]
	[1.14, 4.78, 4.05, 3.51, 8.98, 7.32, 7.89, 2.23, 5.82, 8.62]	[4.47, 4.79, 7.83, 2.59, 5.17, 6.55, 6.06, 8.01, 10.0, 4.34]
	[4.62, 6.17, 1.27, 6.54, 4.96, 6.23, 6.97, 6.43, 4.98, 7.81]	[5.81, 6.76, 9.06, 2.3, 1.65, 9.04, 8.44, 1.76, 9.14, 4.35]
	[1.21, 1.35, 2.49, 2.48, 4.62, 4.77, 2.56, 9.21, 6.75, 2.53]	[5.66, 5.63, 3.94, 4.61, 1.94, 1.62, 6.87, 9.67, 7.85, 6.67]
	[8.97, 6.21, 8.89, 3.14, 7.07, 3.94, 7.53, 9.41, 9.76, 6.08]	[5.1, 3.62, 9.12, 1.44, 5.81, 6.84, 4.13, 3.64, 7.81, 1.1]
	.	.
	.	.
	.	.

Source(s): Own elaboration

Table 2. Sample random vectors for the three analysed cases

	Case 1			Case 2			Case 3		
	st_1	st_2	st_3	st_1	st_2	st_3	st_1	st_2	st_3
1	0.5976	0.7919	0.8022	0.5087	0.5221	0.5373	0.5308	0.5218	0.5417
2	0.7286	0.8435	0.8526	0.5627	0.5506	0.5824	0.5307	0.5214	0.5429
3	0.7192	0.8301	0.8395	0.5587	0.5487	0.5799	0.6261	0.7316	0.6791
4	0.5243	0.5333	0.5575	0.9253	0.9024	0.9634	0.5429	0.5436	0.5762
5	0.6062	0.6733	0.7000	0.8485	0.7885	0.8109	0.5309	0.6645	0.6817
. . .									
9,996	0.7914	0.7273	0.7434	0.9421	0.9238	0.9401	0.5712	0.5462	0.5833
9,997	0.7611	0.6836	0.6989	0.5227	0.5346	0.5508	0.5044	0.5276	0.5422
9,998	0.5413	0.5447	0.5758	0.8733	0.8304	0.9527	0.5177	0.5411	0.5731
9,999	0.5127	0.5271	0.5472	0.5083	0.5250	0.5367	0.6135	0.5899	0.6157
10,000	0.7556	0.6796	0.6947	0.8069	0.7389	0.7610	0.5996	0.5776	0.6197

Source(s): Own elaboration

Table 3. Sample results of simulation for different standardisation methods

Table 4.
Sample results of simulation for different standardisation methods

		1	2	3	4	5	...	9,996	9,997	9,998	9,999	10,000
Case 1	st_1	0.6	0.73	0.72	0.52	0.61	...	0.79	0.76	0.54	0.51	0.76
	st_2	0.79	0.84	0.83	0.53	0.67	...	0.73	0.68	0.54	0.53	0.68
	st_3	0.8	0.85	0.84	0.56	0.7	...	0.74	0.7	0.58	0.55	0.69
	ev	0.73	0.81	0.8	0.54	0.66	...	0.75	0.71	0.55	0.53	0.71
	me_r1	−0.13	−0.08	−0.08	−0.01	−0.05	...	0.04	0.05	−0.01	−0.02	0.05
	me_r2	0.06	0.04	0.03	−0.01	0.01	...	−0.03	−0.03	−0.01	0	−0.03
	me_r3	0.07	0.04	0.04	0.02	0.04	...	−0.01	−0.02	0.02	0.02	−0.02
	mae_r1	0.13	0.08	0.08	0.01	0.05	...	0.04	0.05	0.01	0.02	0.05
	mae_r2	0.06	0.04	0.03	0.01	0.01	...	0.03	0.03	0.01	0	0.03
	mae_r3	0.07	0.04	0.04	0.02	0.04	...	0.01	0.02	0.02	0.02	0.02
	mpe_r1	−22.25	−10.93	−10.72	−2.68	−8.85	...	4.72	6.12	−2.33	−3.18	6.04
	mpe_r2	7.75	4.18	4.08	−0.95	2	...	−3.68	−4.53	−1.7	−0.36	−4.47
	mpe_r3	8.93	5.2	5.15	3.43	5.74	...	−1.43	−2.24	3.8	3.33	−2.2
	mape_r1	22.25	10.93	10.72	2.68	8.85	...	4.72	6.12	2.33	3.18	6.04
	mape_r2	7.75	4.18	4.08	0.95	2	...	3.68	4.53	1.7	0.36	4.47
	mape_r3	8.93	5.2	5.15	3.43	5.74	...	1.43	2.24	3.8	3.33	2.2
	mse_r1	0.02	0.01	0.01	0	0	...	0	0	0	0	0
	mse_r2	0	0	0	0	0	...	0	0	0	0	0
	mse_r3	0.01	0	0	0	0	...	0	0	0	0	0
	rmse_r1	0.13	0.08	0.08	0.01	0.05	...	0.04	0.05	0.01	0.02	0.05
	rmse_r2	0.06	0.04	0.03	0.01	0.01	...	0.03	0.03	0.01	0	0.03
	rmse_r3	0.07	0.04	0.04	0.02	0.04	...	0.01	0.02	0.02	0.02	0.02

(*continued*)

		1	2	3	4	5	...	9,996	9,997	9,998	9,999	10,000
Case 2	st_1	0.51	0.56	0.56	0.93	0.85	...	0.94	0.52	0.87	0.51	0.81
	st_2	0.52	0.55	0.55	0.9	0.79	...	0.92	0.53	0.83	0.52	0.74
	st_3	0.54	0.58	0.58	0.96	0.81	...	0.94	0.55	0.95	0.54	0.76
	ev	0.52	0.57	0.56	0.93	0.82	...	0.94	0.54	0.89	0.52	0.77
	me_r1	−0.01	0	0	−0.01	0.03	...	0.01	−0.01	−0.01	−0.02	0.04
	me_r2	0	−0.01	−0.01	−0.03	−0.03	...	−0.01	0	−0.06	0	−0.03
	me_r3	0.01	0.02	0.02	0.03	−0.01	...	0	0.01	0.07	0.01	−0.01
	mae_r1	0.01	0	0	0.01	0.03	...	0.01	0.01	0.01	0.02	0.04
	mae_r2	0	0.01	0.01	0.03	0.03	...	0.01	0	0.06	0	0.03
	mae_r3	0.01	0.02	0.02	0.03	0.01	...	0	0.01	0.07	0.01	0.01
	mpe_r1	−2.75	−0.45	−0.67	−0.55	3.83	...	0.72	−2.55	−1.39	−2.96	4.71
	mpe_r2	−0.11	−2.66	−2.5	−3.1	−3.48	...	−1.25	−0.27	−6.63	0.32	−4.06
	mpe_r3	2.72	2.95	3.01	3.43	−0.62	...	0.51	2.68	7.06	2.49	−1.04
	mape_r1	2.75	0.45	0.67	0.55	3.83	...	0.72	2.55	1.39	2.96	4.71
	mape_r2	0.11	2.66	2.5	3.1	3.48	...	1.25	0.27	6.63	0.32	4.06
	mape_r3	2.72	2.95	3.01	3.43	0.62	...	0.51	2.68	7.06	2.49	1.04
	mse_r1	0	0	0	0	0	...	0	0	0	0	0
	mse_r2	0	0	0	0	0	...	0	0	0	0	0
	mse_r3	0	0	0	0	0	...	0	0	0	0	0
	rmse_r1	0.01	0	0	0.01	0.03	...	0.01	0.01	0.01	0.02	0.04
	rmse_r2	0	0.01	0.01	0.03	0.03	...	0.01	0	0.06	0	0.03
	rmse_r3	0.01	0.02	0.02	0.03	0.01	...	0	0.01	0.07	0.01	0.01

(*continued*)

Table 4.

		1	2	3	4	5	...	9,996	9,997	9,998	9,999	10,000
Case 3	st_1	0.53	0.53	0.63	0.54	0.53	...	0.57	0.5	0.52	0.61	0.6
	st_2	0.52	0.52	0.73	0.54	0.66	...	0.55	0.53	0.54	0.59	0.58
	st_3	0.54	0.54	0.68	0.58	0.68	...	0.58	0.54	0.57	0.62	0.62
	ev	0.53	0.53	0.68	0.55	0.63	...	0.57	0.52	0.54	0.61	0.6
	me_r1	0	0	−0.05	−0.01	−0.09	...	0	−0.02	−0.03	0.01	0
	me_r2	−0.01	−0.01	0.05	−0.01	0.04	...	−0.02	0	0	−0.02	−0.02
	me_r3	0.01	0.01	0	0.02	0.06	...	0.02	0.02	0.03	0.01	0.02
	mae_r1	0	0	0.05	0.01	0.09	...	0	0.02	0.03	0.01	0
	mae_r2	0.01	0.01	0.05	0.01	0.04	...	0.02	0	0	0.02	0.02
	mae_r3	0.01	0.01	0	0.02	0.06	...	0.02	0.02	0.03	0.01	0.02
	mpe_r1	−0.12	−0.18	−8.44	−2.09	−17.86	...	0.75	−4.03	−5.07	1.16	0.11
	mpe_r2	−1.85	−1.97	7.2	−1.96	5.84	...	−3.79	0.54	−0.53	−2.79	−3.7
	mpe_r3	1.9	2.07	0.02	3.81	8.21	...	2.81	3.22	5.08	1.52	3.35
	mape_r1	0.12	0.18	8.44	2.09	17.86	...	0.75	4.03	5.07	1.16	0.11
	mape_r2	1.85	1.97	7.2	1.96	5.84	...	3.79	0.54	0.53	2.79	3.7
	mape_r3	1.28	1.32	0.02	3.94	8	...	2.21	2.5	5.06	1.71	2.98
	mse_r1	0	0	0	0	0.01	...	0	0	0	0	0
	mse_r2	0	0	0	0	0	...	0	0	0	0	0
	mse_r3	0	0	0	0	0	...	0	0	0	0	0
	rmse_r1	0	0	0.05	0.01	0.09	...	0	0.02	0.03	0.01	0
	rmse_r2	0.01	0.01	0.05	0.01	0.04	...	0.02	0	0	0.02	0.02
	rmse_r3	0.01	0.01	0	0.02	0.06	...	0.02	0.02	0.03	0.01	0.02

Note(s): Where
ev – expected value
Source(s): Own elaboration

Table 4.

Table 5 shows averaged errors in calculating the relative degree of grey incidence depending on the method of standardising variables for each of the three cases.

It turns out that, regardless of the error statistics used, the smallest measuring error occurs in the standardisation by dividing by the average value in the series (the grey colour marks the standardisation methods with the smallest error) – this situation is repeated in all three cases. The method of standardisation used by dividing by the first term is characterised by, on average, significantly lower accuracy than standardising the series by the mean.

The fact that the average error when applying the operator of dividing all terms of the series by the average is significantly lower likely results from the fact that the average value in the series much better represents the series than its first value, which can considerably differ from the other values in the series. The more the first value deviates from the expected value in the series (in this case, the arithmetic mean), the more it will affect the reduction of the accuracy of the operator dividing by the first term in the series.

To demonstrate the influence of the operator choice on the obtained results, we present calculations based on the database shown in the article (Łopatka and Nowak, 2020). The article examined the correlation between the size of funds within the European Union's regional operational programs per capita from 2007 to 2013 in Polish provinces and:

(1) gross domestic product per capita from 2007 to 2013 in a given province,

(2) investment expenditures per capita from 2007 to 2013 in a given province,

(3) internal expenditures on research and development activities per capita from 2007 to 2013 in a given province,

(4) gross value added per worker (in PLN) from 2007 to 2013 in a given province.

Based on the indicated database (seven-year time series for 16 Polish provinces for 5 different variables), the relative degree of grey incidence indicators was calculated using two operators – in the first case, the operator dividing by the first value was applied, and in the second case, the operator dividing by the average was applied. The results of these calculations are presented in Table 6.

Table 7 presents the percentage change in the value of the relative degree of grey incidence indicator when using the operator of division by average relative to the operator of division by the first term.

Analysing Table 7, it turns out that the percentage change in the result is approximately 3.5%. However, in some cases, the change in the value of the relative degree of grey incidence indicator has altered by over 10%. It appears that even minor changes in the value of this indicator can lead to alterations in the ranking of entities. Accordingly, Table 8 illustrates how the method of standardisation might influence the position of Polish provinces in the ranking.

	Case 1			Case 2			Case 3		
	st_1	st_2	st_3	st_1	st_2	st_3	st_1	st_2	st_3
ME	−0.0177	−0.0004	0.0187	−0.013	−0.0034	0.0165	−0.0199	−0.001	0.0209
MPE	−3.1423	−0.3446	2.8409	−2.2862	−0.6883	2.4795	−3.7204	−0.4412	3.0946
MAE	0.0286	0.0139	0.0216	0.0243	0.014	0.0226	0.0336	0.0189	0.0271
MAPE	4.579	2.0776	3.2199	3.7159	2.0368	3.245	5.4699	2.6662	4.2164
MSE	0.0015	0	0.0002	0.0008	0	0.0003	0.0024	0.0004	0.0009
RMSE	0.0286	0.0139	0.0216	0.0243	0.014	0.0226	0.0336	0.0189	0.0271

Source(s): Own elaboration

Table 5. Averaged errors in determining the relative degree of grey incidence depending on the method of standardising variables for each of the three cases

	Standardisation by the first term				Standardisation by the average			
	GDP	CE	R&D	VA	GDP	CE	R&D	VA
a	0.762	0.924	0.615	0.730	0.787	0.929	0.662	0.759
b	0.860	0.962	0.576	0.762	0.873	0.966	0.625	0.787
c	0.803	0.751	0.590	0.752	0.821	0.774	0.634	0.775
d	0.826	0.652	0.574	0.770	0.841	0.685	0.628	0.792
e	0.718	0.844	0.613	0.674	0.736	0.851	0.644	0.698
f	0.831	0.917	0.668	0.804	0.849	0.923	0.710	0.825
g	0.686	0.823	0.632	0.673	0.708	0.819	0.661	0.697
h	0.785	0.710	0.586	0.730	0.798	0.730	0.623	0.749
i	0.808	0.650	0.540	0.754	0.825	0.686	0.597	0.777
j	0.766	0.819	0.598	0.706	0.786	0.832	0.645	0.733
k	0.965	0.724	0.704	0.972	0.974	0.749	0.771	0.975
l	0.789	0.944	0.594	0.757	0.808	0.948	0.640	0.780
m	0.926	0.837	0.623	0.799	0.932	0.847	0.664	0.819
n	0.997	0.999	0.599	0.891	0.995	0.995	0.668	0.907
o	0.740	0.819	0.597	0.708	0.761	0.818	0.634	0.732
p	0.883	0.819	0.654	0.768	0.893	0.824	0.692	0.792

Note(s): Where
GDP - Gross Domestic Product per capita from 2007 to 2013 in a given province
CE - Internal expenditures on research and development activities per capita from 2007 to 2013 in a given province
R&D - Internal expenditures on research and development activities per capita from 2007 to 2013 in a given province
VA - Gross value added per worker (in PLN) from 2007 to 2013 in a given province
a - Lower Silesian province
b - Kuyavian-Pomeranian province
c - Lublin province
d - Lubusz province
e - Łódź province
f - Le-sser Poland province
g - Masovian province
h - Opole province
i - Subcarpathian province
j - Podlaskie province
k - Pomeranian province
l - Silesian province
m - Świętokrzyskie province
n - Warmian-Masurian province
o - Greater Poland province
p - West Pomeranian province
Source(s): Own elaboration

Table 6.
The relative degree of grey incidence indicators for the analysed problem for two standardisation methods

Analysing the data in the table, it becomes apparent that the method of standardisation can significantly influence an object's position in the ranking. For instance, changing the method of standardisation when studying the relationship between EU funds and R&D expenditures results in half of the Polish voivodeships changing their position in the ranking. This shift is not merely of academic interest – quite the opposite. Changes in ranking can have a profound impact on socio-economic policies implemented towards individual voivodeships in Poland by public authorities. Minor discrepancies in rankings can entail consequences amounting to billions of euros. Similar conclusions could be drawn by analysing the results of studies found, for example, in an article where sustainability indicators are determined for 15 sub-provincial cities in China, as seen in the article by Yi *et al.* (2021).

Table 7. Percentage changes in the relative degree of grey incidence indicators as a result of a change in the standardisation method for the analysed issue

	Percentage changes [%]			
	GDP	CE	R&D	VA
a	3.28	0.52	7.68	4.03
b	1.51	0.47	8.45	3.24
c	2.22	3.06	7.38	3.17
d	1.83	5.13	9.32	2.81
e	2.55	0.86	5.01	3.46
f	2.11	0.63	6.29	2.59
g	3.13	−0.46	4.67	3.45
h	1.67	2.86	6.32	2.56
i	2.17	5.62	10.53	3.16
j	2.6	1.64	7.82	3.86
k	0.96	3.48	9.5	0.35
l	2.45	0.39	7.85	3.06
m	0.64	1.27	6.7	2.43
n	−0.13	−0.43	11.36	1.84
o	2.74	−0.15	6.23	3.39
p	1.19	0.63	5.79	3.09

Source(s): Own elaboration

Table 8. Impact of the standardisation method of variables on the ranking position for individual entities

	GDP		Expenditures		R&D		VA	
ranking	St_1	St_2	St_1	St_2	St_1	St_2	St_1	St_2
1	n	n	n	n	k	k	k	k
2	k	k	b	b	f	f	n	n
3	m	m	l	l	p	p	f	f
4	p	p	a	a	g	n	m	m
5	b	b	f	f	m	m	d	d
6	f	f	e	e	a	a	p	p
7	d	d	m	m	e	g	b	b
8	i	i	g	j	n	j	l	l
9	c	c	j	p	j	e	i	i
10	l	l	p	g	o	l	c	c
11	h	h	o	o	l	o	h	a
12	j	a	c	c	c	c	a	h
13	a	j	k	k	h	d	o	j
14	o	o	h	h	b	b	j	o
15	e	e	d	i	d	h	e	e
16	g	g	i	d	i	i	g	g

Note(s): Where
St_1 – standardisation by the first term
St_2 – standardisation by the average
Source(s): Own elaboration

Considering the conducted simulation tests, we can conclude that the method of standardising elements in a series when determining the relative degree of grey incidence significantly affects the test results; therefore, the hypothesis formulated in this article has been confirmed. In addition to verifying the hypotheses of the article, we also point out that in models in which the relative degree of grey incidence is determined as a standardisation method, division by the first term should not be used, but by the arithmetic mean.

5. Findings

In this article, we evaluate the method of GIA, which is based on the relative degree of grey incidence. The influence of the applied methods of variable standardisation on the values of the "relative degree of grey incidence" indicators has been verified. As a result of this article, the flaws of standardisation involving the division by the first term were identified, and a change in the standardisation method was recommended. The recommendation was an effect of the conducted simulation tests. This article may, therefore, contribute to expanding our knowledge about testing the relations between variables with the use of the GIA. This can be reflected in solving practical problems where an important issue is to determine the impact of some variables on others. Limitations of the conducted evaluation involve in particular the limited scope of inference. This is since the obtained results referred to only one of the indicators classified into the GIA. Further research could be focused in particular on extending the scope of evaluation by the absolute degree of grey incidence and the synthetic degree of grey incidence as well as the indicators calculated for the distance-based and panel-based models in the GIA.

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Further reading

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Appendix

```
import random
import numpy as np
import pandas as pd

random.seed(21)

list1 = [random.sample(range(1, 11), 10) for _ in range(100)]
list2 = [random.sample(range(1, 11), 10) for _ in range(100)]
list3 = [[round(random.uniform(1, 10), 2) for _ in range(10)] for _ in
range(100)]
list4 = [[round(random.uniform(1, 10), 2) for _ in range(10)] for _ in
range(100)]
list5 = [random.sample(range(1, 101), 10) for _ in range(100)]
list6 = [random.sample(range(1, 101), 10) for _ in range(100)]

combination = [(x, y) for x in list1 for y in list2]
combination2 = [(x, y) for x in list3 for y in list4]
combination3 = [(x, y) for x in list5 for y in list6]

def gia(a, b):
    a = np.array(a)
    b = np.array(b)

    def standardize(vec, func):
        return vec/func(vec) - (vec/func(vec))[0]

     funcs = [lambda x: x[0], np.mean, np.max]

    stand_2a = [standardize(a, func) for func in funcs]
    stand_2b = [standardize(b, func) for func in funcs]

    s_a = [abs(0.5 * arr[-1] + arr[:-1].sum()) for arr in stand_2a]
    s_b = [abs(0.5 * arr[-1] + arr[:-1].sum()) for arr in stand_2b]

    s_mod = [abs(0.5 * (a[-1] - b[-1]) + a[:-1].sum() - b[:-1].sum())
for a, b in zip(stand_2a, stand_2b)]

    rel = [(1 + sa + sb)/(1 + sa + sb + sm) for sa, sb, sm in zip(s_a,
s_b, s_mod)]

      rel = [round(r, 4) for r in rel]

    return tuple(rel)

results = [gia(x, y) for x, y in combination]
results2 = [gia(x, y) for x, y in combination2]
results3 = [gia(x, y) for x, y in combination3]
```

Source(s): Own elaboration

Figure A1.
Python source code for the simulation