Letter to the Editor: Discussing some Korteweg-de Vries-directional contributions in fluid mechanics, atmospheric science, plasma physics and nonlinear optics concerning HFF 33, 3111 and 32, 1674

Recently, in *International Journal of Numerical Methods for Heat & Fluid Flow*, Wazwaz *et al.* (2023) and Khan (2022) have made some interesting Korteweg-de Vries (KdV)-directional contributions, respectively, on a linear structure of components of the modified KdV hierarchy including the third-order, fifth-order and seventh-order modified KdV equations with the solitons and breathers, as well as on a Hausdorff-fractal isotropically extended Lax-KdV equation for an incompressible fluid.

We express our sincere thanks to the Editors and Reviewers for their valuable comments. This work has been supported by the National Natural Science Foundation of China under Grant Nos. 11871116 and 11772017, and by the Fundamental Research Funds for the Central Universities of China under Grant No. 2019XD-A11.

Erratum: It has come to the attention of the publisher that the article Xin-Yi Gao, Yong-Jiang Guo and Wen-Rui Shan (2024), "Letter to the editor: Discussing some Korteweg-de Vries-directional contributions in fluid mechanics, atmospheric science, plasma physics and nonlinear optics concerning HFF 33, 3111 and 32, 1674", *International Journal of Numerical Methods for Heat & Fluid Flow*, https://doi.org/10.1108/HFF-05-2024-943, contained incorrect affiliations for all authors. These errors were introduced during the publication process. The affiliations for Xin-Yi Gao have been corrected from North China University of Technology, Beijing, China to State Key Laboratory of Information Photonics and Optical Communications, and School of Science, Beijing University of Posts and Telecommunications, Beijing, China and College of Science, North China University of Technology, Beijing, China. The affiliations for Yong-Jiang Guo and Wen-Rui Shan have been corrected from Beijing University of Posts and Telecommunications, Beijing, China to State Key Laboratory of Information Photonics and Optical Communications, and School of Science, Beijing University of Posts and Telecommunications, Beijing, China. The publisher sincerely apologises for these errors and for any confusion caused.

Erratum: It has come to the attention of the publisher that the article Xin-Yi Gao, Yong-Jiang Guo and Wen-Rui Shan (2024), "Letter to the editor: Discussing some Korteweg-de Vries-directional contributions in fluid mechanics, atmospheric science, plasma physics and nonlinear optics concerning HFF 33, 3111 and 32, 1674", *International Journal of Numerical Methods for Heat & Fluid Flow*, https://doi.org/10.1108/HFF-05-2024-943, contained several symbol errors in the equations and text. These errors were introduced during the publication process and have now been corrected as: Between equations (8) and (11a), "equation (5)" in three places have been corrected to "equations (5)". Just below equation (15c), " … has been investigated in Ince (1956)." has been corrected to " … has been investigated in Ince (1956); Zwillinger and Dobrushkin (2022)." In the 11th line above equation (2), the word "2024c" has been added. A reference has been added: Gao, X.Y. (2024c), "Auto-Backlund transformation with the solitons and similarity reductions for a generalized nonlinear shallow water wave equation", *Qualitative Theory of Dynamical Systems*, Vol. 23, p. 181, https://doi.org/10.1007/s12346-024-01034-8.

International Journal of Numerical Methods for Heat & Fluid Flow
Vol. 34 No. 5, 2024
pp. 1929-1936

0961-5539
DOI 10.1108/HFF-05-2024-943

Vitalized by a sense of refreshing the KdV work from Wazwaz *et al.* (2023) and Khan (2022), this Letter will consider the following generalized forced variable-coefficient KdV equation in fluid mechanics, atmospheric science, plasma physics or nonlinear optics (Brugarino, 1989; Kapadia, 2001; Zhang *et al.*, 2013; Chen *et al.*, 2020a, 2020b):

$$\phi_\tau + \rho(\tau)\phi\phi_\chi + a(\tau)\phi_{\chi\chi\chi} + \left[q(\chi,\tau)\phi\right]_\chi + z(\tau)\phi + f(\chi,\tau) = 0, \tag{1}$$

with the subscripts denoting the partial derivatives, $\phi(\chi, \tau)$, the wave amplitude, being a real differentiable function as for the scaled time coordinate τ and scaled space coordinate χ, the external-force term $f(\chi, \tau)$ and dissipative coefficient $q(\chi, \tau)$ representing the differentiable functions as for χ and τ, while $\rho(\tau)$, $a(\tau)$ and $z(\tau)$ corresponding to the effects of the nonlinearity, third-order dispersion and relaxation/perturbation, respectively (Brugarino, 1989; Kapadia, 2001; Zhang *et al.*, 2013; Chen *et al.*, 2020a, 2020b).

Painlevé property for equation (1) under some variable-coefficient constraints has been discussed (Brugarino, 1989; Kapadia, 2001). Brugarino (1989) has also obtained an auto-Bäcklund transformation and a Lax pair for equation (1) and shown that equation (1) is transformable to the KdV equation via some variable-coefficient constraints. Zhang *et al.* (2013) have worked out certain bilinear forms, Bäcklund transformations, Lax pair, infinite conservation laws and soliton solutions for equation (1). Chen *et al.* (2020a) have obtained a bilinear Bäcklund transformation, a non-isospectral Ablowitz-Kaup-Newell-Segur system and some infinite conservation laws for equation (1). Chen *et al.* (2020b) have constructed a reduction from equation (1) to another variable-coefficient KdV equation and gotten certain rational, periodic and mixed solutions for equation (1). Special cases of equation (1) have been seen in fluid mechanics, atmospheric science, plasma physics and nonlinear optics (Brugarino, 1989; Kapadia, 2001; Zhang *et al.*, 2013; Chen *et al.*, 2020a, 2020b). By the way, other nonlinear models in fluid mechanics, atmospheric science, plasma physics and nonlinear optics have also been presented (Chen *et al.*, 2024a, 2024b; Peng *et al.*, 2024; Cheng *et al.*, 2022, 2023a, 2023b; Gao, 2023a, 2024b, 2024c; Gao *et al.*, 2023a, 2023b, 2024; Wu and Gao, 2023; Wu *et al.*, 2023a, 2023b; Shen *et al.*, 2023a, 2023b, 2023c; Zhou and Tian, 2022; Zhou *et al.*, 2023a, 2023b; Feng *et al.*, 2023).

However, to our knowledge, reductions for equation (1) beyond those in Brugarino (1989) and Chen *et al.* (2020b) have not been published as yet. This Letter will find out two sets of the similarity reductions for equation (1) with symbolic computation (Wu *et al.*, 2023c; Shen *et al.*, 2023d, 2023e, 2023f; Zhou *et al.*, 2023c), which are different from those in Brugarino (1989) and Chen *et al.* (2020b).

For the sake of constructing certain similarity reductions for equation (1), similar to those in Clarkson and Kruskal (1989); Gao (2023b, 2024a) and Gao and Tian (2024), the form of $\phi(\chi, \tau)$ can be assumed as

$$\phi(\chi,\tau) = \alpha(\chi,\tau) + \beta(\chi,\tau)r[w(\chi,\tau)], \tag{2}$$

of which $\alpha(\chi, \tau)$, $\beta(\chi, \tau) \neq 0$ and $w(\chi, \tau) \neq 0$ represent some real differentiable functions to be determined, while $r(w)$ denotes a real differentiable function as for w only.

Making use of symbolic computation and inserting assumption (2) into equation (1), we find that

$$m_0 r''' + m_1 r'' + m_2 r r' + m_3 r^2 + m_4 r' + m_5 r + m_6 = 0, \tag{3}$$

with m_i's ($i = 0, \ldots, 6$) meaning the real differentiable functions of χ and τ,

$$m_0 = a(\tau)\beta w_\chi^3, \tag{4a}$$

$$m_1 = 3a(\tau)w_\chi(\beta_\chi w_\chi + \beta w_{\chi\chi}), \tag{4b}$$

$$m_2 = \rho(\tau)\beta^2 w_\chi, \tag{4c}$$

$$m_3 = \rho(\tau)\beta\beta_\chi, \tag{4d}$$

$$m_4 = 3a(\tau)(\beta_\chi w_{\chi\chi} + \beta_{\chi\chi}w_\chi) + \beta[w_\tau + q(\chi,\tau)w_\chi + \rho(\tau)\alpha w_\chi + a(\tau)w_{\chi\chi\chi}], \tag{4e}$$

$$m_5 = z(\tau)\beta + \beta_\tau + [q(\chi,\tau)\beta]_\chi + \rho(\tau)(\beta\alpha)_\chi + a(\tau)\beta_{\chi\chi\chi}, \tag{4f}$$

$$m_6 = f(\chi,\tau) + z(\tau)\alpha + \alpha_\tau + [q(\chi,\tau)\alpha]_\chi + \rho(\tau)\alpha\alpha_\chi + a(\tau)\alpha_{\chi\chi\chi}, \tag{4g}$$

while the prime sign denoting the differentiation, hereby with respect to w.

Because equation (3) should be required as an ordinary differential equation (ODE) about $r(w)$, we order that the ratios of different derivatives and powers of $r(w)$ denote the functions as for w only. Hence, each set of $\alpha(\chi,\tau)$, $\beta(\chi,\tau) \neq 0$ and $w(\chi,\tau) = 0$, which we plan to work out, can bring about a similarity reduction.

Case 1: $w_\chi \neq 0$

As a result of $m_0 \neq 0$, we obtain

$$m_i = \Omega_i(w)m_0, \tag{5}$$

on which $\Omega_i(w)$'s imply some real to-be-determined functions with respect to w only.

The second freedom in remark 2 in Clarkson and Kruskal (1989) simplifies equations (5) with $i = 2$ into

$$\beta(\chi,\tau) = \frac{a(\tau)}{\rho(\tau)}w_\chi^2, \qquad \Omega_2(w) = 1. \tag{6}$$

Based on the first freedom in remark 2 in Clarkson and Kruskal (1989), equations (5) with $i = 3$ can result in

$$w(\chi,\tau) = \sigma_1(\tau)\chi + \sigma_2(\tau), \qquad \Omega_3(w) = 0, \tag{7}$$

and then equations (5) with $i = 1$ may come to

$$\Omega_1(w) = 0, \tag{8}$$

with $\sigma_1(\tau) \neq 0$ and $\sigma_2(\tau)$ as two real differentiable functions of τ.

On account of the first freedom in remark 2 in Clarkson and Kruskal (1989), equations (5) with $i = 4$ should give rise to

$$\alpha(\chi,\tau) = -\frac{q(\chi,\tau)}{\rho(\tau)} - \frac{\sigma_1'(\tau)\chi + \sigma_2'(\tau)}{\rho(\tau)\sigma_1(\tau)}, \qquad \Omega_4(w) = 0, \tag{9}$$

and equations (5) with $i = 5$ would make for

$$q(\chi, \tau) = q_1(\tau)\chi + q_2(\tau), \quad a(\tau) = \mu_0\rho(\tau), \quad \sigma_1(\tau) = \lambda_1 e^{-\int z(\tau)d\tau}, \quad \Omega_5(w) = 0, \tag{10}$$

as a result that the first freedom in remark 2 in Clarkson and Kruskal (1989) helps us transform equations (5) with $i = 6$ into

$$\sigma_2(\tau) = \lambda_2, \quad q_2(\tau) = \mu_2\rho(\tau), \quad z(\tau) = \mu_1\rho(\tau) + q_1(\tau), \tag{11a}$$

$$f(\chi, \tau) = -\mu_1[2\mu_1\rho(\tau) + 3q_1(\tau)]\chi + \mu_2[\mu_1\rho(\tau) + 2q_1(\tau)], \quad \Omega_6(w) = 0, \tag{11b}$$

with $\lambda_1 \neq 0$, λ_2, $\mu_0 \neq 0$, μ_1 and μ_2 as five real constants.

Thus, making use of symbolic computation, we are able to transform equation (3) into the following ODE:

$$r''' + rr' = 0, \tag{12}$$

which could be integrated once with respect to w, i.e.,

$$r'' + \frac{1}{2}r^2 + \eta_1 = 0, \tag{13}$$

with η_1 as a real constant of integration.

In sum, under the following variable-coefficient constraints:

$$\rho(\tau) \neq 0, \quad q(\chi, \tau) = q_1(\tau)\chi + \mu_2\rho(\tau), \quad z(\tau) = \mu_1\rho(\tau) + q_1(\tau) \neq 0, \tag{14a}$$

$$a(\tau) = \mu_0\rho(\tau), \quad f(\chi, \tau) = -\mu_1[2\mu_1\rho(\tau) + 3q_1(\tau)]\chi + \mu_2[\mu_1\rho(\tau) + 2q_1(\tau)], \tag{14b}$$

we find out a set of the similarity reductions for equation (1), i.e.,

$$\phi(\chi, \tau) = (\mu_1\chi - \mu_2) + \mu_0\lambda_1^2 e^{-2\left[\mu_1\int\rho(\tau)d\tau + \int q_1(\tau)d\tau\right]} r[w(\chi, \tau)], \tag{15a}$$

$$w(\chi, \tau) = \lambda_1 e^{-\int z(\tau)d\tau}\chi + \lambda_2, \tag{15b}$$

$$r'' + \frac{1}{2}r^2 + \eta_1 = 0. \tag{15c}$$

ODE (15c), a known ODE, has been investigated in Ince (1956); Zwillinger and Dobrushkin (2022).

In fluid mechanics, atmospheric science, plasma physics and nonlinear optics, with respect to the wave amplitude, under variable-coefficient constraints (14), similarity reductions (15) rely on the nonlinearity coefficient $\rho(\tau)$ and relaxation/perturbation coefficient $z(\tau)$ for equation (1).

Case 2: $w_\chi = 0$

On account of $m_0 = m_1 = m_2 = 0$ and $m_4 \neq 0$, we find that

$$m_3 = \sum\nolimits_1(w)m_4, \tag{16a}$$

$$m_5 = \sum\nolimits_2(w)m_4, \tag{16b}$$

$$m_6 = \sum\nolimits_3(w)m_4, \tag{16c}$$

of which $\sum_h(w)$'s $(h = 1, 2, 3)$ mean some real to-be-determined functions as for w only.

Because the second freedom in remark 2 in Clarkson and Kruskal (1989) makes us simplify equation (16a) into

$$\beta(\chi, \tau) = \frac{w'(\tau)}{\rho(\tau)}[\chi + \theta_0(\tau)], \qquad \sum\nolimits_1(w) = 1, \tag{17}$$

according to the first freedom in remark 2 in Clarkson and Kruskal (1989), equations (16b) and (16c) bring about

$$w(\tau) = \theta_1\int\rho(\tau)d\tau, \qquad \theta_0(\tau) = \theta_2, \qquad \alpha(\chi, \tau) = -\frac{1}{2}(\delta_1\chi + \delta_2), \tag{18a}$$

$$f(\chi, \tau) = \frac{1}{4}\delta_1^2\rho(\tau)\chi + \frac{1}{4}\left[\delta_1\delta_2\rho(\tau) + (\delta_2 - \delta_1\theta_2)z(\tau)\right], \tag{18b}$$

$$q(\chi, \tau) = \frac{1}{2}[\delta_1\rho(\tau) - z(\tau)]\chi + \frac{1}{2}[\delta_2\rho(\tau) - \theta_2 z(\tau)], \qquad \sum\nolimits_2(w) = \sum\nolimits_3(w) = 0, \tag{18c}$$

in which $\theta_1 \neq 0$, θ_2, δ_1 and δ_2 denote four real constants, whereas $\theta_0(\tau)$ represents a real differentiable function of τ.

In short, under the following variable-coefficient constraints:

$$\rho(\tau) \neq 0, \qquad q(\chi, \tau) = \frac{1}{2}[\delta_1\rho(\tau) - z(\tau)]\chi + \frac{1}{2}[\delta_2\rho(\tau) - \theta_2 z(\tau)], \tag{19a}$$

$$f(\chi, \tau) = \frac{1}{4}\delta_1^2\rho(\tau)\chi + \frac{1}{4}\left[\delta_1\delta_2\rho(\tau) + (\delta_2 - \delta_1\theta_2)z(\tau)\right], \tag{19b}$$

we conclude with another set of the similarity reductions for equation (1), i.e.,

$$\phi(\chi, \tau) = -\frac{1}{2}(\delta_1\chi + \delta_2) + \theta_1(\chi + \theta_2)r[w(\tau)], \tag{20a}$$

$$w(\tau) = \theta_1\int\rho(\tau)d\tau, \tag{20b}$$

$$r' + r^2 = 0. \tag{20c}$$

ODE (20c), a known ODE, has been reported in Zwillinger and Dobrushkin (2022), and is hereby solved out as

$$r(w) = \frac{1}{w - \Upsilon},$$

where Υ is a constant.

In fluid mechanics, atmospheric science, plasma physics and nonlinear optics, with respect to the wave amplitude, under variable-coefficient constraints (19), similarity reductions (20) rely on the nonlinearity coefficient $\rho(\tau)$ for equation (1).

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