

Membrane-driven flow and heat transfer of viscoelastic fluids: MHD and entropy generation analysis

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Abstract

Purpose – The complex behavior of viscoelastic fluids and its flow analysis under the impact of transverse magnetic field are becoming increasingly important in numerous emerging applications including biomedical engineering, aerospace engineering, geophysics and industrial applications. Additionally, the thermal analysis and fluid flow driven by propagating membranes will aid significant applications for microscale transport in bio-thermal systems. This study aims to investigate the thermal effects of viscoelastic fluids driven by membrane-induced propagation and transverse magnetic field.

Design/methodology/approach – The propagation of the membranes will work as pump which pushes the fluids from bottom to top against the gravitation force; however, there is backflow due to compression and expansion phases of membrane propagation. The Jeffrey fluid model is employed to analyze the viscoelastic fluid flow, with entropy generation examined and equations solved analytically under low Reynolds number and long-wavelength assumptions.

Findings – The findings reveal that an increase in magnetic field strength impedes fluid flow, while higher values of the Grashof number, heat source parameter and Jeffrey fluid parameter enhance fluid motion. The study's findings have significant implications for optimizing magnetohydrodynamic systems in various emerging applications, including biomedical engineering, aerospace, geophysics and industrial processes.

Originality/value – This study aims to investigate the impact of a transverse magnetic field on the flow and heat transfer characteristics of viscoelastic fluids driven by membrane propagation.

Keywords Propagating membrane, Heat transfer, Viscoelastic fluid, Entropy generation, Magnetohydrodynamics, Biothermal system

Paper type Research paper

1. Introduction

In the fluid dynamics, exploring the non-Newtonian fluid is crucial phenomena due to their unique and intricate functioning in various industrial and engineering applications. In



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contrast to Newtonian fluids like water and air, which exhibit constant viscosity regardless of shear rate, non-Newtonian fluids have viscosities that vary with shear strain rate. This phenomenon is crucial in understanding the mechanism of incompressible, stretchy fluid behavior when the waves traveling along the tube's wall (Böhme and Friedrich, 1983). Lindner *et al.* (2002) examined fluid movement in narrow spaces with both rigid and flexible polymers, noting that flexible polymers create varying pressure while maintaining the same flow. Hosseini *et al.* (2007) introduced a smooth particle hydrodynamic method for non-Newtonian fluid flow, adopting Lagrangian approach and unique viscous term treatment inspired by the Eulerian methods. Based on LBM (Lattice Boltzmann Method), Yoshino *et al.* (2007) applied this to simulate incompressible non-Newtonian fluids, including shear thickening and shear thinning types. Esmael *et al.* (2010) studied the flow of shear-thinning fluids in pipe and highlighted how shear-thinning properties affects the flow characteristics in the confined geometries. By using Adomian decomposition method, Tripathi (2011) analyzed that how relaxation time, fractional parameter and wave amplitude influence the flow. These studies provide insights into the distinctive flow properties of shear-thinning and viscoelastic non-Newtonian fluids, enhancing our understanding of their behavior for varying flow conditions and constraints.

In the continuation of the non-Newtonian fluid, viscoelastic fluid (Jeffrey fluid model) is characterized by both viscous and elastic properties. Jeffrey fluids demonstrate stress relaxation, allowing them to flow like a liquid while also possessing elastic properties that enable them to return to their initial shape. Hayat *et al.* (2007) studied the peristaltic motion of the Jeffrey fluid in a circular tube and found that the net flow rate's oscillatory behavior in the Jeffrey fluid is lower in the comparison of the Maxwell fluid. Nadeem and Akbar (2009) examined how variable viscosity influences the flow of incompressible Jeffrey fluids within an asymmetric channel, applying Reynolds viscosity model. Pandey and Tripathi (2010) studied the transport of a viscoelastic fluid through peristaltic motion in a channel and a circular tube, employing the Jeffrey fluid model. Akbar *et al.* (2011) examined the blood flow in a narrowed, tapered artery where the blockage or stenosis is present. This Jeffrey fluid model helps to capture the complex behavior, accounting for both its viscous and elastic characteristic, which varies with the flow conditions. Ellahi *et al.* (2014) have explored a mathematical model that describes how Jeffrey fluid moves through narrow tapered arteries affected by the atherosclerosis, which provides better understanding of blood flow in arteries with plaque buildup and may improve our understanding of cardiovascular diseases and potential treatments. Liao (2003) applied the Homotopy analysis method to demonstrate that magnetic fields enhance skin friction in shear-thinning fluids flowing over a stretching sheet. Tripathi and Bég (2012) examined the peristaltic flow of couple stress fluid within gap between two coaxial channels which include an annular porous channel influenced by the presence of magnetic field.

Heat transfer is the process by which the thermal energy transferred from one body or substance to another, which occurs primarily three mechanisms i.e., conduction, convection and radiation. Bhatti *et al.* (2024) analyzed third-grade fluid motion through vertical walls with nanofluid infusion, examining effects of magnetic/electric fields, viscosity, temperature rise and joint lubrication. Yadeta and Shaw (2023) investigated blood flow and magnetic nanoparticle transport in a stenosed artery, demonstrating the significance of parameter effects for drug delivery and cardiovascular therapies. Shaw *et al.* (2023) analyzed magnetic drug targeting in Jeffrey fluid, revealing enhanced drug delivery efficiency influenced by memory effects, magnetization and nanoparticle volume fraction. Bhatti *et al.* (2016) explored effects of variable magnetic fields on peristaltic Jeffrey fluid flow in nonuniform rectangular ducts with compliant walls. Kumar *et al.* (2024) examined the impact of the viscoelastic fluid and surface roughness in the diverging channel for physiological fluid.

[Metzner \(1965\)](#) reviewed recent research on heat transfer in non-Newtonian fluids, providing a summary of earlier studies. [Tsai *et al.* \(2008\)](#) investigated the effects of a spatially varying heat source or sink on flow dynamics and heat transfer over an unsteady stretching sheet in a stationary fluid. [Prommas \(2011\)](#) conducted experimental validation of a combined mass and thermal model for convective drying in multi-layered packed beds composed of glass beads, water and air. [Shojaeian and Koşar \(2014\)](#) analyzed that how efficiently heat transfer through different types of fluids with different boundary conditions. [Magesh and Kothandapani \(2021\)](#) considered the Johnson-Segalman fluid in an asymmetrically curved channel and studied peristaltic flow by incorporating heat and mass transfer. [Yusuf *et al.* \(2024\)](#) explored the use of graphene oxide nanoparticles in vacuum residue fluid for enhanced oil recovery, utilizing computational models to optimize efficiency. [Aslam *et al.* \(2024\)](#) developed a hybrid firefly-water cycle algorithm to optimize nonlinear Hall currents and electric double layer effects in multiphase wavy flow, enhancing accuracy and efficiency. [Vajravelu *et al.* \(2011\)](#) investigated heat transfer in Jeffrey fluid flow within a vertical porous layer, considering long-wavelength approximations and low Reynolds number conditions. [Turkylmazoglu and Pop \(2013\)](#) explored how Jeffrey fluid flow and transfer of heat near a stagnation point on a sheet that is either stretching or shrinking, with an external flow moving parallel to it. [Ellahi and Hameed\(2012\)](#) examined thermal properties in both Newtonian and non-Newtonian fluids including Jeffrey and third grade types fluids confined between parallel plate, incorporating velocity slip effects under isothermal and uniform heat flux boundary conditions. [Ellahi \(2013\)](#) investigated the influence of magnetohydrodynamics (MHD) and variable viscosities on the flow of non-Newtonian nanofluids in a pipe under constant pressure. [Elelamy *et al.* \(2020\)](#) presented a mathematical model and numerical simulation for bacterial growth in heart valves, incorporating non-Newtonian fluid flow, nanoparticles and magnetic fields. [Kothandapani and Pushparaj \(2017\)](#) analyzed the combined effects of induced magnetic fields and thermal radiation on Carreau nanofluid peristaltic transport in a tapered asymmetric channel, using analytical solutions. [Bhatti *et al.* \(2022\)](#) analyzed entropy generation and thermal behavior of magnetic hybrid nanofluid (Au-Ag/NPs) in Eyring-Powell fluid, examining its effects on velocity, temperature and pumping characteristics, with potential applications in pharmaceuticals. [Ramadan *et al.* \(2025\)](#) developed a poroelastic model to investigate temperature increases in knee cartilage under cyclic sinusoidal loading. Some studies incorporated the effect of Joule and Newtonian heating at Jeffrey fluid on stretching sheet influenced by magnetic field with heat source/sink ([Idowu *et al.*, 2013](#); [Farooq *et al.*, 2015](#)). [Hayat *et al.* \(2015\)](#) focused on analyzing the momentum and thermal boundary layers in the flow of a viscoelastic Jeffrey fluid over an exponentially stretching sheet. Furthermore, [Sen *et al.* \(2021\)](#) examined the effects of Newtonian cooling, magnetic fields, and nonlinear radiation on Jeffrey fluid flow near a stagnation point, highlighting potential industrial applications in plastic and food processing. Membrane pumping mechanism can have significant impact to control fluid behavior in the various application in biomedical devices, microfluidic system and industrial cooling, where precise fluid control and efficient heat transfer are essential.

Membrane pumping mechanism is a microfluidic solution for enhanced fluid flow; where traditional pumping methods fall short, membrane-based pumps offer a viable alternative. By periodically expanding and contracting a flexible membrane, these pumps create fluid flow through microchannels, particularly in biological and microfluidic applications. Researchers and engineers have continually refined membrane pump design for optimized performance. [Abouelkassem and Staples \(2013\)](#) investigated a bioinspired model shows that rhythmic wall contractions in an inelastic tube with two constriction sites, which is generating unidirectional flow with low Reynolds number approximation. [Cai *et al.* \(2016\)](#)

described advanced valving pumping mechanism designed for an elastic polymer-based centrifugal microfluidic platform. [Tripathi et al. \(2023\)](#) reviewed on the discrete scheme membrane-based pumping mechanism, which highlights the potential of these mechanisms for advanced micro-flow systems in medical, bio-engineering and microfabrication applications. Furthermore, [Bhandari et al. \(2022b\)](#) and [Bhandari and Tripathi \(2022\)](#) analyzed the thermal behavior in a finite-length vertical microchannel. The pumping model evaluated the streamlines and isotherms for key parameters during the contraction and expansion phases of membrane motion. [Bhandari et al. \(2022a\)](#) analyzed transient viscous flow in a microchannel driven by propagating membrane pumping under the influence of electric and magnetic fields. Building on such membrane-driven flow models, [Pandey et al. \(2024\)](#) recently investigated the effects of viscoelastic fluid in a porous medium subjected to an electric field within an asymmetric microchannel.

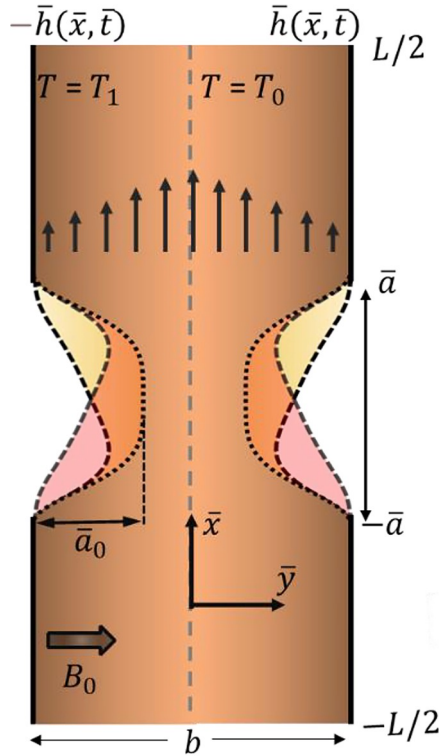
After the comprehensive investigation of existing literature on magneto-viscoelastic flow and heat transfer, and propagating membrane driven flow, this research aims to explore a novel and unexplored model for heat transfer and magneto-viscoelastic flow driven by propagating membranes. A mathematical model is formulated to explore the unique phenomenon of magneto-viscoelastic flow induced by a membrane pumping mechanism. Moreover, entropy generation analysis for the present model is discussed. And the analytical solutions of governing equations for this model have been derived by using the assumptions of low Reynolds number. Present study examines the impact of various parameters like Grashof number, Hartmann number, Heat source parameter, Jeffrey fluid parameter, Brinkman number, relative temperature and membrane shape parameter on the variation in fluid flow characteristics including axial and transverse velocity profile, pressure gradient, pressure distribution, entropy generation, volumetric flow rate, variation in heat transfer, wall shear stress and streamline patterns. The results of this study are anticipated to improve the understanding of membrane-driven flow propagation in non-Newtonian fluids in magnetohydrodynamic framework. The membrane pumping mechanism's future scope includes miniaturization, material innovations and optimization algorithms for enhanced performance in microfluidic and biomedical applications.

2. Formulation of the problem

2.1 Problem definition

Present study examines heat transfer with magneto-viscoelastic flow driven by propagating membranes. The wall geometry is depicted in [Figure 1](#), which shows a vertical microchannel with length L and width b , propagated by single membrane of length $2\bar{a}$. The membrane's propagation causes the fluid to move in both forward and backward directions. The wall deformation is considered periodic, resulting from the motion caused by membrane contraction. The phase of expansion and contraction in membrane motion can be mathematically expressed as follows ([Bhandari et al., 2022a](#)):

$$\bar{h}(\bar{x}, \bar{t}) = \begin{cases} \left(\frac{b}{2}\right)^{\psi} & -\frac{L}{2} \leq \bar{x} < -\bar{a} \\ \left(\frac{b}{2}\right)^{\psi} + 2\bar{a}_0 \left(\left(\frac{\bar{x}}{\bar{a}}\right)^{2M} - 1\right)^3 h_{memb}(\bar{x}, \bar{t}) & -\bar{a} \leq \bar{x} \leq \bar{a} \\ \left(\frac{b}{2}\right)^{\psi} & \bar{a} < \bar{x} \leq \frac{L}{2} \end{cases} \quad (1)$$



Source(s): Figure by authors

Figure 1. Schematic diagram of propagating membrane-driven viscoelastic fluids flow and heat transfer in presence of transverse magnetic field

where, $h_{memb}(\bar{x}, \bar{t}) = (1 - \bar{x}k_0 \cos(\pi f \bar{t})) \sin^2(\pi f \bar{t})$ is the membrane kinematic profile, M is the membrane parameter which define its shape, \bar{a}_0 denotes the amplitude of membrane contraction, \bar{a} specifies the membrane's axial length, k_0 characterizes the optical parameter defining the membrane shape and f denotes the frequency; this represents the rhythmic propagation of the membrane, contributing to the development of pressure within the microchannel.

2.2 Governing equations

In this study, an incompressible, two-dimensional, transient viscoelastic fluid flow under laminar conditions, driven by membrane contraction along a finite and vertical microchannel with an applied transverse magnetic field, has been examined. To analyze the heat transfer, energy equation is considered. The length of the microchannel is assumed to be large in comparison to the width of the microchannel $\delta = \frac{b}{L} \ll 1$. The governing equation for the additional stress $\bar{\tau}$ for the Jeffery fluid is given as (Pandey and Tripathi, 2010):

$$\bar{\tau} = \frac{\mu}{1 + \lambda_1} \{ \dot{\gamma} + \lambda_2 \ddot{\gamma} \}, \quad (2)$$

where $\dot{\gamma}$, μ , λ_1 and λ_2 are the rate of strain, viscosity, ratio of relaxation to retardation times, and dots denote the differentiation with respect to time. On simplification of equation (2), the stress term $\bar{\tau}_{\bar{x}\bar{y}}$ reduces to following form:

$$\bar{\tau}_{\bar{x}\bar{y}} = \frac{\mu}{1 + \lambda_1} \left[1 + \lambda_2 \left(\bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right) \right] \left(\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{u}}{\partial \bar{y}} \right), \psi \quad (3)$$

The governing equations for mass, momentum and energy conservation are considered as (Tripathi and Bég, 2012; Vajravelu et al., 2011):

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \psi \quad (4)$$

$$\rho \left(\frac{\partial}{\partial \bar{t}} + \bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right) \bar{u} = - \frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{\bar{x}\bar{y}}}{\partial \bar{y}} - \sigma B_0^2 \bar{u} + \rho g \alpha (\bar{T} - T_0), \psi \quad (5)$$

$$\rho \left(\frac{\partial}{\partial \bar{t}} + \bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right) \bar{v} = - \frac{\partial \bar{p}}{\partial \bar{y}} + \frac{\partial \bar{\tau}_{\bar{y}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{\bar{y}\bar{y}}}{\partial \bar{y}}, \psi \quad (6)$$

$$\rho C_p \left(\frac{\partial}{\partial \bar{t}} + \bar{u} \frac{\partial}{\partial \bar{x}} + \bar{v} \frac{\partial}{\partial \bar{y}} \right) \bar{T} = k \left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right) + \varphi, \psi \quad (7)$$

where, \bar{u} and \bar{v} represent the velocities in the \bar{x} and \bar{y} directions, respectively. The other parameters are defined as follows: ρ is the density, \bar{t} is time, \bar{p} is pressure, α is the coefficient of linear thermal expansion, φ is the heat source parameter, k is the thermal conductivity, \bar{T} is temperature, C_p is the specific heat at constant pressure, B_0 is the magnetic field and $\bar{\tau}_{\bar{x}\bar{x}}$, $\bar{\tau}_{\bar{x}\bar{y}}$, $\bar{\tau}_{\bar{y}\bar{x}}$, $\bar{\tau}_{\bar{y}\bar{y}}$ are the components of the extra stress.

2.3 Dimensional analysis

The following nondimensional variables are incorporated as:

$$x = \frac{\bar{x}}{L}, y = \frac{\bar{y}}{b}, t = \frac{\bar{t}}{L/u_0}, u = \frac{\bar{u}}{u_0}, v = \frac{\bar{v}}{u_0 \delta}, p = \frac{\bar{p} b^2}{\mu L u_0}, h = \frac{\bar{h}}{b},$$

$$\theta = \frac{T - T_0}{T_1 - T_0}, \psi \beta = \frac{b^2 \varphi}{k(T_1 - T_0)}, \psi Ha = B_0 b \sqrt{\frac{\sigma}{\mu}} \left(Gr = \frac{\rho g b^2 \alpha (T_1 - T_0)}{\mu u_0}, \psi Re = \frac{\rho b u_0}{\mu}, \psi \delta = \frac{b}{L}, \tau = \frac{b \bar{\tau}}{\mu u_0} \right), \psi \quad (8)$$

Here, u_0 denotes the reference velocity, δ represents the ratio of the width and length, θ is the non-dimensional form of temperature, β denotes heat source parameter. Additionally, Gr and Re are Grashof number and Reynolds number, respectively. The symbols used without bar are same meaning as without dimension. By taking the approximations i.e., $\delta \ll 1$ and $Re \ll 1$, the fundamental equations (4)–(7) are transformed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \psi \quad (9)$$

$$\frac{\partial p}{\partial x} = \frac{1}{1 + \lambda_1} \frac{\partial^2 u}{\partial y^2} - Ha^2 u + Gr\theta, \psi \quad (10)$$

$$\frac{\partial p}{\partial y} = 0, \psi \quad (11)$$

$$\frac{\partial^2 \theta}{\partial y^2} + \beta = 0, \psi \quad (12)$$

From [equation \(11\)](#), it can be observed that pressure remains unaffected by the variation in y , that means pressure is only function of x and time t . The [equations \(9\)–\(12\)](#) are simplified using the provided boundary conditions:

$$\frac{\partial \theta}{\partial y} \Big|_{y=0} = 0, \theta \Big|_{y=h} = 1,$$

$$\frac{\partial u}{\partial y} \Big|_{y=0} = 0, u \Big|_{y=h} = 0, v \Big|_{y=0} = 0, v \Big|_{y=h} = \frac{\partial h}{\partial t} \cdot \psi \quad (13)$$

The above boundary conditions are considered based on the physical model which are defined as:

- a minimum temperature at the microchannel wall ($y = h$) while the maximum temperature at the center ($y = 0$) are considered;
- no-slip condition at the wall ($y = h$) and a regularity condition ($\partial u / \partial y = 0$) at the center ($y = 0$) are considered; and
- the transverse velocity at the center of the microchannel remains zero, indicating no fluid movement, while at the wall, it is determined by the rate of change of the microchannel's wall, reflecting dynamic boundary conditions due to the propagation of membranes fitted to the microchannel.

2.4 Analytical solutions

The temperature can be calculated by integrating [equation \(12\)](#) and using the boundary conditions defined in [equation \(13\)](#) as follows:

$$\theta = 1 + \frac{1}{2} \beta (h^2 - y^2) \cdot \psi \quad (14)$$

The expression for the axial velocity can be determined by solving [equation \(10\)](#) with the boundary conditions defined in [equation \(13\)](#) as:

$$u = \frac{\cosh \left(\frac{y}{h} \sqrt{1 + \lambda_1} \right)}{\cosh \left(\sqrt{1 + \lambda_1} \right)} \left(\frac{1}{Ha^2} \frac{\partial p}{\partial x} + \frac{Gr\beta}{Ha^4(1 + \lambda_1)} - \frac{Gr}{Ha^2} \right) - \frac{1}{2} \frac{\partial p}{\partial x} + \frac{1}{2} \frac{Gr\beta}{Ha^2} \left(h^2 - y^2 \right) \left(- \frac{Gr\beta}{Ha^4(1 + \lambda_1)} + \frac{Gr}{Ha^2} \right) \cdot \psi \quad (15)$$

The solution of transverse velocity can be obtained from [equation \(9\)](#) and [equation \(15\)](#) with the boundary conditions defined in [equation \(13\)](#) as:

$$v = \frac{1}{Ha^2} \frac{\partial^2 p}{\partial x^2} \frac{\sinh y}{\sqrt{1+\lambda_1}} \left(\frac{\cosh h Ha \sqrt{1+\lambda_1}}{\cosh h Ha \sqrt{1+\lambda_1}} \right) \left(\frac{\cosh y Ha \sqrt{1+\lambda_1}}{\cosh y Ha \sqrt{1+\lambda_1}} \right) \left(\frac{1}{Ha^2} \frac{\partial p}{\partial x} + \frac{Gr\beta}{Ha^4(1+\lambda_1)} - \frac{Gr}{Ha^2} \frac{\partial h}{\partial x} \right) - \frac{1}{Ha^2} \frac{\partial^2 p}{\partial x^2} \frac{\sinh y}{\sqrt{1+\lambda_1}} \left(\frac{\cosh h Ha \sqrt{1+\lambda_1}}{\cosh h Ha \sqrt{1+\lambda_1}} \right) \left(\frac{\cosh y Ha \sqrt{1+\lambda_1}}{\cosh y Ha \sqrt{1+\lambda_1}} \right) \left(-y \right) \left(-\frac{Gr\beta}{Ha^2} \frac{\partial h}{\partial x} \right) \quad (16)$$

By using boundary condition $v|_{y=h} = \frac{\partial h}{\partial t}$ in [equation \(16\)](#), the expression for rate of wall deformation can be evaluated as:

$$\frac{\partial h}{\partial t} = \frac{1}{Ha^2} \frac{\partial^2 p}{\partial x^2} \left(h - \frac{\tanh h Ha \sqrt{1+\lambda_1}}{Ha \sqrt{1+\lambda_1}} \right) + \tanh^2 \left(h \left(Ha \left(\sqrt{1+\lambda_1} \right) \right) \right) \frac{\partial h}{\partial x} \frac{1}{Ha^2} \frac{\partial p}{\partial x} + \frac{Gr\beta}{Ha^4(1+\lambda_1)} - \frac{Gr}{Ha^2} - \frac{Gr\beta}{Ha^2} \frac{\partial h}{\partial x} h^2 \quad (17)$$

The axial pressure gradient is calculated by integrating [equation \(17\)](#) with respect to x and can be expressed as:

$$\frac{\partial p}{\partial x} = \frac{Ha^2 \left\{ \int_0^x \frac{\partial h}{\partial t} dx - h - \frac{\tanh \left(h \left(Ha \left(\sqrt{1+\lambda_1} \right) \right) \right)}{Ha \left(\sqrt{1+\lambda_1} \right)} \left(-\frac{Gr\beta}{Ha^4(1+\lambda_1)} - \frac{Gr}{Ha^2} \right) + \frac{Gr\beta}{Ha^2} h^3 - G_0(t) \right\}}{h - \frac{\tanh \left(h \left(Ha \left(\sqrt{1+\lambda_1} \right) \right) \right)}{Ha \left(\sqrt{1+\lambda_1} \right)}} \quad (18)$$

In this context, $G_0(t)$ is an arbitrary constant which can further be evaluated as:

$$G_0(t) = \frac{\Delta p_L - \int_0^l Ha^2 \left\{ \int_0^x \frac{\partial h}{\partial t} dx - h - \frac{\tanh \left(h \left(Ha \left(\sqrt{1+\lambda_1} \right) \right) \right)}{Ha \left(\sqrt{1+\lambda_1} \right)} \left(-\frac{Gr\beta}{Ha^4(1+\lambda_1)} - \frac{Gr}{Ha^2} \right) + \frac{Gr\beta}{Ha^2} h^3 \right\} dx}{\int_0^l \frac{Ha^2}{\frac{\tanh \left(h \left(Ha \left(\sqrt{1+\lambda_1} \right) \right) \right)}{Ha \left(\sqrt{1+\lambda_1} \right)} - h} dx} \cdot \psi \quad (19)$$

The volumetric flow rate Q quantifies the volume of fluid flowing through a designated point in a pipe or channel within a specified time interval and is determined as:

$$Q = \int_0^h u dy = \frac{1}{Ha \sqrt{1+\lambda_1}} \left(\frac{1}{Ha^2} \frac{\partial p}{\partial x} + \frac{Gr\beta}{Ha^4(1+\lambda_1)} - \frac{Gr}{Ha^2} \tanh \left(h \left(Ha \left(\sqrt{1+\lambda_1} \right) \right) \right) - \frac{1}{Ha^2} \frac{\partial p}{\partial x} h - \frac{Gr\beta}{Ha^2} \frac{h^3}{6} - \frac{Grh}{Ha^2} \frac{\beta}{Ha^2(1+\lambda_1)} - 1 \right) \cdot \psi \quad (20)$$

Stream function (ψ) can also be computed as:

$$\psi = \frac{1}{Ha^2} \frac{\partial p}{\partial x} \left(\frac{\sinh y Ha \sqrt{1+\lambda_1}}{\cosh h Ha \sqrt{1+\lambda_1}} \right) (-y) + \frac{1}{Ha \sqrt{1+\lambda_1}} \frac{\sinh y Ha \sqrt{1+\lambda_1}}{\cosh h Ha \sqrt{1+\lambda_1}} \left(\frac{Gr\beta}{Ha^4(1+\lambda_1)} - \frac{Gr}{Ha^2} + \frac{Gr\beta h^2}{Ha^2} y - \frac{Gr\beta y^3}{Ha^2} - \frac{Gr y}{Ha^2} \frac{\beta}{Ha^2(1+\lambda_1)} - 1 \right) \quad (21)$$

The wall shear stress is an essential factor in characterizing fluid flow, as it is affected by resistive forces, and is defined and computed as:

$$\tau|_{y=h} = \frac{\partial u}{\partial y}|_{y=h} = Ha \sqrt{1+\lambda_1} \left(\frac{1}{Ha^2} \frac{\partial p}{\partial x} + \frac{Gr\beta}{Ha^4(1+\lambda_1)} - \frac{Gr}{Ha^2} \right) \tanh h Ha \sqrt{1+\lambda_1} \left(- \frac{Gr\beta h}{Ha^2} \right) \cdot \psi \quad (22)$$

The Nusselt number quantifies the heat transfer rate caused by fluid motion in a thermofluidic system, and is mathematically expressed as:

$$Nu = \frac{\partial h}{\partial x} \frac{\partial \theta}{\partial y}|_{y=h} \cdot \psi \quad (23)$$

2.5 Entropy generation

Entropy generation refers to the rise in entropy within a system because of the irreversibilities like heat transfer, friction and mixing, indicating the degree of energy dissipation, which can be defined as:

$$\overline{EG} = \frac{k}{T_0} \left(\frac{\partial T}{\partial x}^2 + \frac{\partial T}{\partial y}^2 \right) + \left(\frac{\mu}{T_0} \left(\frac{\partial \bar{u}}{\partial x}^2 + \frac{\partial \bar{v}}{\partial y}^2 \right) + \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) \cdot \psi \quad (24)$$

Further it can be expressed in the nondimensional form as:

$$EG = \frac{\overline{EG}}{EG_0} \cdot \psi \quad (25)$$

Here, $EG = \frac{k(\Delta T)^2}{T_0^2 b^2}$, and $\Delta T = (T_1 - T_0)$ represent the temperature difference between the wall and the center of the channel, and the normalized form of entropy generation can further be expressed as:

$$EG = \frac{\partial \theta}{\partial y}^2 + \frac{Br}{\Delta T} \frac{\partial u}{\partial y}^2 \cdot \psi \quad (26)$$

Here, $Br = \frac{\mu u_0^2}{k \Delta T}$ is Brinkmann number and $\Delta T = \frac{\Delta T}{T_0}$ is relative temperature.

3. Results and discussion

This study investigates the thermal behavior of Jeffrey fluid flow in the presence of a magnetic field, induced by rhythmic membrane propagation with varying time phase lags. Jeffrey fluid and membrane-based microchannels are chosen for their broad applicability. The governing equations are solved analytically, and MATLAB software is used to generate graphs and evaluate the effects of physical parameters ranges including Hartmann number

($Ha = 1-3$), Grashof number ($Gr = 1-5$), heat source parameter ($\beta = 1-7$), Jeffrey fluid parameter ($\lambda_1 = 1-7$), membrane shape parameter ($M = 2-6$), relative temperature ($\Delta T = 0.1-0.9$) and Brinkmann number ($Br = 0.1-0.9$) (Kumar *et al.*, 2024; Bhandari *et al.*, 2022b). The results include an analysis of entropy generation, pressure distribution, pressure gradient, heat transfer rate, axial and transverse velocities, volumetric flow rate and wall shear stress. The dimensionless form for the membrane wall is expressed as:

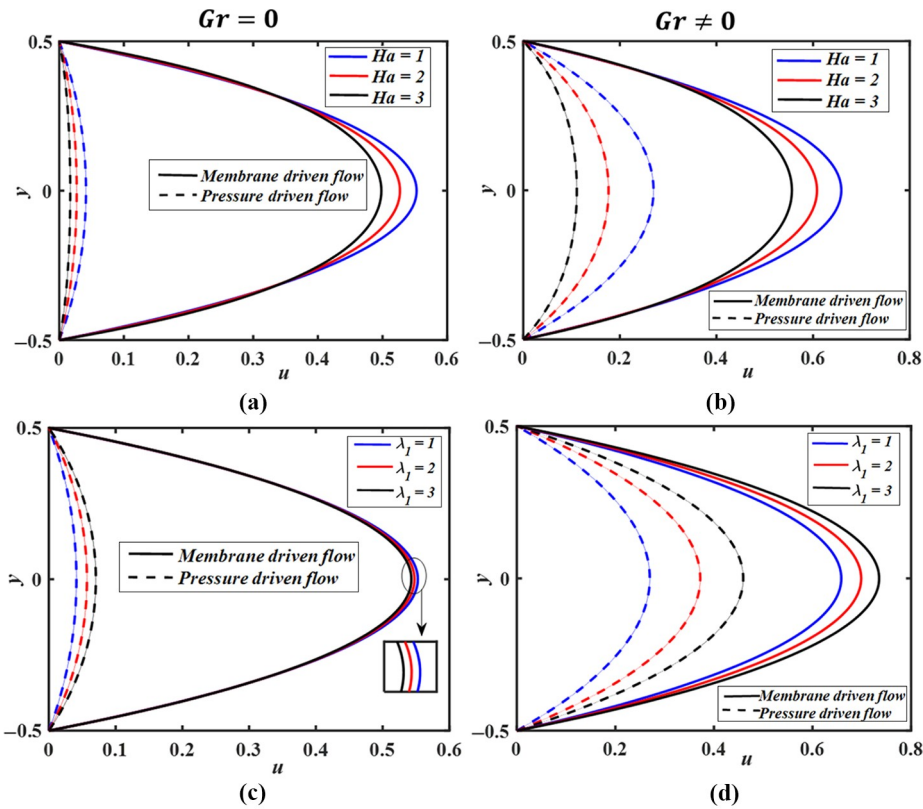
$$h(x, t) = \begin{cases} \frac{1}{2} & -\frac{L}{2} \leq x < -a \\ \frac{1}{2} + 2a_0 \left(\frac{x}{a} \right)^{2M} - 1 & -a \leq x \leq a \\ \frac{1}{2} & a < x \leq \frac{L}{2} \end{cases}, \psi \quad (27)$$

The outcomes of this study can be examined as follows.

3.1 Velocity profile

Figures 2(a-d) provides the comparative analysis for the fluid driven by two mechanisms: pressure-driven flow (due to a pressure gradient created at the inlet of the channel) and fluid flow driven by a membrane. In Figures 2(a, b), the axial velocity profile is analyzed for the various values of Hartmann number in the presence and absence of the buoyancy force. The results clearly show that an increase in the Hartmann number, a dimensionless parameter representing the effect of the Lorentz force, leads to a decrease in axial velocity. This phenomenon occurred for membrane-driven and pressure-driven flow (Bhatti *et al.*, 2016). In the membrane-driven flow, we have considered the external pressure at the inlet to be zero, and for the pressure-driven flow, we have neglected the impact of the flexible membrane by substituting the amplitude of the membrane as zero. Furthermore, Figure 2(c, d) also demonstrate the impact of axial velocity without buoyancy forces, and with buoyancy forces. Figure 2(c) depicts that in the case of pressure-driven flow, an increase in the Jeffrey fluid parameter (λ_1) results in a significant increase in axial velocity. In contrast, when the flow is influenced by membrane action, a rise in λ_1 leads to a notable decrease in axial velocity. These observations strongly indicate that the Jeffrey fluid parameter exerts a more substantial effect on flow characteristics in the absence of buoyancy forces. Figure 2(d) demonstrates that in the presence of buoyancy forces ($Gr \neq 0$), increasing the Jeffrey fluid parameter enhances the axial velocity in both pressure-driven and membrane-driven flows. This observation aligns with previous findings (Bhatti *et al.*, 2016; Vajravelu *et al.*, 2011) and emphasizes the significant influence of buoyancy effects on fluid behavior in microchannels.

Figure 3(a) illustrates that the axial velocity increases with a higher Grashof number in both pressure-driven and membrane-driven flows. This behavior aligns with findings in prior studies (Vajravelu *et al.*, 2011), as the Grashof number represents buoyancy effects. Enhanced buoyancy promotes fluid acceleration, thereby elevating axial velocity in both flow mechanisms. Figure 3(b) shows that an increase in heat source parameters enhanced axial velocity, highlighting the strong impact of thermal energy on the flow's dynamics. This indicates that thermal inputs enhance fluid movement along the axis, underscoring the critical role of heat generation in modifying the flow behavior. Figure 4 presents a comparative analysis of axial velocity profiles for the pressure-driven ($a_0 = 0$) and membrane-driven flows ($\rho_0 = 0$). The solid lines represent the pressure-driven flow and the dots (\bullet) denote the membrane-driven flow. The results reveals that both flow types exhibit

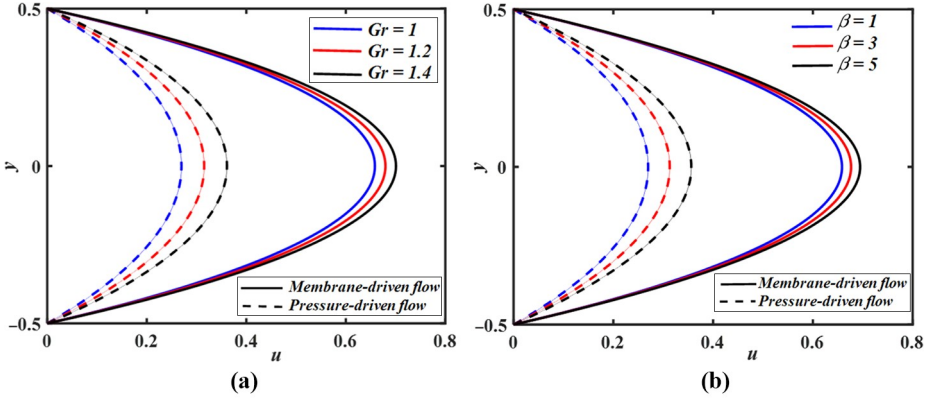


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Figure 2. Comparative analysis of axial velocity between pressure-driven flow and membrane driven flow for different values of (a, b) Hartmann number (Ha) and (c, d) Jeffrey fluid parameter (λ_1), at fixed values of $\beta = 1$, $t = 0.25$ and $x = 0.75$

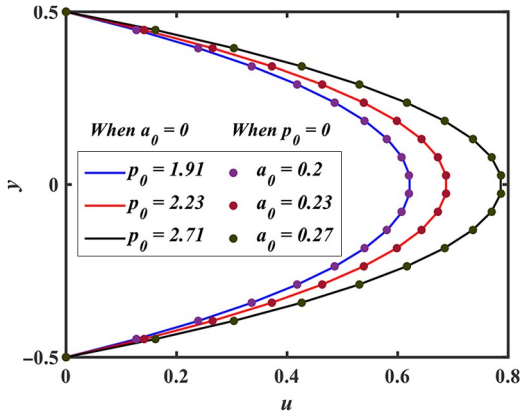
similar behavior when inlet pressure is set at $p_0 = 1.91, 2.23, 2.71$ and the membrane amplitude is $a_0 = 0.2, 0.23, 0.27$. This finding highlights the comparable influence of these parameters on flow dynamics, providing valuable insights for optimizing fluid systems in various applications.

Figures 5(a-d) analyze the impact of numerous parameters like the Hartmann number (Ha), Grashof number (Gr), heat source parameter (β) and Jeffrey fluid parameter (λ_1) on transverse velocity. Figure 5(a) illustrates the effect of the Hartmann number (Ha) on the transverse velocity profile. At lower Ha values, the transverse velocity profile diminished and smoother, reflecting weaker magnetic damping. Figure 5(b) depicts the influence of the Grashof number (Gr), revealing that as Gr increases, the transverse velocity also increases (Bhandari and Tripathi, 2022). Further, Figures 5(c, d) illustrate that the transverse velocity rises with an increase in both the heat source parameter and the Jeffrey fluid parameter. The heat source parameter amplifies thermal and buoyancy-driven forces (Bhandari and Tripathi, 2022), while the Jeffrey fluid parameter adjusts the fluid's



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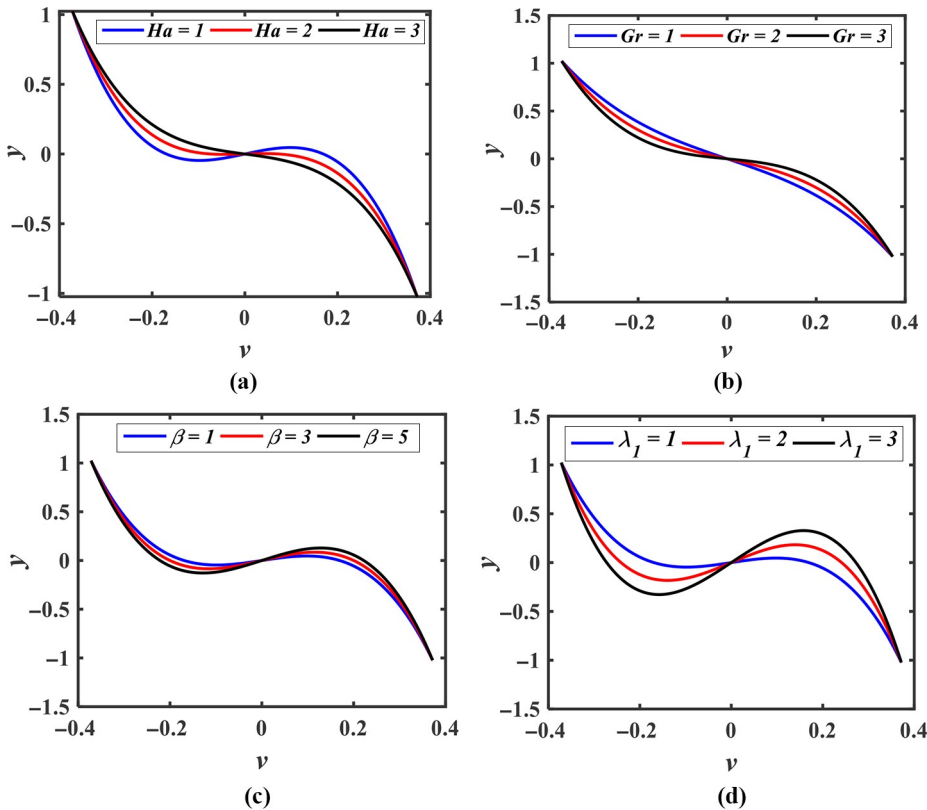
Figure 3. Comparative analysis of axial velocity between pressure-driven flow and membrane driven flow for different values of (a) Grashof number (Gr) and (b) heat source parameter (β), at fixed values of $Ha = 1, \lambda_1 = 1, t = 0.25$ and $\chi = 0.75$



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Figure 4. Comparative analysis of axial velocity between pressure-driven flow and membrane driven flow for different values of inlet pressure (p_0), and amplitude of the membrane (a_0), at fixed values of $Ha = 1, \lambda_1 = 1, Gr = 1, \beta = 1, t = 0.25$ and $\chi = 0.75$

behaviour, enabling more pronounced velocity variations. Physically, the heat source parameter intensifies thermal and buoyancy-driven forces within the fluid, promoting enhanced fluid motion. Meanwhile, the Jeffrey fluid parameter characterizes the fluid's viscoelastic properties, influencing its responsiveness to the driving forces. As this parameter increases, the fluid becomes more adaptable to these thermal and buoyancy effects, resulting in greater transverse velocity variations. This interplay highlights how thermal energy sources and fluid elasticity synergistically affect the flow dynamics, leading to notable velocity augmentation.

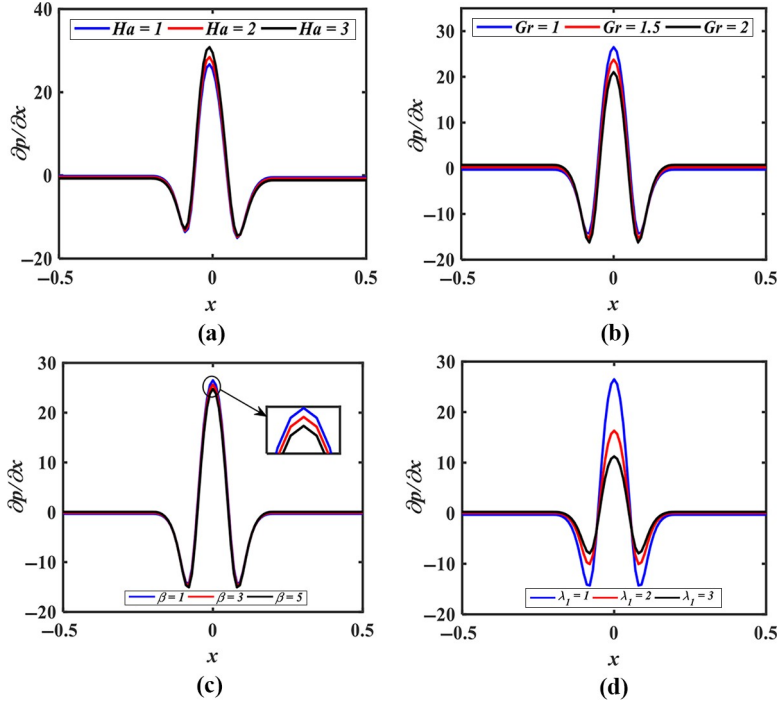


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Figure 5. Transverse velocity profiles for different values of (a) Hartmann number (Ha), (b) Grashof number (Gr), (c) heat source parameter (β) and (d) Jeffrey fluid parameter (λ_1) for fixed values of $M = 2$, $a = 0.2$, $a_0 = 0.2$, $k_0 = 9.95$, $t = 0.25$, $x = 0.5$

3.2 Pressure gradient

Figures 6(a-d) illustrate the changes in the axial pressure gradient for the of Hartmann number (Ha), Grashof number (Gr), heat source parameter (β) and Jeffrey fluid parameter (λ_1). Figure 6(a) elucidates the variation in axial pressure gradient for different values of Hartmann number (Ha). The higher values of Ha enhance the magnetic field's damping effect on fluid flow, necessitating a greater pressure gradient to sustain the same flow rate due to the suppression of fluid motion (Bhandari *et al.*, 2020). Figure 6(b) shows that higher the Grashof number implying the stronger buoyancy forces which enhance fluid motion, leading to a reduced pressure gradient required to maintain the same flow rate. Figure 6(c) shows that as the heat source parameter increases, fluid temperature rises, reducing viscosity and lowering the pressure gradient. Additionally, Jeffrey fluid parameter (λ_1) influences the fluid's viscoelastic behavior, which is the ratio of relaxation to retardation time, affecting its overall flow dynamics and response to external forces. As shown in Figure 6(d), enhancing the ratio of relaxation times amplifies the elastic effects, influencing flow behavior and the



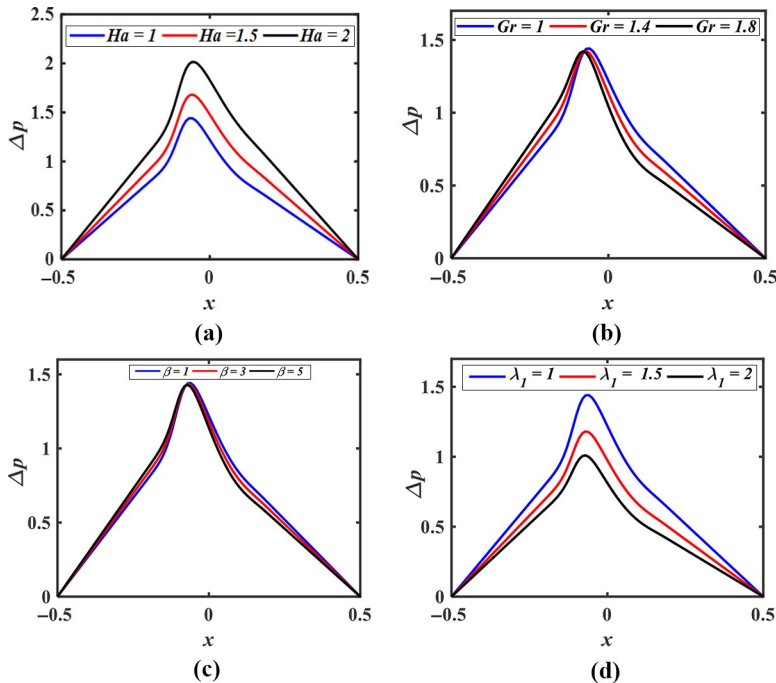
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Figure 6. Pressure gradient for different values of (a) Hartmann number (Ha), (b) Grashof number (Gr), (c) heat source parameter (β) and (d) Jeffrey fluid parameter (λ_1) at the transition phase with the fixed values $M = 2$, $a = 0.2$, $a_0 = 0.2$, $k_0 = 9.95$

pressure gradient see. In membrane-based pumping systems, a higher ratio results in a reduction of the pressure gradient, as the elastic properties of the fluid become more prominent. This change alters the overall flow dynamics, highlighting the significant impact of relaxation time ratios on the pumping performance and pressure distribution in the system.

3.3 Pressure distribution

3.3.1 Compression phase. Pressure distribution is the variation of pressure within a microchannel during fluid flow, which helps to explain the fluid behavior under various conditions. In a symmetrical microchannel utilizing membrane-based pumping, the pressure distribution during the compression phase ($t = 0.25$) is significantly affected by the membrane's motion. The maximum pressure occurs at the microchannel's center, where the membrane is integrated. **Figures 7(a-d)** illustrate how various parameters influence this pressure distribution. **Figure 7(a)** highlights the impact of the Hartmann number (Ha), showing that an increase in Ha leads to enhance pressure distribution. **Figure 7(b)** examines the effect of the Grashof number (Gr) during the compression phase (Bhandari *et al.*, 2020). As Gr increases, the pressure distribution rises, reflecting the dominance of buoyant forces over viscous forces, which enhances natural convection and elevates pressure (Bhandari *et al.*, 2022b). Similarly, **Figure 7(c)** demonstrates that as the heat source parameter (β) increases, the pressure distribution also



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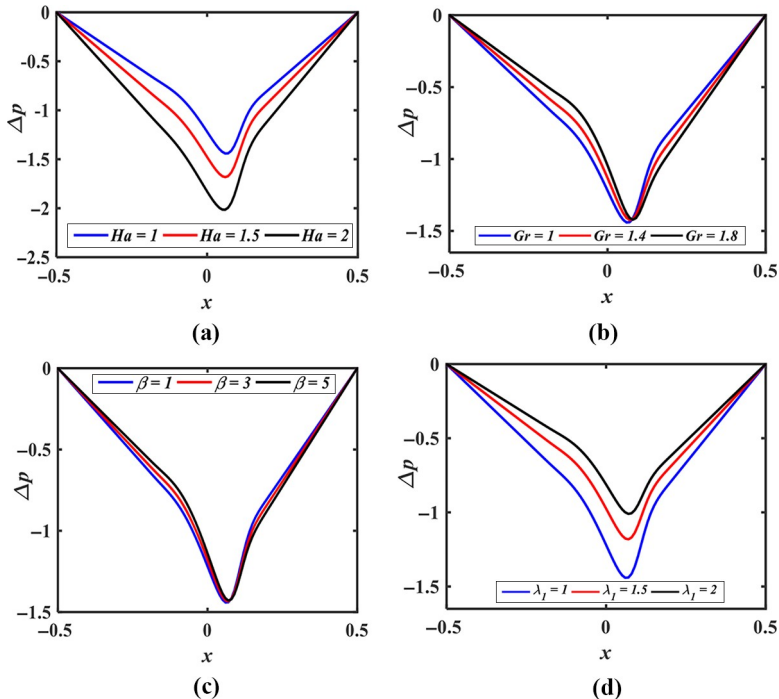
Figure 7. Pressure distribution at compression phase ($t = 0.25$) for different value of (a) Hartmann number (Ha), (b) Grashof number (Gr), (c) heat source parameter (β) and (d) Jeffrey fluid parameter (λ_1) at expansion phase with the fixed values $M = 2$, $a = 0.2$, $a_0 = 0.2$, $k_0 = 9.95$

risers. This indicates that a higher β leads to elevated temperature, stronger buoyancy-driven convection, and increased pressure (Bhandari *et al.*, 2022b). Finally, Figure 7(d) explores the effect of the Jeffrey fluid parameter (λ_1), revealing that a higher λ_1 significantly diminishes the pressure distribution. This underscores the influence of elastic effects, as a larger Jeffrey fluid parameter intensifies pressure during membrane compression.

3.3.2 Expansion phase. During the expansion phase ($t = 0.75$), the results are similar to those in Figures 8(a-d), but exhibit an inverse effect, as illustrated in Figures 8(a-d). This difference is attributed to the increased cross-sectional area available for fluid flow during expansion phase. The impact of various parameters including the Hartmann number (Ha), Grashof number (Gr), heat source parameter (β) and Jeffrey fluid parameter (λ_1) is evident in this phase. In Figure 8(a), with an increase in Ha , a rise in pressure distribution can be seen, which behaves oppositely compared to the contraction phase ($t = 0.25$) (Bhandari *et al.*, 2020). This is due to the movement between the magnetic field and fluid flow. For other parameters like Grashof number (Gr), heat source parameter (β) and Jeffrey fluid parameter (λ_1), an increase in their values reduces pressure distribution during the expansion phase ($t = 0.75$), which again shows a reversed trend compared to the contraction phase ($t = 0.25$). This reduction suggests that buoyancy forces, heat generation and elastic effects have a less pronounced impact on the fluid during expansion, requiring less pressure to drive the flow.

3.4 Volumetric flow rate

The volumetric flow rate is analyzed for a viscoelastic fluid in membrane-based pumping, considering factors such as Hartmann number (Ha), Grashof number (Gr), heat source parameter (β) and Jeffrey fluid parameter (λ_1) for viscoelastic fluid in membrane-based pumping. In Figure 9(a), as Ha increases, the Lorentz force (due to magnetic field) becomes stronger, decreasing the fluid flow leads to a reduction in the volumetric flow rate. Figure 9 (b) shows that a higher Grashof number (Gr), which corresponds to stronger buoyancy forces, enhances the volumetric flow rate. The heat source parameter (β) represents internal heat generation within the fluid. An increase in β raises the fluid's temperature, influencing viscosity, density and flow rate. Figure 9(c) demonstrates that an increase in β leads to a higher volumetric flow rate. Bhandari *et al.* (Bhandari *et al.*, 2022b) reported a comparable trend, noting that an increase in the Grashof number was associated with a rise in the volumetric flow rate. Finally, Figure 9(d) illustrates the role of the Jeffrey fluid parameter (λ_1) in defining the viscoelastic properties of the Jeffrey fluid. A higher λ_1 suggests more pronounced viscoelastic effects, which can enhance resistance to deformation, thereby lowering the volumetric flow rate. These parameters collectively influence the dynamics of fluid flow in the system.

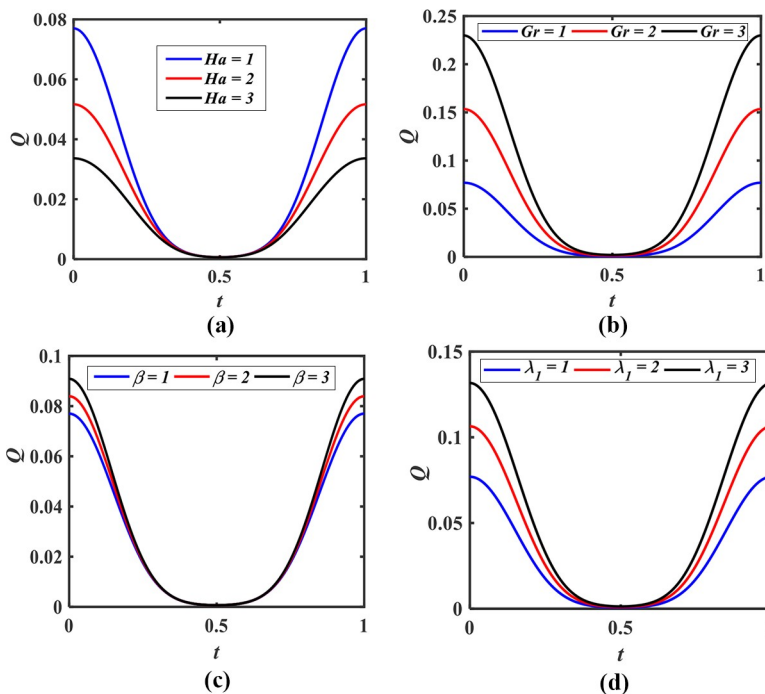


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Figure 8. Pressure distribution at expansion phase for different value of (a) Hartmann number (Ha), (b) Grashof number (Gr), (c) heat source parameter (β) and (d) Jeffrey fluid parameter (λ_1) at expansion phase ($t = 0.75$) with the fixed values $M = 2$, $a = 0.2$, $a_0 = 0.2$, $k_0 = 9.95$

3.5 Heat transfer rate (Nusselt number)

The Nusselt number (Nu) is a dimensionless quantity that quantifies the ratio of convective to conductive heat transfer at a fluid boundary. It is commonly used to assess heat transfer efficiency, especially in fluid flow applications like heat exchangers. Mathematically, it can be expressed as: $Nu = \frac{\partial h}{\partial x} \frac{\partial \theta}{\partial y} \Big|_{y=h}$. The Nusselt number is analyzed for different membrane shape parameters (M) and heat source parameter (β) to understand their effect on heat transfer. In Figure 10(a), the Nusselt number remains zero throughout the channel except where membranes are attached, indicating that membrane-based pumping enhances heat transfer in those areas. The largest variation occurs when the membrane shape parameter is $M = 6$, where fluctuations are more pronounced due to the high amplitude at $t = 0.5$. Figure 10(b) shows the effect of the heat source parameter (β) on the Nusselt number, with convective heat transfer increasing as β rises (Bhandari and Tripathi, 2022). When $\beta = 6$, more significant fluctuations are observed, similar to those seen with membrane shape parameters, demonstrating that both membrane shapes and heat sources significantly influence heat transfer dynamics within the system.



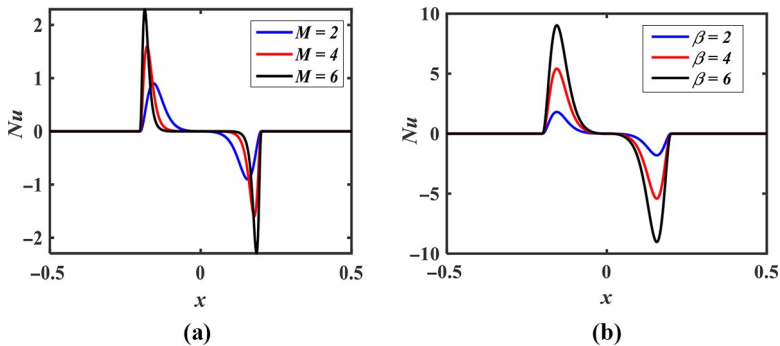
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Figure 9. Volumetric flow rate for different values of (a) Hartmann number (Ha), (b) Grashof number (Gr), (c) heat source parameter (β) and (d) Jeffrey fluid parameter (λ_1) at expansion phase ($t = 0.75$) with the fixed values $M = 2$, $a = 0.2$, $a_0 = 0.2$, $k_0 = 9.95$

3.6 Entropy generation

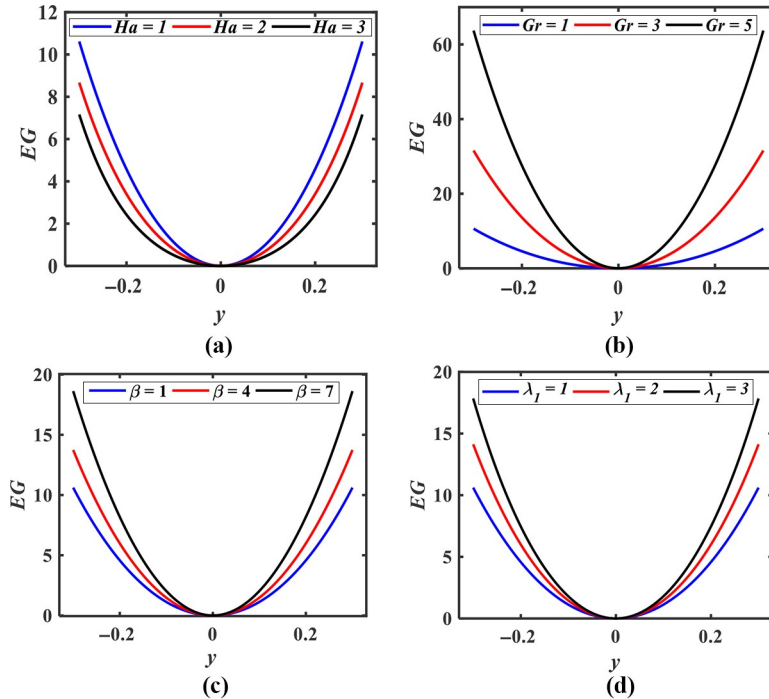
Entropy generation (*EG*) quantifies a system’s irreversibility, reflecting the loss of usable energy caused by inefficiencies such as heat transfer and fluid friction. It is mathematically expressed as: $EG = \frac{\partial \theta}{\partial y}^2 + \frac{Br}{\Delta T} \frac{\partial u}{\partial y}^2$. The first term, $\frac{\partial \theta}{\partial y}^2$, represents thermal dissipation, while the second term, $\frac{\partial u}{\partial y}^2$, accounts for viscous dissipation. The Brinkman number

$Br = \frac{\mu u_0^2}{k \Delta T}$ quantifies viscous heating in the fluid, and $\Delta T = \frac{\Delta T}{T_0}$ represents relative temperature. The variation in entropy generation with parameters such as Hartmann number (*Ha*), Grashof number (*Gr*), heat source parameter (β), Jeffrey fluid parameter (λ_1), membrane shape parameter (*M*), Brinkman number (*Br*) and relative temperature (ΔT) is analyzed through Figures 11(a-d) and 12(a-c). Figure 11(a) shows that increasing the Hartman number (*Ha*), which demonstrates the impact of a magnetic field on fluid flow, typically suppresses fluid motion due to magnetic damping. This reduction in flow decreases viscous dissipation, leading to lower entropy generation associated with viscous effects. In contrast, Figure 11(b) demonstrates that a higher Grashof number (*Gr*) results in stronger buoyancy forces, which increase fluid motion. Figures 11(c, d) further reveal that both the heat source parameter (β) and Jeffrey fluid parameter (λ_1) increases the entropy generation. The presence of a heat source parameter raises the fluid’s temperature, enhancing thermal dissipation (Bhandari and Tripathi, 2022), while a higher λ_1 value, representing the fluid’s viscoelasticity, adds to the complexity of the flow and increases viscous dissipation, thus contributing to higher entropy. Moving to Figures 12(a-c), Figure 12(a) indicates that increasing the membrane shape parameter (*M*) enhances fluid flow, leading to increased entropy generation (Bhandari and Tripathi, 2022). As the membrane deforms, the fluid motion becomes more intense, driving higher dissipation rates. Figures 12(b, c) examine the impact of the Brinkman number (*Br*) and relative temperature (ΔT) on entropy generation profile. A higher Brinkman number leads to an increased entropy generation, signifying greater energy loss due to frictional heating. However, an increase in relative temperature (ΔT) shows the opposite effect, reducing entropy generation (Bhandari and Tripathi, 2022). This inverse relationship suggests that higher temperatures lower the



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Figure 10. Variation in heat transfer rate (Nusselt number) for different value of (a) membrane shape parameter (*m*) and (b) heat source parameter (β) at the fixed values $a = 0.2, a_0 = 0.2, k_0 = 9.95, t = 0.5$



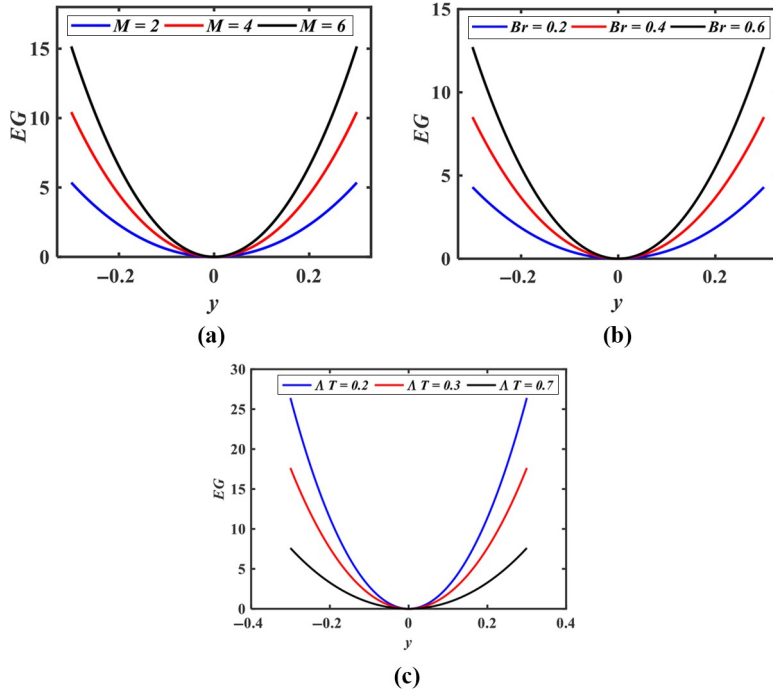
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Figure 11. Entropy generation for different values of (a) Hartmann number (Ha), (b) Grashof number (Gr), (c) heat source parameter (β) and (d) Jeffrey fluid parameter (λ_1) with the fixed values $M = 2$, $\alpha = 0.2$, $\alpha_0 = 0.2$, $k_0 = 9.95$, $Br = 0.5$, $\Delta T = 0.5$, $t = 0.25$, $x = 0.5$

temperature gradients within the fluid, reducing thermal dissipation and overall entropy production.

3.7 Wall shear stress

Figures 13(a-d) show the variation in wall shear stress (τ) along the wall for different parameter values, including Hartmann number (Ha), Grashof number (Gr), heat source parameter (β) and the Jeffrey fluid parameter (λ_1). In Figure 13(a), it is observed that the wall shear stress peaks in the contraction region due to membrane movement. The effect of varying Ha on wall shear stress is examined, revealing that an increase in Ha , which indicates the magnetic field's influence on fluid flow, results in higher wall shear stress (Bhandari *et al.*, 2020). This is attributed to the stronger Lorentz force at higher Ha , which increases flow resistance. Figure 13(b) shows that higher Grashof number, representing stronger buoyancy force, reduce local wall shear stress, which implies fluid flow is enhanced (Bhandari and Tripathi, 2022). Figure 13(c) indicates that increasing the heat source parameter decreases wall shear stress, likely due to the thermal effects reducing viscosity and weakening fluid resistance (Bhandari and Tripathi, 2022). Finally, Figure 13(d) demonstrates that as the Jeffrey fluid parameter (λ_1) increases, the viscoelastic nature of the Jeffrey fluid leads to a reduction in wall shear stress, as the fluid becomes more resistant to deformation.

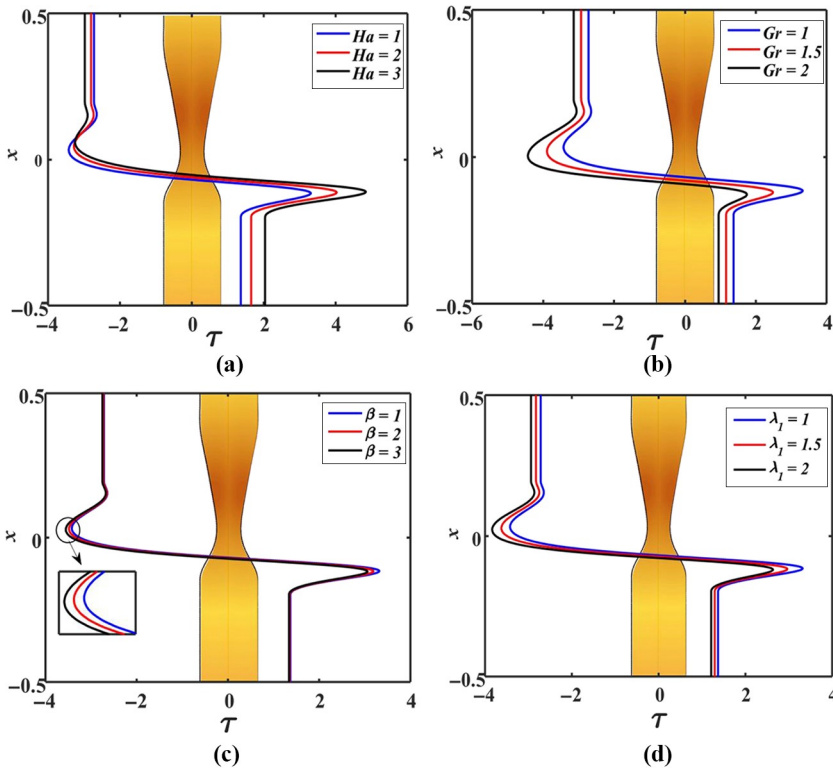


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Figure 12. Entropy generation for different values of (a) membrane shape parameter (m), (b) Brinkman number (Br) and (c) relative temperature (AT) at the time $t = 0.25$ and position $x = 0.5$ for the fixed values $a = 0.2$, $a_0 = 0.2$, $k_0 = 9.95$, $Ha = 1$, $\beta = 1$, $Gr = 1$, $\lambda_1 = 1$

3.8 Streamlines

Contour plots for the stream function during the transition phase ($t = 0.5$) for varying values of the Hartmann number (Ha), Grashof number (Gr), heat source parameter (β), Jeffrey fluid parameter (λ_1), membrane shape parameter (M) and time variation (t) are depicted in Figures 14(a-f). Figure 14(a) shows how the Hartmann number affects the streamlines. As Ha increases, the magnitude of the stream function decreases, indicating that higher Hartmann number suppresses fluid flow due to the magnetic damping effect. Figure 14(b) highlights the effect of the Grashof number, where an increase in Gr leads to a rise in the stream function magnitude, reflecting enhanced fluid flow. Figures 14(c, d) illustrate the impact of the heat source parameter (β) and Jeffrey fluid parameter (λ_1) on the streamlines. In Figure 14(c), a higher heat source parameter increases the stream function, suggesting that internal heating enhances fluid motion. In Figure 14(d), the Jeffrey fluid parameter, which define the characteristics of viscoelastic fluids i.e., Jeffrey fluids, is shown to amplify the stream function. Higher λ_1 values make the fluid more elastic, allowing it to store and release energy efficiently, promoting more vigorous fluid flow in the presence of heat transfer. In Figure 14(e), the membrane shape parameter (M) is examined. As the membrane shape parameter increases, so does the stream function, altering the membrane shape can modify fluid motion, offering a potential means of controlling flow patterns. Figure 14(f) shows stream patterns at different time phases. During the contraction phase, the stream function



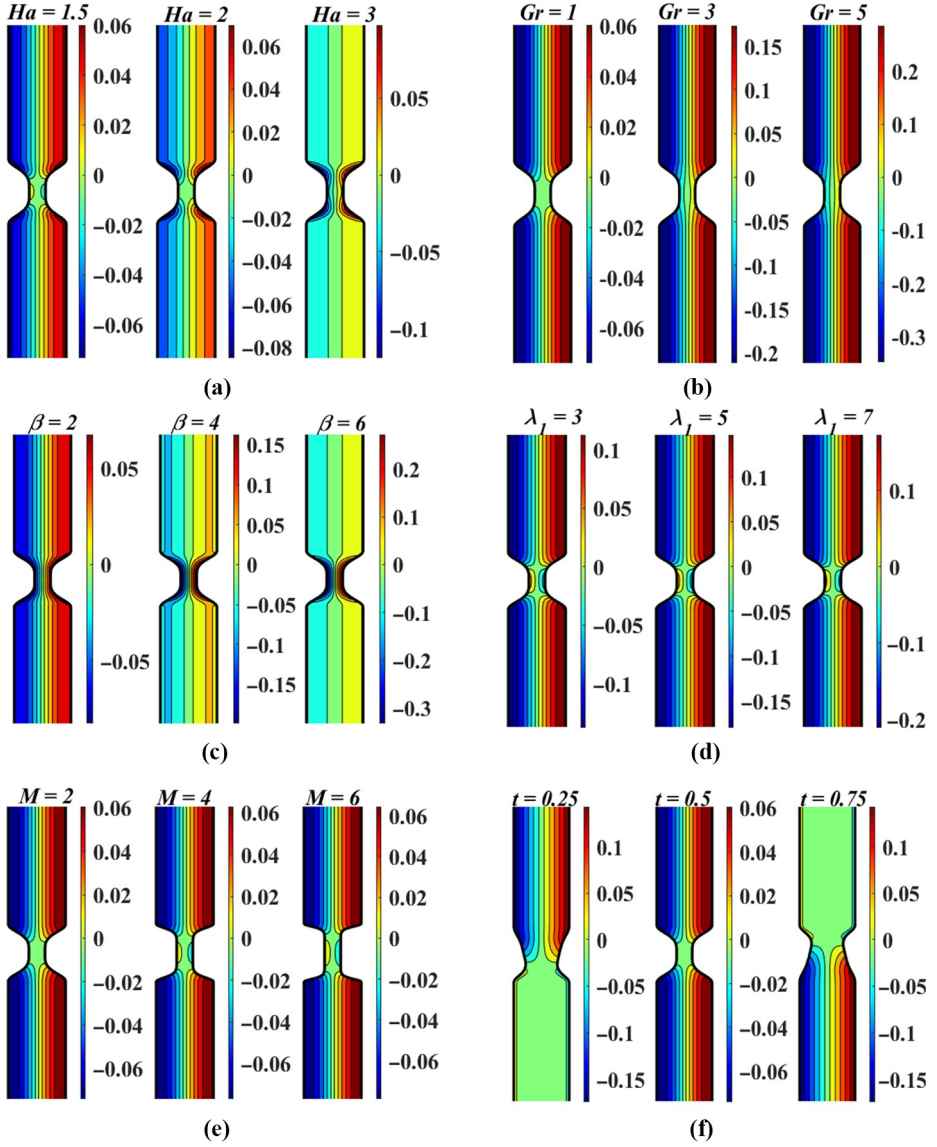
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Figure 13. Wall shear stress at compression phase ($t = 0.25$) for different values of (a) Hartmann number (Ha), (b) Grashof number (Gr), (c) heat source parameter (β) and (d) Jeffrey fluid parameter (λ_1) at the fixed values $M = 2$, $a = 0.2$, $a_0 = 0.2$, $k_0 = 9.95$

magnitude is higher, indicating that the fluid is being compressed and accelerated. In contrast, during the transition phase, the stream function is lower, signaling a shift in flow dynamics. In the expansion phase, the stream function decreases as the cross-sectional area through which the fluid flows increases, suggesting a reduction in flow rate. This reverse behavior in the stream function indicates that as the microchannel expands, the fluid slows down, reflecting the fluid's transition through different flow states.

3.9 Isotherms

Figure 15(a, b) presents contour plots of isotherms for varying heat source parameters (β) and time phases (t). Figure 15(a) examines the effect of heat source parameter at the contraction phase ($t = 0.25$). The results show that as the heat source parameter rises, the contour plots exhibit a noticeable increase in the magnitude of the contour plots of isotherms. This indicates that higher heat source values lead to a more intense thermal response in the flow. Figure 15(b) explores the impact of different time phases including the contraction, transition and expansion phases, on the contour plots. It is also observed that the temperature reaches



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Figure 14. Streamlines at transition phase ($t = 0.5$) for different values of (a) Hartmann number (Ha) when $M = 2$, $Gr = \lambda_1 = \beta = 1$, (b) Grashof number (Gr) when $Ha = \lambda_1 = \beta = 1$, $M = 2$, (c) heat source parameter (β) when $M = 2$, $Gr = Ha = \lambda_1 = 1$, (d) Jeffrey fluid parameter (λ_1) when $M = 2$, $Ha = Gr = \beta = 1$, (e) shape of the membrane (M) when $Ha = Gr = \beta = 1$ and (f) different time phase when $M = 2$, $Ha = Gr = \beta = 1$

maximum at the center of the microchannel, suggesting that higher heat source parameters intensify the temperature gradient.

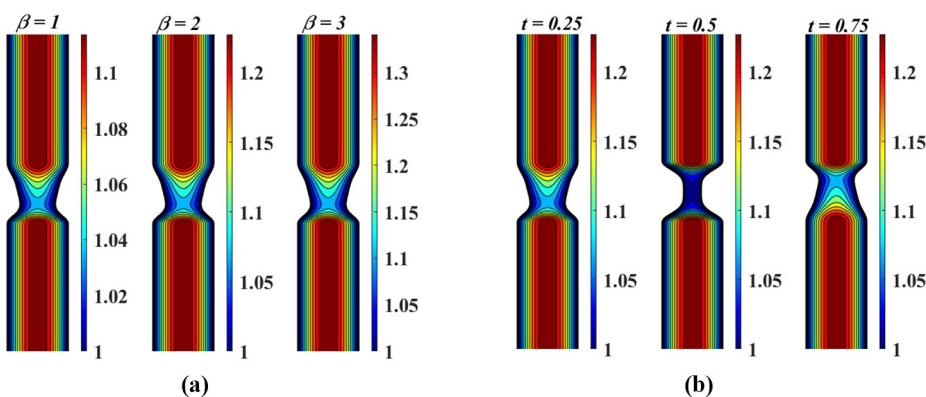
3.10 Velocity distribution

Figures 16(a, b) illustrate the contour plots for the axial velocity and transverse velocity at the different times $t = 0.25$, $t = 0.5$ and $t = 0.75$. In Figure 16(a), the axial velocity profile illustrates the fluid speed distribution along the channel length. It is noted that during the transition phase ($t = 0.5$), the axial velocity is lower compared to the compression and expansion phases, due to maximum compression of the microchannel. Additionally, Figure 16(b) shows that the transverse velocity is minimal during the transition phase when compared to the compression and expansion phases. The transverse velocity also varies where the membrane is fitted within the microchannel.

4. Conclusions

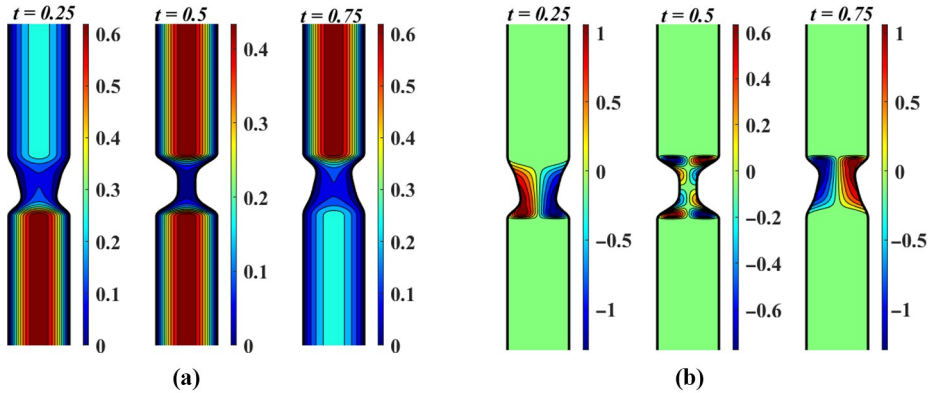
This study investigated the membrane-driven flow and heat transfer of viscoelastic fluids under magnetic field influence. This study demonstrates the significant impact of transverse magnetic field, membrane deformation on viscoelastic fluid flow and heat transfer. A comparative analysis between membrane driven flow and pressure driven flow in absence and presence of buoyancy force has also been presented. Moreover, entropy generation has been discussed. The key findings include:

- The axial velocity decreases as the Hartmann number increases, but it increases with higher values of the Grashof number, heat source parameter and Jeffrey fluid parameter.
- The pressure increases with an increase in the Hartmann number, but decreases as the Grashof number, heat transfer rate and Jeffrey fluid parameter rise.
- The heat transfer rate (Nusselt number) increases with higher values of the membrane shape parameter and heat source parameter.
- The wall shear stress increases with the Hartmann number but decreases as the Grashof number, heat transfer rate and Jeffrey fluid parameter increase.



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Figure 15. Isotherms (θ) for different values of (a) heat source parameter (β) and (b) for different time phase (t) when $Gr = Ha = \lambda_1 = 1$, $M = 2$



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Figure 16. Contour plots for (a) axial velocity and (b) transverse velocity for compression phase ($t = 0.25$), transition phase ($t = 0.5$) and expansion phase ($t = 0.75$) at fixed values of $Ha = \lambda_1 = 1$, $M = Gr = 2$

- Entropy generation decreases with an increase in the Hartmann number and relative temperature, but increases with higher values of the Grashof number, heat transfer rate, Jeffrey fluid parameter, Brinkman number and membrane shape parameter. Increasing the Hartman number reduces the magnitude of the stream function, indicating that stronger magnetic fields suppress fluid flow due to magnetic damping, while higher Grashof numbers and heat source parameters enhance the flow.
- As the heat source parameter rises, the isotherms intensify, showing a more significant thermal response in the flow, with maximum temperatures concentrated at the center of the microchannel during the contraction phase.

The findings provide valuable insights for optimizing membrane-based systems, enhancing heat transfer efficiency and advancing applications in biomedical and industrial fields. This study establishes a foundation for future research on the thermal behavior of non-Newtonian fluids under magnetic fields. Future investigations can build upon this work by exploring complex geometries, varying magnetic field configurations and different non-Newtonian fluids. The ultimate goal is to optimize membrane-driven systems for industrial applications in biomedical engineering, aerospace and renewable energy, improving efficiency and sustainability.

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