# Erratum: High-order algorithm to simulate thermal convection for a thermo-dependent viscoplastic fluid flow

It has come to the attention of the publisher that article Al-Bahrani, B.K. and Al-Muslimawi, A.H. (2025), "High-order algorithm to simulate thermal convection for a thermo-dependent viscoplastic fluid flow". *International Journal of Numerical Methods for Heat and Fluid Flow*, Vol. 35 No. 9 pp. 3322–3345, https://doi.org/10.1108/HFF-02-2025-0115, had errors in several equations with incorrect brackets/parentheses, overlapping numbers, extra symbol within the mathematical and inline equations only in the PDF version of the article which is now corrected.

The table below provides details of the affected equations which have now been corrected in the PDF:

| Equation no. | Incorrect equation | Correct equation |
|---|---|---|
| (1) | $\nabla.u = 0, \psi$ | $\nabla.u = 0,$ |
| (2) | $\frac{\partial up}{\partial tp} = p\frac{1}{Rep}\nabla.\overline{\tau}p - Rep(u.\nabla)u - \nabla, \psi$ | $\frac{\partial u}{\partial t} = \frac{1}{Re}\left(\nabla \cdot \overline{\tau} - Re(u \cdot \nabla)u - \nabla p\right),$ |
| (3) | $\frac{\partial T}{\partial tp} = p\frac{1}{Pep}\nabla^2 Tp - Pep(u.\nabla)T + Brp(\overline{\tau}p.\nabla u).\psi$ | $\frac{\partial T}{\partial t} = \frac{1}{Pe}\left(\nabla^2 T - Pe(u.\nabla)T + Br(\overline{\tau} : \nabla u)\right).$ |
| (4) | $\overline{\tau} = \mu(\dot{\gamma})\overline{\gamma} \cdot \psi\psi$ | $\overline{\tau} = \mu(\dot{\gamma})\overline{\dot{\gamma}}.$ |
| (5) | $\overline{\tau} = \begin{cases} \left(\mu_p + \frac{\tau_0}{\lvert\gamma\rvert}\left[1 - ep^{(-a\lvert\gamma\rvert)}\right]\right)\overline{\dot{\gamma}} & \lvert\overline{\tau}\rvert > \tau_0, \psi \\ \overline{\dot{\gamma}} = 0 & \lvert\overline{\tau}\rvert \le \tau_0, \psi \end{cases}$ | $\overline{\tau} = \begin{cases} \left(\mu_p + \frac{\tau_0}{\lvert\dot{\gamma}\rvert}\left[1 - e^{(-a\lvert\dot{\gamma}\rvert)}\right]\right)\overline{\dot{\gamma}} & \lvert\overline{\tau}\rvert > \tau_0, \\ \overline{\dot{\gamma}} = 0 & \lvert\overline{\tau}\rvert \le \tau_0, \end{cases}$ |
| (6) | $\mu = e\overline{p}^{b(T-T_0)}\mu(\gamma) \cdot \psi\psi$ | $\mu = e^{-b(T-T_0)}\mu(\dot{\gamma}).$ |
| (7) | $\frac{\partial}{\partial tp} + p\frac{1}{Ma^2}(\nabla \cdot u) = 0, \psi\psi$ | $\frac{\partial p}{\partial t} + \frac{1}{Ma^2}(\nabla \cdot u) = 0,$ |
| (8) | $\frac{\partial}{\partial tp} + \xi p + p\frac{1}{Ma^2}(\nabla \cdot u) = 0, \psi$ | $\frac{\partial p}{\partial t} + \xi p + \frac{1}{Ma^2}(\nabla \cdot u) = 0$ |
| (9) | $u^{m+\frac{1}{2}} = u^m + p\frac{\Delta tp}{2Rep} - Re(u^m \cdot \nabla)u^m + p\frac{1}{2}\nabla \cdot \overline{\tau}^{m+\frac{1}{2}} + \nabla \cdot \overline{\tau}^{mp} - \nabla^m, \psi$ | $u^{m+\frac{1}{2}} = u^m + \frac{\Delta t}{2Re}\left(-Re(u^m \cdot \nabla)u^m + \frac{1}{2}\left(\nabla \cdot \overline{\tau}^{m+\frac{1}{2}} + \nabla \cdot \overline{\tau}^m\right) - \nabla p^m\right),$ |
| (10) | $T^{m+\frac{1}{2}} = T^m + p\frac{\Delta tp}{2Pep} - Pe(u^m \cdot \nabla)T^m + \frac{1}{2}\nabla^2 T^{m+\frac{1}{2}} + \nabla^2 T^m + Br(\overline{r}^{mp} : p\nabla u^m) \leftarrow, \psi$ | $T^{m+\frac{1}{2}} = T^m + \frac{\Delta t}{2Pe}\left(-Pe(u^m \cdot \nabla)T^m + \frac{1}{2}\left(\nabla^2 T^{m+\frac{1}{2}} + \nabla^2 T^m\right) + Br(\overline{\tau}^m : \nabla u^m)\right),$ |
| (11) | $m + \frac{1}{2} = p\frac{mp - \frac{\Delta tp}{2Ma^2}\left(\nabla \cdot u^{m+\frac{1}{2}}\right)}{1 + p\frac{\xi\Delta tp}{2}}, \psi$ | $p^{m+\frac{1}{2}} = \frac{p^m - \frac{\Delta t}{2Ma^2}\left(\nabla \cdot u^{m+\frac{1}{2}}\right)}{1 + \frac{\xi\Delta t}{2}},$ |
| (12) | $u^{m+1} = u^m + p\frac{\Delta tp}{Rep} - Repu^{m+\frac{1}{2}} \cdot \nabla u^{m+\frac{1}{2}} + p^{\frac{1}{2}} (\nabla \cdot \overline{\tau}^{m+1} + \nabla \cdot \overline{\tau}^m) - \nabla^{m+\frac{1}{2}}, \psi$ | $u^{m+1} = u^m + \frac{\Delta t}{Re}\left(-Re\left(u^{m+\frac{1}{2}} \cdot \nabla\right)u^{m+\frac{1}{2}} + \frac{1}{2}(\nabla \cdot \overline{\tau}^{m+1} + \nabla \cdot \overline{\tau}^m) - \nabla p^{m+\frac{1}{2}}\right),$ |

(*continued*)

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Continued

| Equation no. | Incorrect equation | Correct equation |
| --- | --- | --- |
| (13) | $T^{m+1} = T^m + p\dfrac{\Delta tp}{Pep} - Pepu^{m+\frac{1}{2}}, \nabla T^{m+\frac{1}{2}} + \dfrac{1}{2}(\nabla^2 T^{m+1} + \nabla^2 T^{mp}) + Brp\bar{\tau}^{m+\frac{1}{2}}; p\nabla u^{m+\frac{1}{2}}, \psi Br\left(\bar{\tau}^{m+\frac{1}{2}} : \nabla u^{m+\frac{1}{2}}\right)\Bigg),$ | $T^{m+1} = T^m + \dfrac{\Delta t}{Pe}\left(-Pe\left(u^{m+\frac{1}{2}} \cdot \nabla\right)T^{m+\frac{1}{2}} + \dfrac{1}{2}(\nabla^2 T^{m+1} + \nabla^2 T^m)\right.$ |
| (14) | ${}^{m+1} = {}^{mp} - \Delta tp\,\xi p^{m+\frac{1}{2}} + p\dfrac{1}{Ma^2}(\nabla \cdot u^{m+1}).\psi$ | $p^{m+1} = p^m - \Delta t\left(\xi p^{m+\frac{1}{2}} + \dfrac{1}{Ma^2}(\nabla \cdot u^{m+1})\right).$ |
| (15) | $\dfrac{2Re}{\Delta tp}M + p\dfrac{1}{2}S_{up}\Delta u^{m+\frac{1}{2}} = p - S_{up} + ReN(u^m)u^{mp} + L^{\top m}, \psi$ | $\left[\dfrac{2Re}{\Delta t}M + \dfrac{1}{2}S_u\right]\Delta u^{m+\frac{1}{2}} = -[S_u + ReN(u^m)]u^m + L^{\top}p^m,$ |
| (16) | $\dfrac{2Pep}{\Delta tp}M + p\dfrac{1}{2}S_{Tp}\Delta T^{m+\frac{1}{2}} = p - PeN(u^m) + S_{Tp}T^{mp} + Br\Phi(u^m), \psi$ | $\left[\dfrac{2Pe}{\Delta t}M + \dfrac{1}{2}S_T\right]\Delta T^{m+\frac{1}{2}} = -[PeN(u^m) + S_T]T^m + Br\Phi(u^m),$ |
| (17) | $Mp^{m+\frac{1}{2}} = p\dfrac{M^{mp} - \dfrac{\Delta t}{2Ma2}Lu^{m+\frac{1}{2}}}{1 + p\dfrac{\xi\Delta tp}{2}}, \psi$ | $M_p p^{m+\frac{1}{2}} = \dfrac{M_p p^m - \dfrac{\Delta t}{2Ma^2}Lu^{m+\frac{1}{2}}}{1 + \dfrac{\xi\Delta t}{2}},$ |
| (18) | $\dfrac{Rep}{\Delta tp}M + p\dfrac{1}{2}S_{up}\Delta u^{m+1} = p - S_u u^{mp} - ReNpu^{m+\frac{1}{2}}u^{m+\frac{1}{2}} + L^{\top m+\frac{1}{2}}, \psi$ | $\left[\dfrac{Re}{\Delta t}M + \dfrac{1}{2}S_u\right]\Delta u^{m+1} = -S_u u^m - ReN\left(u^{m+\frac{1}{2}}\right)u^{m+\frac{1}{2}} + L^{\top}p^{m+\frac{1}{2}},$ |
| (19) | $\dfrac{Pe}{\Delta tp}M + p\dfrac{1}{2}S_{Tp}\Delta T^{m+1} = p - PeNpu^{m+\frac{1}{2}}T^{m+\frac{1}{2}} - S_T T^{mp} + Br\Phi pu^{m+\frac{1}{2}}, \psi$ | $\left[\dfrac{Pe}{\Delta t}M + \dfrac{1}{2}S_T\right]\Delta T^{m+1} = -PeN\left(u^{m+\frac{1}{2}}\right)T^{m+\frac{1}{2}} - S_T T^m + Br\Phi\left(u^{m+\frac{1}{2}}\right),$ |
| (20) | $M\Delta p^{m+1} = p - \Delta tp\xi Mp^{m+\frac{1}{2}} - \dfrac{1}{Ma^2}Lu^{m+1}, \psi$ | $M_p\Delta p^{m+1} = -\Delta t\left(\xi M_p p^{m+\frac{1}{2}} - \dfrac{1}{Ma^2}Lu^{m+1}\right),$ |
| (21) | $\dfrac{2Rep}{\Delta tp}M + p\dfrac{1}{2}S_{up}\Delta u^{m+\frac{1}{2}} = p - S_{up} + ReN(u^m)u^{mp} + L^{\top m}, \psi$ | $\left[\dfrac{2Re}{\Delta t}M + \dfrac{1}{2}S_u\right]\Delta u^{m+\frac{1}{2}} = -[S_u + ReN(u^m)]u^m + L^{\top}p^m,$ |
| (22) | $\dfrac{2Pep}{\Delta tp}M + p\dfrac{1}{2}S_{Tp}\Delta T^{m+\frac{1}{2}} = -PeN(u^m) - S_{Tp}T^m + Br\Phi(u^m), \psi$ | $\left[\dfrac{2Pe}{\Delta t}M + \dfrac{1}{2}S_T\right]\Delta T^{m+\frac{1}{2}} = [-PeN(u^m) - S_T]T^m + Br\Phi(u^m),$ |
| (23) | $\dfrac{Rep}{\Delta tp}M + p\dfrac{1}{2}S_{up}\Delta u^{\star} = p - S_u u^{mp} - ReNpu^{m+\frac{1}{2}}u^{m+\frac{1}{2}} + L^{\top mp}, \psi$ | $\left[\dfrac{Re}{\Delta t}M + \dfrac{1}{2}S_u\right]\Delta u^{\star} = -S_u u^m - ReN\left(u^{m+\frac{1}{2}}\right)u^{m+\frac{1}{2}} + L^{\top}p^m,$ |
| (24) | $\dfrac{Pe}{\Delta tp}M + \dfrac{1}{2}S_{Tp}\Delta T^{m+1} = p - PeNu^{m+\frac{1}{2}}T^{m+\frac{1}{2}} - S_T T^{mp} + Br\Phi pu^{m+\frac{1}{2}}, \psi$ | $\left[\dfrac{Pe}{\Delta t}M + \dfrac{1}{2}S_T\right]\Delta T^{m+1} = -PeNu^{m+\frac{1}{2}}T^{m+\frac{1}{2}} - S_T T^m + Br\Phi\left(u^{m+\frac{1}{2}}\right),$ |
| (25) | $K\Delta p^{m+1} = p - \dfrac{2Re}{\Delta tp}Lu^{\star}, \psi$ | $K\Delta p^{m+1} = -\dfrac{2Re}{\Delta t}Lu^{\star},$ |
| (26) | $\dfrac{2Re}{\Delta tp}M\left(pu^{m+1} - u^{\star}\right) = L^{\top}\Delta p^{m+1}, \psi$ | $\dfrac{2Re}{\Delta t}M(u^{m+1} - u^{\star}) = L^{\top}\Delta p^{m+1},$ |

On page 3325, the incorrect equation $Re = p\frac{\rho U^{2-n}D^{np}}{\mu_0}$, $Pe = p\frac{\rho CpUlp}{kp}$, $Br = p\frac{\mu_0 U^2}{k\Delta Tp}$, has been corrected to $Re = \frac{\rho U^{2-n}D^n}{\mu_0}$, $Pe = \frac{\rho C_p Ul}{k}$, $Br = \frac{\mu_0 U^2}{k\Delta T}$,.

On page 3326, the incorrect equation $\overline{\tau}p$, $\overline{\gamma}p$, $|\gamma| = \sqrt{\frac{1}{2}\Pi_\gamma} = \left(\sqrt{\frac{1}{2}\overline{\gamma} : \overline{\gamma}}\right)$, $f = p\frac{\tau_{wp}}{\frac{1}{2}\rho u_e^2} = p\frac{D\Delta p}{2l\rho u_{ep}^2}$, $Nu = p\frac{H_x D}{k}$. has been corrected to $\overline{\tau}$, $\overline{\gamma}$. $|\dot{\gamma}| = \sqrt{\frac{1}{2}\Pi\dot{\gamma}} = \left(\sqrt{\frac{1}{2}\overline{\dot{\gamma}} : \overline{\dot{\gamma}}}\right)$, $fp = \frac{\tau_w}{\frac{1}{2}\rho u_e^2} = \frac{D\Delta p}{2l\rho u_e^2}$, $Nu = \frac{H_x D}{k}$. respectively in the PDF.

On page 3327, the below equations in the steps 1 and 2 for Continuity equation, Momentum equation, Energy equation has been corrected in the PDF:

| Continuity equation | Momentum equation | Energy equation |
|---|---|---|
| ${}^{m+\frac{1}{2}} = p^{mp} - \frac{\Delta tp}{2}\xi p^{mp} + \frac{1}{Ma^2}(\nabla \cdot u^m) \leftarrow,$ | $u^{m+\frac{1}{2}} = u^m + p\frac{\Delta tp}{2Rep} - Re(u^m \cdot \nabla)u^m + \nabla \cdot \overline{\tau}^{mp} - \nabla^m,$ | $T^{m+\frac{1}{2}} = T^m + p\frac{\Delta tp}{2Pep} - Pe(u^m \cdot \nabla)T^m$<br>$+ \nabla^2 T^m + Br(\overline{\tau}^{mp} : p\nabla u^m),$ |
| ${}^{m+1} = {}^{mp} - \Delta tp\xi p^{m+\frac{1}{2}} + p\frac{1}{Ma^2}(\nabla \cdot u^m) \leftarrow$ | $u^{m+1} = u^m + p\frac{\Delta tp}{Rep}\left(\left(-Repu^{m+\frac{1}{2}}\nabla u^{m+\frac{1}{2}} + \nabla \cdot \overline{\tau}^{m+\frac{1}{2}} - \nabla^m\right)\right).$ | $T^{m+1} = T^m + p\frac{\Delta tp}{Pep} - Pe(u^m \cdot \nabla)T^{m+\frac{1}{2}}$<br>$+ \nabla^2 T^{m+\frac{1}{2}} + Br(\overline{\tau}^{mp} : p\nabla u^m).$ |

On page 3329, the below incorrect inline equations have now been corrected in the PDF:

$$M_{ij} = p\int_\Omega \psi_i\psi_j rd\Omega,\ M_{pij} = p\int_{\Omega p}\left(\phi_i\phi_j rd\Omega,\ N_{ij}(u) = \rho\int_{\Omega p}\left(\psi_{ip}u_k^l\psi_{lp}\frac{\partial\psi_j}{\partial x_k}rd\Omega,\right.\right.$$

$$\psi\psi(L_k)_{ij} = p\int_\Omega \phi_i\nabla_k\psi_j rd\Omega,\ (S_u)_{ij} = (S_{lq})_{ijp} + (V_{lqp})_{ij},\ \psi(S_{Tp})_{ij}$$

$$= \rho\int_\Omega\left(\nabla\psi_i \cdot \nabla\psi_{jp} + \frac{\psi_{ip}}{rp}\frac{\partial\psi}{\partial rp}rd\Omega,\right.$$

$$\Phi_i = p\frac{1}{2}\int_{\Omega p}\mu(\gamma)_{ip}\left[\frac{\partial\psi_{qp}}{\partial x_{kp}}u_l^{qp} + p\frac{\partial\psi_q}{\partial x_l}u_{kp}^{qp^2} + 2\frac{\psi_q}{r}u_{rp}^{qp2}\right]rd\Omega,\ k,\ l = 1,\ 2.$$

$$\nabla\psi_{ip} = \frac{\partial\psi_{ip}}{\partial rp}, \frac{\partial\psi_i}{\partial zp}^{\top},$$

$$(S_{lq})_{ij} = p\int_{\Omega p}\mu(\gamma)\,\chi_{lkp}\frac{\partial\psi_{ip}}{\partial x_{kp}}\frac{\partial\psi_{jp}}{\partial x_{kp}}rd\Omega, \psi if pl = q, \; (S_{lqb})_{ij} = p\int_{\Omega p}\mu(\gamma) \leftarrow \frac{\partial\psi_{ip}}{\partial x_{qp}}\frac{\partial\psi_j}{\partial x_l})rd\Omega, \psi if pl \neq q,$$

$$\chi_{lk} = 2if pl \; = k \, and \, 1 \quad if pl \neq k, \psi$$

$$(V_{lqb})_{ijp} = p\frac{2\psi_i\psi_j}{r^2}, \quad \psi if pl \quad = q = 1, \; (V_{lq})_{ij} = 0, \psi if pl \neq q, \quad such that \quad l, k, q = 1, 2$$

On page 3330, the below incorrect inline equations have now been corrected in the PDF:

$$\Delta u^{m+\frac{1}{2}} = u^{m+\frac{1}{2}} - u^m, \Delta u^{m+1} = u^{m+1} - u^m, \Delta p^{m+1} = {}^{m+1} - {}^m, \psi$$
$$\Delta T^{m+\frac{1}{2}} = T^{m+\frac{1}{2}} - T^m, \Delta T^{m+1} = T^{m+1} - T^m \cdot \psi$$

$$K_{ij} = p\int_{\Omega p}\left(\nabla\phi_i \cdot \nabla\phi_j rd\Omega \cdot \psi\right.$$

On page 3331, the incorrect inline equations $u_r = 0, u_z = u_{fully-developed}\ (r) \cdot \psi,\ u_{sp} = \beta_s\tau_{rz}|_{wall} \cdot \psi$ have been corrected in the PDF.

Additionally, on the first line of page 3332, “620 triangular” is corrected to “1620 triangular”.

This error was introduced during the article publication process, for which the publisher apologises.