

Mixed convection flow and heat transfer of a Williamson tetra-hybrid (SWCNT–MWCNT–SiO₂–TiO₂) nanofluid across a rough wedge

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Abstract

Purpose – The purpose of this study is to investigate the flow and heat transfer enhancement of a Williamson tetra-hybrid nanofluid over a rough wedge in the presence of mixed convection and a periodic magnetic field. Special attention is given to the combined effects of surface roughness, non-Newtonian fluid behaviour and magnetic modulation on momentum and thermal transport. The study aims to evaluate how tetra-hybrid nanoparticles improve thermal efficiency compared to conventional fluids.

Design/methodology/approach – The governing nonlinear partial differential equations for Williamson tetra-hybrid nanofluid flow over a rough wedge are transformed into a nondimensional nonsimilar system using appropriate transformations. The resulting equations are solved numerically using an implicit finite-difference scheme coupled with the quasilinearisation technique. Surface roughness and magnetic field effects are incorporated through sinusoidal wall-velocity oscillations and applied magnetic forces.

Findings – The results demonstrate that surface roughness induces pronounced sinusoidal variations in the skin friction coefficient and heat transfer rate along the wedge. An increase in the roughness-frequency parameter insignificantly amplifies the amplitude of these oscillations, indicating stronger fluctuations in both momentum and thermal transport. Further, the application of a periodic magnetic field markedly intensifies these sinusoidal responses, leading to enhanced overall heat transfer performance.

Originality/value – This study presents a novel analysis of mixed convection Williamson tetra-hybrid nanofluid flow over a rough wedge under the influence of a periodic magnetic field, a combination that remains largely unexplored in existing literature. The originality lies in the integration of non-Newtonian rheology, tetra-hybrid nanoparticle formulation and sinusoidal surface roughness within a unified numerical framework. The findings provide new insights into the enhancement of heat transfer and

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momentum transport, offering practical relevance for advanced thermal management and engineering applications.

Keywords Tetra-hybrid Williamson nanofluid, Wedge, Periodic magnetic field, Surface roughness, Quasilinearisation

Paper type Research paper

Nomenclature

B_0 = Magnetic field strength;
 C_{pf} = Specific heat capacity;
 C_f = Skin friction;
 Ec = Viscous dissipation parameter;
 f = Dimensionless stream function;
 G, F = Temperature and velocity dimensionless profiles;
 g = Acceleration gravity;
 Gr = Grashof number;
 L = Characteristic length;
 m = Wedge angle;
 M = Magnetic strength;
 Nu = Heat transfer rate;
 n = Frequency;
 Pr = Prandtl number;
 Re = Reynolds number;
 Ri = Mixed convection parameter;
 T = Temperature;
 T_w = Wall temperature;
 T_∞ = Ambient temperature;
 u, v = Components of velocities;
 u_∞ = Mainstream velocity; and
 We = Williamson number.

Greek symbols

α = Wall roughness parameter;
 β = Hartree pressure gradient;
 β_f = Volumetric expansion coefficient;
 ϕ = Volume fraction of nanoparticles;
 ρ_f = Fluid density;
 κ = Thermal conductivity;
 ν_f = Kinematic viscosity of fluid;
 σ_f = Electrical conductivity;
 ψ = Stream function;
 ϖ = Sphericity;
 ε = Ratio of velocity; and
 η_∞ = Edge of the boundary layer.

Subscripts

f = Base fluid;
 ∞ = Away from the boundary layer;
w = Wall condition;

ξ, η = Derivatives w.r.t. these variables; and
tethnf = tetra-hybrid nanofluids.

1. Introduction

Non-Newtonian fluids play a crucial role in the mechanisms governing heat and mass transfer in scientific and industrial systems. This is due to their markedly distinct behaviour compared to Newtonian fluids. These fluids exhibit distinct mechanisms of heat transfer, making their study essential for applications requiring enhanced thermal performance. Typical examples of non-Newtonian fluids include coolants, lubricants, blood, food-processing slurries and several complex industrial fluids. These fluids do not conform to Newtonian expectations. This behaviour arises from the intricate microstructural attributes conferred during the fabrication of materials such as polymers, pigments, food products and biological entities. Consequently, non-Newtonian fluids continue to captivate scientists and engineers due to their numerous significant and advantageous properties. The rheological behaviour is crucial for determining process efficiency and the movement of materials within this category of fluids. The Williamson model (Amjad, 2022; Muhammad *et al.*, 2022; Sherani *et al.*, 2025; Akbar *et al.*, 2025) provides a comprehensive constitutive framework for pseudoplastic, shear-thinning fluids, characterised by a consistent reduction in apparent viscosity with increasing shear rate. The model is extensively used for polymeric, biological and industrial fluids due to its precise measurement of shear-dependent viscosity. It can be used in several critical technical domains, including biomedical engineering for blood flow modelling, polymer processing techniques such as extrusion and coating, and lubrication systems where shear-thinning behaviour enhances mechanical and operational efficiency. Yusuf *et al.* (Yusuf, 2020) have recently investigated the magnetohydrodynamic (MHD) effects on Williamson nanofluid across a convective stretching plate, incorporating chemical reactions and entropy generation. Tazin and Ahmmad (Tamanna and Sarder Firoz, 2025) have investigated the effects of periodic magnetic components, thermal radiation, Brownian motion and chemical reactions characterised by Arrhenius activation energy on the time-dependent flow of a radiative micro-non-Newtonian (Williamson) fluid over a stretched surface.

The motivation for this work comes from the growing use of non-Newtonian fluids and carbon nanotubes (CNTs) in contemporary engineering devices. While many other carbon allotropes behave as semiconductors, CNTs exhibit very high electrical conductivity. Their unique nanostructure and strong carbon-carbon bonds provide them with better tensile strength and thermal conductivity, making them particularly effective at managing heat. Water and ethanol are examples of traditional heat transfer fluids that do not transfer heat very well, making them less useful in high-performance systems. Adding CNTs to base fluids, on the other hand, greatly improves their thermophysical properties, thereby enhancing heat transfer efficiency. Because of these qualities, CNT-based fluids have been widely studied for use in biomedical devices, industrial processing, energy systems and cooling technologies for space. Single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs) are two structural varieties of CNTs. These nanotubes have structures and surface properties that can be altered, making them ideal for a wide variety of applications. Because of their excellent electrical and mechanical properties, CNTs are also an important component of many functional devices, such as gas storage systems, ultracapacitors and solar energy components.

Hamzeh and Muhammad (Hamzeh Taha and Muhammad Khairul Anuar, 2023) performed a computational analysis of the flow and thermal transfer properties of porous Williamson hybrid nanofluid over an exponentially shrinking sheet, including MHD effects. Composed of a combination of SWCNTs and MWCNTs, the hybrid nanoparticles have a unique structure. There is a slight decrease in the Nusselt number and a 72.2% increase in skin

friction when the magnetic parameter is increased in the SWCNT+MCWNT hybrid nanofluid compared to the SWCNT nanofluid alone. Ruchi *et al.* (Jain *et al.*, 2024) investigated the thermal and flow properties of single-walled and multi-walled carbon nanotubes (SWCNTs and MWCNTs) and CuO nanoparticles dispersed in water to analyse inclined MHD flow, integrating Hall, Soret and Dufour effects in a porous medium for a Williamson fluid over a stretching/shrinking sheet. Their comparison of CuO–water, SWCNT–water and MWCNT–water nanofluids revealed that hybrid carbon nanotube formulations outperformed nanofluids composed solely of SWCNT or MWCNT in terms of skin friction performance and local Nusselt numbers. Patil and Goudar (Patil and Goudar, 2023) examined the unsteady, coupled convective flow of a Williamson ternary hybrid nanofluid over a rotating sphere, accounting for several slip effects and entropy generation. The formulation involved the incorporation of SWCNTs and MWCNTs with TiO₂ nanoparticles into the base fluid. Their results showed that the CNT–TiO₂ ternary hybrid nanofluid exhibited higher heat transfer rates than single- or binary-nanoparticle fluids at the same 6% volume fraction. It also showed an 8% increase in energy transport compared to the SWCNT nanofluid. Patil and Hadapad (Patil and Shankar, 2023) demonstrate that incorporating hybrid nanoparticles effectively enhances thermal processes across multiple domains. The Eyring–Powell nanofluid serves as a viable alternative for chemical engineering applications that conventional non-Newtonian fluids inadequately address. This formulation contains iron oxide nanoparticles and a combination of SWCNTs and MWCNTs integrated into the base fluid. Due to their distinctive nanostructure and robust carbon-carbon bonding, CNTs exhibit excellent electrical conductivity, exceptional strength and superior thermal conductivity.

By integrating four distinct nanoparticles, tetra-hybrid nanofluid research enhances thermal and rheological characteristics, thereby advancing conventional fluid mechanics. These fluids enable precise control of thermal conductivity, entropy generation and flow characteristics, making them highly advantageous for biological thermal management and targeted medication administration. Tetra-hybrid formulations are being used in heat exchangers, automotive systems and biotechnological processes due to their superior heat transfer capabilities. Recent research by Mahmood *et al.* (2025) demonstrated a significant improvement in heat transport enabled by tetra-hybrid nanofluids. Sakkaravarthi *et al.* (2024) used the Levenberg–Marquardt neural network method to improve entropy generation in the flow of Casson-type tetra-hybrid nanofluid under electromagneto-hydrodynamic conditions. Priyadharshini and Vanitha Archana (2024) investigated the effects of thermal conduction and radiation in a porous medium, highlighting the importance of tetra-hybrid nanofluids in solar-powered charging systems. Considering temperature-dependent characteristics, Farayola *et al.* (2024) investigated the behaviour of a TiO₂–SiO₂–ZnO₂–Fe₂O₃/PAO tetra-hybrid nanofluid as it moved along a vertically porous surface while being subjected to suction. A methodology was proposed by Sajida *et al.* (2023) to evaluate the impact of solar radiation on a magnetised Xue-type tetra-hybrid nanoliquid flowing across an expanding sheet in the photovoltaic layers of offshore solar modules. In addition, Sajid *et al.* (2023) investigated the Tiwari–Das tetra-hybrid nanofluid model for arterial blood flow using a Cross non-Newtonian formulation. When doing so, they incorporated heat sources/sinks, viscous dissipation, Joule heating and nonlinear thermal radiation. Dinesh Kumar *et al.* (2025) conducted an ANFIS-PSO analysis of axisymmetric tetra-hybrid nanofluid flow, which includes Cu, CNT, graphene and TiO₂ nanoparticles suspended in WEG-blood, under the influence of linear heat radiation and an inclined magnetic field, highlighting its relevance for biomedical applications. Using Casson, Maxwell and tangent hyperbolic formulas, Amudhini and De (2025) examined the MHD flow of a tetra-hybrid

non-Newtonian nanofluid across an inclined stretched sheet with different slip conditions in a porous medium. Additionally, their study evaluated the combined effects of radiation and the Soret-Dufour phenomenon, as well as the entropy generated in nanofluids containing Al_2O_3 , Cu, SiO_2 and TiO_2 dissolved in water.

In a non-Darcy porous stretched cylinder, [Amudhini and De \(2024\)](#) showed that heat diffusion significantly affects the unsteady maximum hydrodynamic drag (MHD) flow of a tetra-hybrid nanofluid (Al_2O_3 -Cu- SiO_2 - TiO_2 /water). The model encompassed chemical reactions, activation energy and heat production. This study clarifies the ability of nanofluids to enhance heat and mass transfer in diverse systems, including solar collectors, chemical reactors, cooling technologies and medical devices. [Paul and Das \(2024\)](#) investigated the electro-pumping characteristics of Phan–Thien–Tanner blood flow, integrating tetra-hybrid nanoparticles within a ciliated arterial channel, while accounting for entropy generation and the influence of ciliary motion on the pumping mechanism. Kamel *et al.* ([Guedri *et al.*, 2026](#)) investigated the thermal performance of an Ag– TiO_2 –GO–Co./EG tetra-hybrid nanofluid flowing through porous media under an external magnetic field. Their model incorporates viscous dissipation and internal heat generation, enabling direct comparison of various formulations of hybrid, tri-hybrid and tetra-hybrid nanofluids. [Nisha and De \(2025\)](#) investigated a tetra-hybrid Sisko nanofluid with MHD motion, thermal radiation and internal heat sources. The organisms used were gyrotactic bacteria, arranged in a porous vertical cone-and-plate configuration. Their findings indicate that tetra-hybrid nanoparticles effectively enhance thermal conductivity and chemical process efficiency in Sisko-type fluids.

Motivated by the limitations of existing studies, the present work introduces a comprehensive analysis of mixed convection flow and heat transfer in a Williamson tetra-hybrid nanofluid over a rough wedge under the influence of a periodic magnetic field. The novelty of this study lies in the simultaneous incorporation of non-Newtonian Williamson fluid behaviour, a tetra-hybrid nanoparticle mixture (SWCNT, MWCNT, SiO_2 and TiO_2) and sinusoidal surface roughness within a unified modelling framework. To the best of the authors' knowledge, the available literature has not investigated this combined configuration. The primary objective of this study is to examine the influence of roughness-induced oscillations and magnetic effects on the momentum and thermal transport characteristics and to evaluate the effectiveness of tetra-hybrid nanofluids in enhancing heat transfer performance for advanced engineering and thermal management applications. This study addresses the identified research gap through the following:

- A comparative analysis of Williamson and Newtonian fluid behaviours is conducted.
- Evaluation of performance variations among mono-, hybrid-, ternary- and tetra-hybrid nanofluid formulations.
- Investigation of the effects of a periodic magnetic field on energy transport.
- Examination of the influence of surface roughness on flow characteristics.
- Assessment of the role of nanoparticle sphericity.
- Analysis of the impact of a fixed nanoparticle volume fraction of 5%.

2. Mathematical descriptions and analysis

[Figure 1](#) illustrates the flow structure, coordinate system and rheological model used in this study, and [Figure 2](#) presents the nanoparticle structures. The research examines a two-dimensional, incompressible Williamson non-Newtonian tetra-hybrid nanofluid that flows down the rough surface of a wedge in the x-direction, with the y-axis orientated vertically

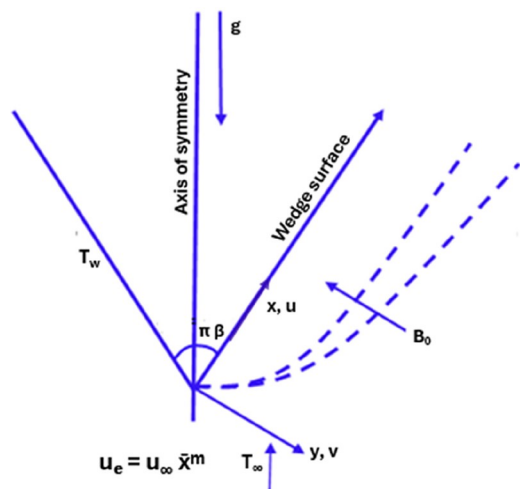


Figure 1. Schematic diagram and flow geometry

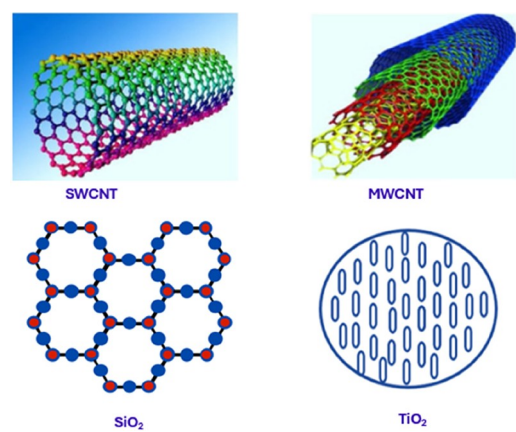


Figure 2. Nanoparticles structures

upwards. A deterministic approach models surface roughness as a sinusoidal wave pattern (Chang, 1995). The two components of the velocity field, u and v , are defined along the x - and y -axes, respectively. A sinusoidal magnetic field $B_0 \sin^2\left(\frac{\pi x}{L}\right)$ emanates from the exterior along the y -axis. The wedge has a specific half-angle, $\left(\frac{\pi\beta}{L}\right)$ and the gravitational force g acts downward along the surface of the wedge. This gravitational force influences the fluid's movement by exerting a downward drag. The mainstream velocity is denoted as $u_e = u_\infty (\bar{x})^m$, where 'm' is the wedge angle parameter, $\phi = \pi\beta$ denotes the total wedge angle and β is the Hartree pressure gradient. The relationship between the wedge angle parameter and the pressure gradient is expressed as $m = \frac{\beta}{2-\beta} \geq 0$ (Kumari et al., 2001; Singh et al., 2009).

Mixed convection resulting from temperature is likely to either heat or cool the left side of the wedge, contingent upon the thermal state of the fluid. The fluid adjacent to the wall is at a higher temperature, while the fluid further from the wall is at a lower temperature. Boussinesq's approximation (Schlichting and Gersten, 2000) considers density variations induced by temperature fluctuations. Given these facts, the governing equations for fluid dynamics and heat transfer, as shown in equations (1)–(3), along with the boundary conditions specified in equation (4), are thoroughly elucidated in references (Chang, 1995; Kumari *et al.*, 2001; Singh *et al.*, 2009; Schlichting and Gersten, 2000).

2.1 Mass

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \quad (1)$$

2.2 Momentum

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \nu_{tethnf} \left[\frac{\partial^2 u}{\partial y^2} + \sqrt{2} \Gamma \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \right] - \frac{\sigma_{tethnf} B_0^2}{\rho_{tethnf}} (u - u_e) \sin^2 \left(\pi \frac{x}{L} \right) + g \beta_{tethnf} (T - T_\infty) \cos \left(\frac{\pi \beta}{2} \right), \quad (2)$$

2.3 Energy

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa_{tethnf}}{(\rho C_p)_{tethnf}} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma_{tethnf} B_0^2}{(\rho C_p)_{tethnf}} (u - u_e)^2 \sin^2 \left(\pi \frac{x}{L} \right). \quad (3)$$

The corresponding pertinent boundary conditions are (Chang, 1995):

$$\begin{aligned} \text{at } y = 0: \quad u &= u_w = u_0 \left(1 + \alpha \sin \left(n \frac{x}{L} \right) \right), \quad v = 0, \quad T = T_w, \\ \text{as } y \rightarrow \infty: \quad u &\rightarrow u_e(x) = u_\infty \left(\frac{x}{L} \right)^m, \quad T \rightarrow \infty. \end{aligned} \quad (4)$$

The nomenclature mentions the physical quantities. The sinusoidal waveform $u_w = u_0 \left(1 + \alpha \sin \left(n \frac{x}{L} \right) \right)$ represents the sinusoidal fluctuations in wall velocity produced by surface roughness at the notional mean surface at $y = 0$.

Introducing nonsimilar transformations (Kumari *et al.*, 2001; Singh *et al.*, 2009):

$$\begin{aligned} \bar{x} &= \frac{x}{L}, \quad \eta = \left[\frac{m+1}{2} \frac{u_e}{x \nu_f} \right]^{1/2} y, \quad \psi(x, y) = \left[\frac{2}{m+1} x \nu_f u_e \right]^{1/2} f(\xi, \eta), \quad u = \frac{\partial \psi}{\partial y}, \\ v &= -\frac{\partial \psi}{\partial x}, \quad u = u_e F, \quad G(x, y) = \frac{T - T_\infty}{T_w - T_\infty}, \quad F = f_\eta, \\ v &= -\left[\frac{2}{m+1} \frac{\nu_f u_e}{x} \right]^{1/2} \frac{1}{2} \{ (m+1)(f + 2\bar{x}f_{\bar{x}}) + (m-1)\eta F \}. \end{aligned} \quad (5)$$

The nonsimilar transformations (5) make sure that [equation \(1\)](#) is trivially achieved. However, [equations \(2\)–\(3\)](#) and boundary conditions (4) have been modified in the following manner:

$$\left. \begin{aligned} D_1 \left(1 + We M_1 \left(\frac{m+1}{2} \right)^{1/2} F_\eta \right) F_{\eta\eta} + D_4 \left(f + \frac{2\bar{x}}{m+1} f_{\bar{x}} \right) F_\eta - \frac{2\bar{x}}{m+1} D_4 F F_{\bar{x}} \\ + \frac{2}{m+1} [D_4 m (1 - F^2) + D_2 D_4 M_2 Ri G + D_3 M_3 Re M^2 (1 - F) \sin^2(\pi\bar{x})] = 0, \end{aligned} \right\} \quad (6)$$

$$\left. \begin{aligned} D_5 G_{\eta\eta} + D_6 Pr \left(f + \frac{2\bar{x}}{m+1} f_{\bar{x}} \right) G_\eta - \frac{2\bar{x}}{m+1} D_6 Pr F G_{\bar{x}} \\ + \frac{2}{m+1} D_3 M_4 Pr Ec Re M^2 \sin^2(\pi\bar{x}) (1 - F)^2 = 0, \end{aligned} \right\} \quad (7)$$

The corresponding transformed boundary conditions are ([Chang, 1995](#)):

$$\begin{aligned} \text{at } \eta = 0 : \quad F &= \varepsilon(1 + \alpha \sin(n\bar{x})), \quad G = 1, \\ \text{as } \eta \rightarrow \infty : F &\rightarrow 1, \quad G \rightarrow 0. \end{aligned} \quad (8)$$

$$\begin{aligned} \text{where, } M_1 &= (\bar{x})^{\frac{3m-1}{2}}; \quad M_2 = (\bar{x})^{1-2m}; \quad M_3 = (\bar{x})^{1-m}; \quad M_4 = (\bar{x})^{1+m}; \quad D_1 = \frac{\mu_{\text{tetra}}}{\mu_f}; \quad D_2 = \frac{\beta_{\text{tetra}}}{\beta_f}; \\ D_3 &= \frac{\sigma_{\text{tetra}}}{\sigma_f}; \quad D_4 = \frac{\rho_{\text{tetra}}}{\rho_f}; \quad D_5 = \frac{\kappa_{\text{tetra}}}{\kappa_f}; \quad D_6 = \frac{(C_p)_{\text{tetra}}}{(C_p)_f}; \quad We = \sqrt{2} \Gamma u_\infty \left(\frac{u_\infty}{\nu_f L} \right)^{1/2}; \quad Re = \frac{u_\infty L}{\nu_f}; \\ M^2 &= \frac{\sigma_f B_0^2 \nu_f}{\rho_f u_\infty^2}; \quad Ri = \frac{g \beta_f (T_w - T_\infty) L \cos(\frac{\pi\theta}{2})}{u_\infty^2}; \quad \varepsilon = \frac{u_0}{u_\infty}; \quad Pr = \frac{\kappa_f}{(\mu C_p)_f}; \quad Ec = \frac{u_\infty^2}{(C_p)_f (T_w - T_\infty)}. \end{aligned}$$

In the present study, the mixed convection flow and heat transfer of a Williamson tetra-hybrid (SWCNT-MWCNT-SiO₂-TiO₂) nanofluid over a rough wedge are examined, where the surface roughness is idealised as a periodic variation in wall velocity to account for the kinematic disturbances caused by small-scale roughness elements in the near-wall region. This modelling approach is routinely used in boundary-layer investigations, as it yields a tractable model for periodic momentum disturbances generated by geometrical undulations or patterned surfaces, without the need to explicitly resolve the detailed surface microstructure. Hence, the sinusoidal form is a mathematical substitute for the flow oscillations due to roughness, not a direct reconstruction of the geometry.

In this context, surface roughness is mainly taken into account by the momentum boundary condition that causes a spatial periodic fluctuation of the velocity field along the wedge surface. The effect of heat transfer is, however, indirect and occurs through alterations in the velocity distribution, which subsequently impact the convective transport of thermal energy in the tetra-hybrid nanofluid. It should be noted that additional roughness-related mechanisms like turbulence amplification, form drag, wake formation and roughness-driven eddy generation are not examined, since the analysis is constrained to the laminar mixed convection regime of a Williamson fluid.

A more complete treatment, which includes direct geometric roughness modelling, turbulence effects, or enhanced drag mechanisms, would require higher-order turbulence modelling or direct numerical simulation and is outside the scope of the present work. Thus, the adopted formulation can be considered a controlled parametric framework to study the coupled effects of periodic roughness-induced disturbances, magnetic field interactions and

nanoparticle loading on flow and thermal behaviour and not a complete physical representation of all roughness phenomena.

The equations (6) and (7) can be further simplified by introducing $\xi = (\bar{x})^{(1-m)/2}$ (Kumari *et al.*, 2001; Singh *et al.*, 2009), which represents the dimensionless distance along the wedge ($\xi > 0$):

$$\text{So that, } \bar{x} f_{\bar{x}} = \bar{x} f_{\xi} \xi_{\bar{x}} = \left(\frac{1-m}{2} \right) \xi f_{\xi} \text{ and } \bar{x} F_{\bar{x}} = \left(\frac{1-m}{2} \right) \xi F_{\xi}. \quad (9)$$

Using the relationships in equation (9), we may transform equations (6) and (7) into the following forms:

$$\left. \begin{aligned} D_1 \left(1 + We \xi^{\frac{3m-1}{1-m}} \left(\frac{m+1}{2} \right)^{1/2} F_{\eta} \right) F_{\eta\eta} + D_4 \left(f - \frac{m-1}{m+1} \xi f_{\xi} \right) F_{\eta} - \frac{m-1}{m+1} \xi D_4 F F_{\xi} \\ + \frac{2}{m+1} \left[D_4 m (1-F^2) + D_2 D_4 \xi^{\frac{2(1-2m)}{1-m}} Ri G + D_3 \xi^2 Re M^2 (1-F) \sin^2 \left(\pi \xi^{\frac{2}{1-m}} \right) \right] = 0, \end{aligned} \right\} \quad (10)$$

$$\left. \begin{aligned} D_5 G_{\eta\eta} + D_6 Pr \left(f - \frac{m-1}{m+1} \xi f_{\xi} \right) G_{\eta} + \frac{m-1}{m+1} D_6 Pr F \xi G_{\xi} \\ + \frac{2}{m+1} D_3 \xi^{\frac{2(1+m)}{1-m}} Pr Ec Re M^2 \sin^2 \left(\pi \xi^{\frac{2}{1-m}} \right) (1-F)^2 = 0, \end{aligned} \right\} \quad (11)$$

The nondimensional boundary conditions are as follows:

$$\begin{aligned} \text{at } \eta = 0 : \quad F = \varepsilon (1 + \alpha \sin(n \xi^{\frac{2}{1-m}})), \quad G = 1, \\ \text{as } \eta \rightarrow \infty : F \rightarrow 1, \quad G \rightarrow 0. \end{aligned} \quad (12)$$

2.4 Physical quantities

2.4.1 Surface drag coefficient. The viscosity of a tetra-hybrid nanofluid causes it to exert a force on a wedge surface when it flows over it. The surface drag coefficient is defined as follows:

$C_f = \frac{\tau_w}{\rho_f u_e^2}$, the wall shear stress is determined by:

$$\tau_w = 2 \mu_{tetrahf} \left(\frac{\partial u}{\partial y} \right)_{y=0}. \quad (13)$$

The nondimensional version is written as follows:

$$\Rightarrow Re^{1/2} C_f = 2 D_1 \left(\frac{m+1}{2} \right)^{1/2} \xi^{\frac{2m+1}{m+1}} F_{\eta}(\xi, 0). \quad (14)$$

2.4.2 Nusselt number. Using convection, the Nusselt number can determine the extent to which heat is transferred from the tetra-hybrid nanofluid to the wedge surface. It is defined as follows:

$$Nu = \frac{x q_w}{\kappa_f (T_w - T_\infty)}, \quad (15)$$

with wall heat flux is given by.

$Nu = -\kappa_{\text{tetmef}} \frac{\partial T}{\partial y}$. The nondimensional form is expressed as follows:

$$\Rightarrow Re^{-1/2} Nu = -D_5 \xi^{\frac{m+1}{m-1}} \left(\frac{m+1}{2} \right)^{1/2} G_\eta(\xi, 0).$$

3. Numerical solution procedure

We use a hybrid numerical approach that integrates quasilinearisation with the implicit finite difference method to address the coupled system of equations (6) – (7) and the boundary conditions specified in equation (8). This approach linearises the nonlinear equations, yielding a sequence of linear equations (16)–(17) with corresponding boundary conditions (13) (Patil, 2012; Patil and Roy, 2010; Patil et al., 2013a; Patil et al., 2013b):

$$F_{\eta\eta}^{i+1} + C_1^i F_\eta^{i+1} + C_2^i F^{i+1} + C_3^i F_\xi^{i+1} + C_4^i G^{i+1} = C_5^i, \quad (16)$$

$$G_{\eta\eta}^{i+1} + E_1^i G_\eta^{i+1} + E_2^i G_\xi^{i+1} + E_3^i F^{i+1} = E_4^i. \quad (17)$$

Corresponding iterative boundary conditions are as follows:

$$\left. \begin{aligned} \eta = 0 & : F^{i+1} = 2\varepsilon \left(1 + \alpha \sin \left(n \xi^{\frac{2}{1-m}} \right) \right), \quad G^{i+1} = 1, \\ \eta = \eta_\infty & : F^{i+1} = 1, \quad G^{i+1} = 0. \end{aligned} \right\} \quad (18)$$

In the iterative indexing system, index i denotes the known functions, whereas index $i + 1$ represents the updated unknown functions. The implicit finite difference method (Inouye and Tate, 1974; Patil et al., 2014) is used to solve equations (16) – (17) alongside the boundary conditions specified in equation (18). A central difference scheme is used in the y-direction, while a backward difference scheme is applied in the x-direction. The discretised equations are organised into a linear algebraic system via a block tridiagonal matrix structure. The Varga algorithm (Varga, 2000) is used to determine the eventual response to this process, which is iteratively executed at each computational level. The optimisation of the numerical solution uses step sizes $d\xi$ and $d\eta$, which are equal to 0.01, selected to ensure no substantial variation occurs outside the designated range. To ensure convergence, the solution domain is extended to $\eta_{\max} = 10$. Iterations continue until the difference between consecutive iterations is less than 0.00001, indicating convergence:

$$\Rightarrow \max \left\{ |(G_\eta)_w^{(i+1)} - (G_\eta)_w^{(i)}|, |(F_\eta)_w^{(i+1)} - (F_\eta)_w^{(i)}| \right\} < 10^{-5}. \quad (19)$$

Here, the coefficients appear in equations (11) and (12) are as follows:

$$C_1^i = D_1 \left[f - \frac{m-1}{m+1} \xi f_\xi \right] df f 1 + \left[D_4 \frac{m-1}{m+1} \xi F F_\xi + \frac{2D_4 m}{m+1} (1 - F^2) + \frac{2D_2 D_4}{m+1} Ri \xi^{\frac{2(1-2m)}{1-m}} G \right. \\ \left. + \frac{2D_3}{m+1} M^2 Re \xi^2 (1 - F) \sin^2 \left(\pi \xi^{\frac{2}{m-1}} \right) \right] df 1,$$

$$df1 = -\frac{D_1 We \left(\frac{m+1}{2}\right)^{1/2} \xi^{\frac{3m-1}{1-m}}}{Q_1^2}, df1 = \frac{D_1}{Q_1^2},$$

$$C_2^i = \frac{1}{Q_1(m+1)} \left[D_4(m-1) \xi F_\xi - 4m D_4 F - 2D_3 M^2 \operatorname{Re} \xi^2 \sin^2 \left(\pi \xi^{\frac{2}{1-m}} \right) \right]$$

$$C_3^i = \frac{m-1}{m+1} \frac{D_4 \xi F}{Q_1}, C_4^i = \frac{2 Ri D_2 D_4}{Q_1(m+1)} \xi^{\frac{2(1-2m)}{1-m}}, Q_1 = D_1 \left(1 + We \xi^{\frac{3m-1}{1-m}} \left(\frac{m+1}{2} \right)^{1/2} F_\eta \right)$$

$$C_5^i = C_1^i F_\eta - \frac{1}{Q_1} \left[D_1 \left(f - \frac{m-1}{m+1} \xi f_\xi \right) F_\eta + \frac{2D_4 m}{m+1} (1 + F^2) - D_4 \frac{m-1}{m+1} \xi F F_\xi \right. \\ \left. + \frac{2D_3}{m+1} M^2 \operatorname{Re} \xi^2 \sin^2 \left(\pi \xi^{\frac{2}{1-m}} \right) \right]$$

$$E_1^i = \frac{D_6}{D_5} \operatorname{Pr} \left(f - \frac{m-1}{m+1} \xi f_\xi \right), E_2^i = \frac{D_6}{D_5} \operatorname{Pr} \frac{m-1}{m+1} \xi F$$

$$E_3^i = \frac{D_6}{D_5} \operatorname{Pr} \frac{m-1}{m+1} \xi G_\xi + \frac{4}{m+1} \frac{D_3}{D_5} \operatorname{Pr} M^2 \operatorname{Re} Ec \xi^{\frac{2(1+m)}{1-m}} \sin^2 \left(\pi \xi^{\frac{2}{1-m}} \right) (F-1)$$

$$E_4^i = \frac{\operatorname{Pr}}{D_5(m+1)} \left[D_6 \operatorname{Pr} (m-1) \xi F G_\xi + 2D_3 M^2 \operatorname{Re} Ec \xi^{\frac{2(1+m)}{1-m}} \sin^2 \left(\pi \xi^{\frac{2}{1-m}} \right) (F^2-1) \right].$$

4. Validation of results

To assess the effectiveness of our approach, we compared the results of Kumari *et al.* (Kumari *et al.*, 2001) and Singh *et al.* (Singh *et al.*, 2009) with the current observations of $Re^{-1/2}Nu$ and $Re^{1/2}C_f$ (as shown in Table 1) for the scenario involving a Newtonian fluid without periodic MHD and a nanofluid. The results of (Kumari *et al.*, 2001; Singh *et al.*, 2009) strongly corroborate the present findings.

5. Results and analysis

This section examines how various significant parameters influence the profiles and gradients of flow characteristics in Williamson mixed convection tetra-hybrid nanofluids. The parameter values are chosen within the specified ranges $Ri(-2 \leq Ri \leq 10)$,

Table 1. Evaluation of $Re^{-1/2}Nu$ and $Re^{1/2}C_f$ for the Newtonian scenario without a periodic magnetic field and nanofluid (Kumari *et al.*, 2001; Singh *et al.*, 2009)

M	Kumari <i>et al.</i> (Kumari <i>et al.</i> , 2001)		Singh <i>et al.</i> (Singh <i>et al.</i> , 2009)		Present results	
	$Re^{1/2}C_f$	$Re^{-1/2}Nu$	$Re^{1/2}C_f$	$Re^{-1/2}Nu$	$Re^{1/2}C_f$	$Re^{-1/2}Nu$
0.0	0.46975	0.42079	0.46975	0.42046	0.46974965	0.42077128
0.0909	0.65501	0.44770	0.65490	0.44742	0.65497868	0.44760262
0.1429	0.73202	0.45728	0.73196	0.45705	0.73198836	0.45709378
0.2000	0.80214	0.46534	0.80208	0.46511	0.80210986	0.46515230
0.3333	0.92766	0.47840	0.92767	0.47815	0.92767318	0.47820199

$We(0 \leq We \leq 0.5)$, $\varepsilon(0.1 \leq \varepsilon \leq 0.4)$, $M(0 \leq M \leq 4)$, $\alpha(0.0 \leq \alpha \leq 0.5)$, $n(1 \leq n \leq 300)$, $s(3 \leq s \leq 8.6)$, and $\phi(0 \leq \phi \leq 0.05)$. The values of Pr and Ec are fixed at 7.0 and 0.1, respectively, throughout the investigation. The results of the graphical analysis of the velocity and temperature profiles, along with their gradients, are shown for significant physical parameters such as Ri , We , M , ε , α and n . To determine these parameters, spherical nanoparticles with a sphericity measure (S) of 3.0 were used. Tables 2–5 present the findings for the various alternative nanoparticle compositions. At $m = 0$, the wedge transforms into a flat horizontal plate, and at $m = 0.3333$, it converts into a vertical plate. The investigation focuses on m -values between 0 and 0.2, namely, to $\varphi = 5^\circ$ ($m = 0.0141$), $\varphi = 30^\circ$ ($m = 0.0909$), $\varphi = 45^\circ$ ($m = 0.1429$) and $\varphi = 60^\circ$ ($m = 0.2000$). For this investigation, we assigned m a value of 0.2000 ($\varphi = 60^\circ$). Tables 6–8 present the thermophysical properties of the tetra-hybrid nanofluid, the shapes of the nanoparticles considered in this study and the corresponding physical properties of the tetra-hybrid nanofluid, respectively.

The behaviour of nanofluids can be either Newtonian or non-Newtonian, depending on a variety of factors. The criteria encompass the type, concentration, shape and aggregation state of the nanoparticles, in addition to the characteristics of the base fluid. For the investigation, we focus on carbon-based and oxide nanoparticles, known for their capacity to exhibit non-Newtonian behaviour. This is in accordance with the Williamson fluid model, which describes the behaviour of these nanoparticles. The carbon-based nanoparticles examined, both SWCNTs and MWCNTs, are among the most frequently reported additives that exhibit significant non-Newtonian behaviour. Because they are long and not uniform in shape, they assist the fluid construct's directional micro-networks. Depending on how they are spread out, this can produce fluid shear thinning or shear thickening. Similarly, silica (SiO_2) and titanium dioxide (TiO_2) nanoparticles, which are based on oxides, also exhibit non-Newtonian behaviour at moderate volume fractions. As the concentration increases, these particles tend to stay together, forming microstructures that strengthen shear thinning. Their surface properties also make it more likely that particles will stick together in chains, altering their flow behaviour from Newtonian. The nanoparticle volume fraction is varied from $\phi = 0.0$ to $\phi = 0.05$ to maintain realism in the model, ensure dispersion remains constant and maintain non-Newtonian behaviour.

When describing the roughness of a wedge surface and its influence on fluid flow, the small-amplitude parameter and the frequency parameter n are essential. With a value of $\alpha = 0$, the surface is considered to be smooth, whereas higher values ($\alpha \neq 0$) indicate that the surface is rougher. In engineering systems, wedge-type components are widely used for various purposes, including mechanical locking, force transfer, alignment and optical steering. In situations like these, surface defects can also affect fluid flow.

This study defines roughness as a sinusoidal waveform characterised by low amplitude and high frequency. This waveform accurately depicts microtextures resulting from machining, wear or additive manufacturing. According to the definition of a roughness profile in ISO 4287:1997, this formulation meets the requirements. The appropriate parameter values must be used to guarantee that the boundary layer profiles and gradients are computed accurately. The frequency parameter n is transformed into a composite number that encompasses the physical roughness frequency and other critical flow parameters because of the nonsimilarity transformation of the governing equations. This demonstrates the interconnectedness of the system's impact from all these elements.

5.1 Variations of We and Ri on profiles and their corresponding gradients

Figures 3–6 display the influence of the mixed convection parameter, Ri and the Williamson number, We , on the velocity $F(\xi, \eta)$ and temperature $G(\xi, \eta)$ profiles, as well as their

Table 2. The values of $Re^{-1/2}Nu$ and corresponding enhanced heat transfer rate ENu for the variations in the nanoparticle volume fraction and particle shape are presented below at $\xi = 1.0$

$SWCNT(\phi_1) \backslash \text{shape}$	$Re^{-1/2}Nu$				ENu			
	Sphere	Brick	Cylinder	Platelets	Blade	Sphere	Brick	Cylinder
$SWCNT(\phi_1)$	0.00	1.32799	1.32799	1.32799	1.32799	—	—	—
	0.01	1.32787	1.34616	1.35490	1.38169	-0.09	0.55	1.37
	0.02	1.32795	1.34105	1.37862	1.43202	-0.03	0.98	2.71
	0.03	1.32846	1.34714	1.40138	1.47963	0.04	1.44	3.91
	0.04	1.32786	1.35208	1.42320	1.52404	-0.09	1.81	5.01
0.05	1.32513	1.35659	1.40907	1.44357	1.56598	-0.22	2.15	6.11
$MWCNT(\phi_2)$	0.01	1.32660	1.34426	1.35206	1.37917	-0.10	0.35	1.23
	0.02	1.32437	1.33767	1.37466	1.42746	-0.27	0.73	2.41
	0.03	1.32261	1.34142	1.39550	1.47248	-0.41	1.01	3.45
	0.04	1.32026	1.34486	1.38739	1.51474	-0.58	1.27	4.47
0.05	1.31733	1.34789	1.39927	1.43358	1.55428	-0.80	1.50	5.37
$SiO_2(\phi_3)$	0.01	1.32297	1.32887	1.33052	1.33933	-0.38	-0.20	0.07
	0.02	1.31811	1.32201	1.33327	1.34988	-0.74	-0.45	0.11
	0.03	1.31319	1.31893	1.32910	1.36108	-1.11	-0.68	0.08
	0.04	1.30814	1.31598	1.32930	1.37070	-1.49	-0.90	0.10
0.05	1.30283	1.31273	1.32927	1.34018	1.38023	-1.89	-1.15	0.10
$TiO_2(\phi_4)$	0.01	1.33545	1.34100	1.35050	1.37901	0.56	0.98	1.70
	0.02	1.34349	1.35383	1.37203	1.42889	1.17	1.95	3.32
	0.03	1.34971	1.36519	1.39296	1.47700	1.64	2.80	4.89
	0.04	1.35539	1.37657	1.41187	1.52237	2.06	3.66	6.32
0.05	1.36084	1.38751	1.43156	1.46158	1.56593	2.47	4.48	7.80

Table 3. The values of $Re^{1/2}Cf$ and corresponding ECf for mono-, hybrid-, ternary- and tetra-nanofluids at 5% volume fraction when compared to the base fluid at $\xi = 1.0$

5% volume fraction shape	$Re^{1/2}Cf$			ECf (in %)		
	Sphere	Brick	Cylinder	Platelets	Blade	Blade
Base fluid($\phi = 0$)	8.01377	8.01377	8.01377	8.01377	8.01377	—
SWCNT	8.89384	8.91192	8.94412	8.96513	9.03604	11.87
MWCNT	8.75001	8.76942	8.80227	8.82217	8.89162	10.09
SiO ₂	8.85575	8.86172	8.87268	8.88004	8.90473	10.81
TiO ₂	9.12635	9.14282	9.17334	9.19082	9.25459	14.69
SWCNT + MWCNT	9.70484	9.74413	9.81314	9.85665	10.00780	23.00
SWCNT + SiO ₂	9.80761	9.83462	9.87933	9.90699	9.99616	23.62
SWCNT + TiO ₂	10.07153	10.11108	10.17812	10.21958	10.36315	27.00
MWCNT + SiO ₂	9.65939	9.68675	9.72964	9.75691	9.84652	21.41
MWCNT + TiO ₂	9.92950	9.96864	10.03541	10.07704	10.21966	25.75
SiO ₂ + TiO ₂	10.03878	10.06385	10.10806	10.13548	10.23151	26.13
SWCNT + MWCNT + SiO ₂	10.69642	10.74574	10.82458	10.87537	11.03652	35.71
SWCNT + SiO ₂ + TiO ₂	10.91914	10.96876	11.04730	11.09817	11.26589	37.85
SWCNT + SiO ₂ + TiO ₂	11.06709	11.11687	11.19733	11.24847	11.41581	38.10
SWCNT + MWCNT + TiO ₂	10.41088	10.46164	10.54464	10.59881	10.78007	30.55
SWCNT + MWCNT + SiO ₂ + TiO ₂	11.44171	11.50182	11.59761	11.66006	11.86317	43.53

Table 4. The values of $Re^{-1/2}Nu$ and corresponding ENu for mono, hybrid, ternary and tetra nanofluids at 5% volume fraction when compared to the base fluid at $\xi = 1.0$

5% volume fraction shape	$Re^{-1/2}Nu$				ENu (in %)			
	Sphere	Brick	Cylinder	Platelets	Blade	Sphere	Brick	Cylinder
Base fluid($\phi = 0$)	1.32799	1.32799	1.32799	1.32799	1.32799	–	–	–
SWCNT	1.32513	1.35659	1.40907	1.44357	1.56598	–0.22	2.15	6.11
MWCNT	1.31733	1.34789	1.39927	1.43358	1.55428	–0.80	1.50	5.37
SiO ₂	1.30283	1.31273	1.32927	1.34018	1.38023	–1.89	–1.15	0.10
TiO ₂	1.36084	1.38751	1.43156	1.46158	1.56593	2.47	4.48	7.80
SWCNT + MWCNT	1.31312	1.37582	1.48346	1.55732	1.83120	–1.12	3.60	11.71
SWCNT + SiO ₂	1.29764	1.33642	1.40107	1.44326	1.58891	–2.29	0.63	5.50
SWCNT + TiO ₂	1.35309	1.41100	1.51059	1.57817	1.82340	1.89	6.25	13.75
MWCNT + SiO ₂	1.29099	1.32844	1.39260	1.43446	1.57814	–2.79	0.03	4.87
MWCNT + TiO ₂	1.34651	1.40403	1.50174	1.56922	1.81183	1.39	5.73	13.08
SiO ₂ + TiO ₂	1.33105	1.36702	1.42718	1.46863	1.61871	0.23	2.94	7.47
SWCNT + MWCNT + SiO ₂	1.28306	1.35100	1.46733	1.54382	1.81436	–3.38	1.73	10.49
MWCNT + SiO ₂ + TiO ₂	1.31482	1.37922	1.49092	1.56408	1.83100	–0.99	3.86	12.27
SWCNT + SiO ₂ + TiO ₂	1.32091	1.38590	1.49780	1.57275	1.84181	–0.53	4.36	12.79
SWCNT + MWCNT + TiO ₂	1.33766	1.42664	1.58394	1.69286	2.10465	0.73	7.43	19.27
SWCNT + MWCNT + SiO ₂ + TiO ₂	1.30337	1.39819	1.56220	1.67287	2.07809	–1.85	5.29	17.64

Table 5. Grid independence test for different mesh sizes for $Ri = 10.0$, $we = 0.5$ $m = 0.2$, $Pr = 7.0$, $Re = 10.0$, $\phi_1 = 0.02$, $\phi_2 = 0.02$, $\phi_3 = 0.02$, $\phi_4 = 0.02$, $\varepsilon = 0.1$, $M = 0.5$, $ec = 0.1$, $n = 100$, $\alpha = 0.1$ and $\xi = 1.0$

Grid size	Skin friction coefficient ($Re^{1/2}Cf$)	Rate of heat transfer ($Re^{-1/2}Nu$)
100 x100	7.70149	1.39037
200 x 200	7.74057	1.38278
300 x 300	7.73813	1.38227
400 x 400	7.73777	1.38288
500 x 500	7.73735	1.38287
600 x 600	7.73754	1.38299
700 x 700	7.73821	1.38213
800 x 800	7.73736	1.38307

Table 6. Thermophysical properties of tetra-hybrid nanofluid (*tethnf*) with base fluid water (Patil and Goudar, 2023; Patil and Shankar, 2023)

Physical properties	$SWCNT(\phi_1)$	$MWCNT(\phi_2)$	$SiO_2(\phi_3)$	$TiO_2(\phi_4)$	H_2O
$\rho(kg/m^3)$	2600	1600	2200	4250	997.1
$Cp(J/kgK)$	425	796	745	686.2	4179
$k(W/mK)$	6600	315	1.4	8.9538	0.613
$\beta(K^{-1})$	15×10^{-6}	19×10^{-6}	90×10^{-6}	90×10^{-6}	210×10^{-6}
$\sigma(\Omega m)^{-1}$	1×10^6	1×10^6	3.5×10^6	2.4×10^6	5.5×10^{-6}

Table 7. The details of the nanoparticles’ shapes (Mohana et al., 2024) considered in the study

Shapes	S
Sphere	3
Bric	3.7
Cylinder	4.9
Platelets	5.7
Blade	8.6

respective gradients $Re^{1/2}Cf$ and $Re^{-1/2}Nu$. The computation is performed for $Re = 10$, $Pr = 7.0$, $Ec = 0.1$, $M = 1$, $\alpha = 0.1$, $\varepsilon = 0.1$, $n = 10$, $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0.02$ and $m = 0.2$. Further, the Williamson (non-Newtonian) fluid is represented by $We = 0.5$, whereas the Newtonian fluid case corresponds to $We = 0$. As depicted in Figures 3 and 4, a comparison of the fluid velocity between the Newtonian fluid and the Williamson fluid reveals that the Williamson fluid flows more slowly in the direction of the stream. This phenomenon is because the viscosity between the layers of fluid in a Williamson fluid is higher than that in a Newtonian fluid, which results in reduced friction between the fluid and the surface.

Ri represents an opposing buoyancy force when negative, and when positive, it acts as a buoyant force that facilitates buoyancy-driven flow. The fluid’s velocity (Figure 3) escalates with increasing Ri values, while its temperature (Figure 4) diminishes. The aiding buoyancy force increases with Ri , thereby imparting additional momentum to the fluid. An increase in

Table 8. Thermo-physical properties of tetra-hybrid nanofluid (Paul and Das, 2024; Guedri et al., 2026)

Property	Formula
Density	$\rho_{tetnrf} = (1 - \phi_4)((1 - \phi_3)((1 - \phi_2)((1 - \phi_1)\rho_f + \phi_1\rho_{s1}) + \phi_2\rho_{s2}) + \phi_3\rho_{s3}) + \phi_4\rho_{s4}$
Dynamic viscosity	$\mu_{tetnrf} = \frac{(1 - \phi_1)^{2.5}(1 - \phi_2)^{2.5}(1 - \phi_3)^{2.5}(1 - \phi_4)^{2.5}}{k_{s4} + 2k_{tnrf} - 2\phi_4(k_{tnrf} - k_{s4})} \times \frac{k_{s4} + 2k_{tnrf} + \phi_4(k_{tnrf} - k_{s4})}{k_{s3} + 2k_{tnrf} - 2\phi_3(k_{tnrf} - k_{s3})} \times$
Thermal conductivity	$\frac{k_{tetnrf}}{k_f} = \left\{ \frac{k_{s2} + 2k_{nf} - 2\phi_2(k_{nf} - k_{s3})}{k_{s2} + 2k_{nf} + \phi_2(k_{nf} - k_{s3})} \times \frac{k_{s1} + 2k_f - 2\phi_1(k_f - k_{s1})}{k_{s1} + 2k_f + \phi_1(k_f - k_{s1})} \right\}$
Heat capacity	$(\rho Cp)_{tetnrf} = (1 - \phi_4) \left((1 - \phi_3) \left((1 - \phi_2) \left((1 - \phi_1)(\rho Cp)_f + \phi_1(\rho Cp)_{s1} \right) + \phi_2(\rho Cp)_{s2} \right) + \phi_3(\rho Cp)_{s3} \right) + \phi_4(\rho Cp)_{s4}$
Thermal expansion	$(\rho\beta)_{tetnrf} = (1 - \phi_4) \left((1 - \phi_3) \left((1 - \phi_2) \left((1 - \phi_1)(\rho\beta)_f + \phi_1(\rho\beta)_{s1} \right) + \phi_2(\rho\beta)_{s2} \right) + \phi_3(\rho\beta)_{s3} \right) + \phi_4(\rho\beta)_{s4}$
Electrical conductivity	$\frac{\sigma_{tetnrf}}{\sigma_f} = \left\{ \frac{\sigma_{s4} + 2\sigma_{tnrf} - 2\phi_4(\sigma_{tnrf} - \sigma_{s4})}{\sigma_{s4} + 2\sigma_{tnrf} + \phi_4(\sigma_{tnrf} - \sigma_{s4})} \times \frac{\sigma_{s3} + 2\sigma_{tnrf} - 2\phi_3(\sigma_{tnrf} - \sigma_{s3})}{\sigma_{s3} + 2\sigma_{tnrf} + \phi_3(\sigma_{tnrf} - \sigma_{s3})} \times \right.$
	$\left. \frac{\sigma_{s2} + 2\sigma_{nf} - 2\phi_2(\sigma_{nf} - \sigma_{s3})}{\sigma_{s2} + 2\sigma_{nf} + \phi_2(\sigma_{nf} - \sigma_{s3})} \times \frac{\sigma_{s1} + 2\sigma_f - 2\phi_1(\sigma_f - \sigma_{s1})}{\sigma_{s1} + 2\sigma_f + \phi_1(\sigma_f - \sigma_{s1})} \right\}$

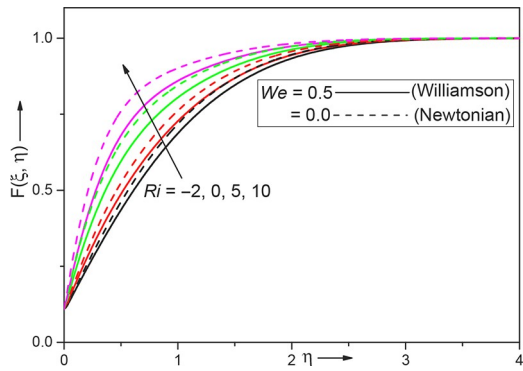


Figure 3. Variations of We and Ri on the distribution of the velocity profile

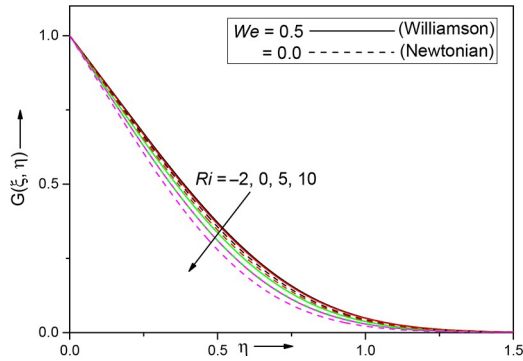


Figure 4. Variations of We and Ri on distribution of Temperature profile

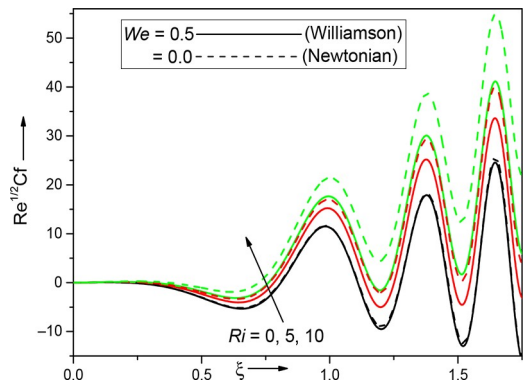


Figure 5. Variations of We and Ri on surface drag

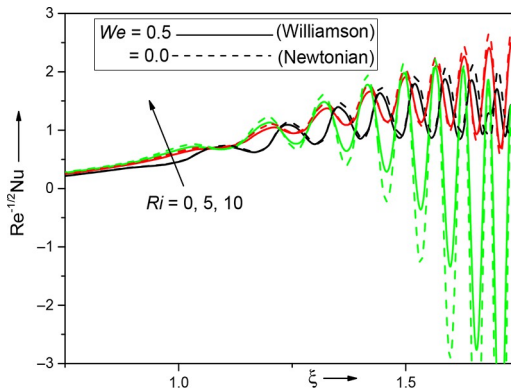


Figure 6. Variations of We and Ri on Nusselt number

Ri augments the assisting buoyancy force, thereby contributing additional momentum to the fluid. Thus, this results in an increase in the fluid velocity and a decrease in the boundary-layer temperature. An increase in the fluid velocity removes hotter fluid particles from the boundary layer, allowing cooler fluid particles to approach the wall. The fluid's temperature decreases as a direct consequence of this. In particular, the fluid velocity is significantly higher for $Ri = 10$ than for $Ri = 0$, due to the stronger wall-driven velocity, which drags the fluid near the wall more firmly than the mainstream. Thus, the convected heat in the boundary layer leads to a decline in temperature, as shown in Figure 5.

The $Re^{1/2}Cf$ (Figure 6) increases along the length of the cylinder ξ in the presence of wall roughness. Due to the roughness of the wedge surface, sinusoidal fluctuations are observed in the $Re^{1/2}Cf$ and the size of these variations increases with increasing values of Ri while decreasing for the Williamson fluid case ($We = 0$) when compared to the Newtonian fluid case ($We = 0.5$), as shown in Figure 5.

Also, the effects of the Newtonian fluid case ($We = 0$) are more prominent than those due to Ri . However, the rate of heat transfer characteristics also exhibits an oscillatory nature, as shown in Figure 6. It is observed to increase significantly with increasing Ri away from the wedge surface. This increase is due to the combined effect of the applied periodic magnetic

field strength $\sin^2(\pi\xi\frac{2}{1-m})$ and wall roughness. Also, the effects on $Re^{-1/2}Nu$ due to the Newtonian fluid case ($We = 0$) are more prominent as compared to that due to the Williamson fluid ($We = 0.5$).

5.2 Impact of periodic magnetic field and velocity ratio on $Re^{1/2}Cf$ and $Re^{-1/2}Nu$

Figures 7–8 illustrate the plots of $Re^{1/2}Cf$ and $Re^{-1/2}Nu$, respectively, for varying values of magnetic field strength M and velocity ratio ϵ for $Re = 10$, $Pr = 7.0$, $Ec = 0.1$, $Ri = 10$, $We = 0.5$, $\alpha = 0.1$, $n = 5$, $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0.02$ and $m = 0.2$. It is observed that, in general, both the gradients $Re^{1/2}Cf$ and $Re^{-1/2}Nu$ are influenced more significantly by the magnetic field (M) than by the velocity ratio ϵ . The dramatic acceleration seen in Figure 7 is primarily attributable to the strength of the magnetic field (M) rather than to the velocity ratio ϵ . In Figure 7, the effects of M and ϵ on the $Re^{1/2}Cf$ are observed, which is oscillatory in nature with its mean increasing along the wedge wall length ξ . The higher the M value, the higher the $Re^{1/2}Cf$ value. Further, it is reduced on account of a velocity ratio ϵ increase from

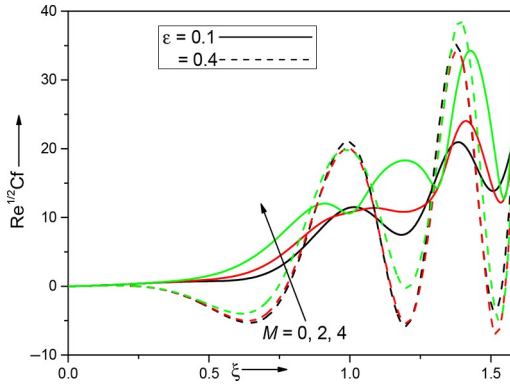


Figure 7. Variations of M and ε on surface drag

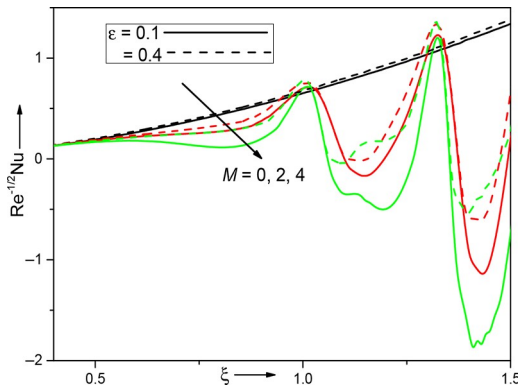


Figure 8. Variations of M and ε on Nusselt number

0.1–0.4. Also, the sinusoidal variations in $Re^{1/2}Cf$ even in the absence of a magnetic strength ($M = 0$) are owing to surface roughness.

In Figure 8, it is observed that $Re^{-1/2}Nu$ is impacted more significantly by the sinusoidal variations in the presence of a periodic magnetic field (M). The higher the value of M , the lower the value of $Re^{-1/2}Nu$. However, in the absence of a magnetic field ($M = 0$), oscillatory behaviour is not observed. If we put $M = 0$ in equations (6) and (7), the coefficient of M , i.e. $\sin^2(\pi \xi^{\frac{2}{1-m}}) = 0$. This expression is responsible for the oscillatory behaviour in gradients. Also, $Re^{-1/2}Nu$ values are decreased on account of a velocity ratio increase from 0.1–0.4. The ε establishes the proportion between the wall velocity and the freestream velocity. The case of $\varepsilon < 1$ signifies that the freestream velocity predominates over the wall velocity, whereas the case of $\varepsilon \geq 1$ implies that the wall velocity prevails over the freestream velocity. However, in this case, freestream velocity predominates over the wall velocity.

5.3 Wall roughness on $Re^{1/2}Cf$

Figures 9–11 illustrate in detail how the wall-roughness amplitude (α) and the roughness-frequency parameter (n) influence the skin-friction coefficient along the wedge surface for

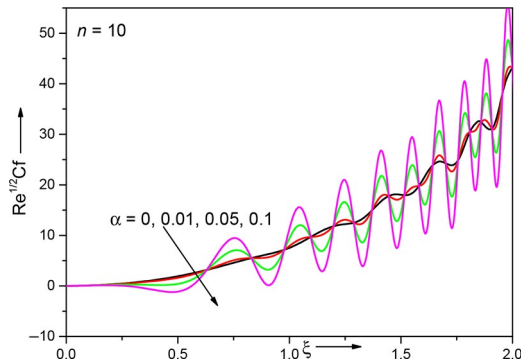


Figure 9. Impact of α on $Re^{1/2}Cf$ when $n = 10$

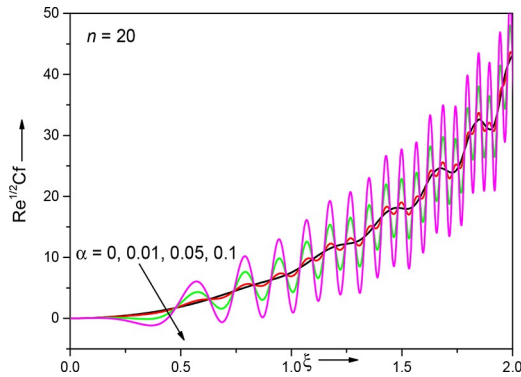


Figure 10. Impact of α on $Re^{1/2}Cf$ when $n = 20$

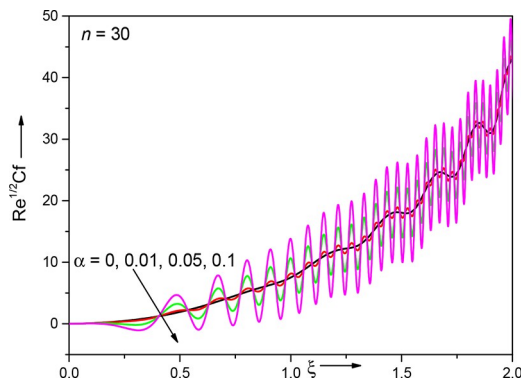


Figure 11. Impact of α on $Re^{1/2}Cf$ when $n = 30$

$Re = 10$, $Pr = 7.0$, $Ec = 0.1$, $Ri = 10$, $We = 0.5$, $\varepsilon = 0.2$, $M = 1.0$, $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0.02$ and $m = 0.2$.

The $Re^{1/2}Cf$ is depicted in [Figure 9](#) for a range of roughness amplitudes, including 0.01, 0.05 and 0.1, with a fixed roughness frequency of 10. A surface that is completely smooth is represented by the case value of $\alpha = 0$. Even when the roughness levels are mild, the skin friction coefficient exhibits significant fluctuations along the length of the wedge. The periodic surface undulations cause these oscillations, and the average amount of skin friction is always higher than it would be for a flat surface. When the roughness amplitude increases, the discrepancy between the smooth-surface baseline and the roughness amplitude increases, indicating that the near-wall shear is modulated more effectively. As the roughness frequency parameter n increases, the distance between the roughness peaks and valleys on the wedge surface decreases. This indicates that the wall has more surface undulations per unit length, making it appear more like a highly corrugated or micro-textured surface than a gently wavy one. As the fluid moves over this surface, the boundary layer speeds up, slows down and speeds up again, because each roughness feature changes how fast the fluid moves.

At low n , the flow has enough space between peaks to partially recover and set up a smoother velocity gradient before the next disruption. At higher n values (e.g., $n = 20$ -[Figure 10](#) and $n = 30$ -[Figure 11](#)), on the other hand, the roughness elements come up quickly, which does not give the boundary layer enough time to relax. This causes the fluid particles near the wall to stretch and compress in cycles, thereby making changes in wall shear stress more pronounced. As a result, the skin friction coefficient exhibits tightly packed, higher-frequency oscillations near the surface. From a physical point of view, each roughness peak acts as a small barrier that temporarily thickens the boundary layer, while each trough allows the layer to thin in certain areas. When these features are close together, the flow passes through a series of zones where shear strength alternates. The result is a boundary layer that is always changing, which makes the wall shear gradients sharper and makes the flow more sensitive to slight changes in shape. This is why a greater n makes the skin friction profile oscillate more quickly and more intensively.

In this model, surface roughness is represented as a sinusoidal waveform, which shows the tiny peaks and valleys produced by machining, wear, coatings or additive manufacturing techniques. As the flow passes over these bumps, each bump and hole pushes the fluid against the wall, compressing and periodically moving small parcels of fluid. These repetitive disturbances cause the sinusoidal changes in the skin friction coefficient. The roughness amplitude controls how strong each disturbance is, and the roughness frequency controls how often these disturbances happen. Together, these two factors account for the oscillatory behaviour observed in [Figures 9–11](#).

[Figures 12, 13 and 14](#) illustrate the influence of the roughness frequency parameter n and the small roughness amplitude parameter α on the surface drag along the wedge surface ξ . For each figure, the amplitude α has been set at 0.01, 0.05 and 0.1, while the value of n varies from 1 to 10, 20, 30 and 50.

In all circumstances, the surface drag exhibits a clear oscillatory pattern, and its average value slowly increases along the wedge. This phenomenon is a direct result of the wall's sinusoidal surface roughness. The wall has tiny peaks and troughs that alternate, indicating that the flow in the boundary layer must change as it flows downstream. When the fluid encounters a peak in the roughness wave, it is pushed closer to the wall, increasing the local velocity gradient and increasing the shear stress. When the fluid reaches a trough, the boundary layer relaxes for a short time, which lowers the local wall shear. These regular compressions and relaxations give rise to the sinusoidal oscillations observed in the skin friction profile.

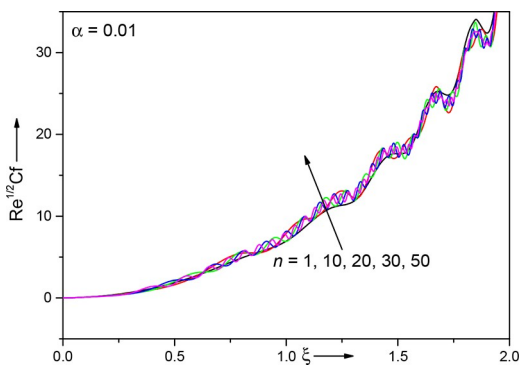


Figure 12. Impact of n on $Re^{1/2}Cf$ when $\alpha = 0.01$

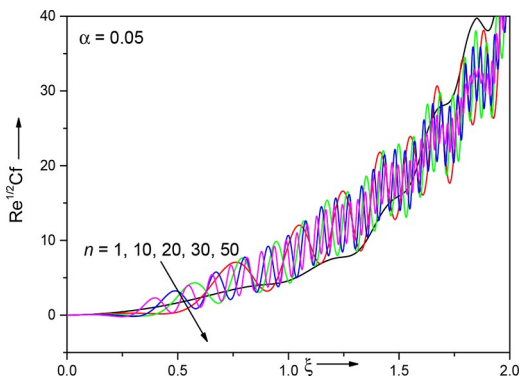


Figure 13. Impact of n on $Re^{1/2}Cf$ when $\alpha = 0.05$

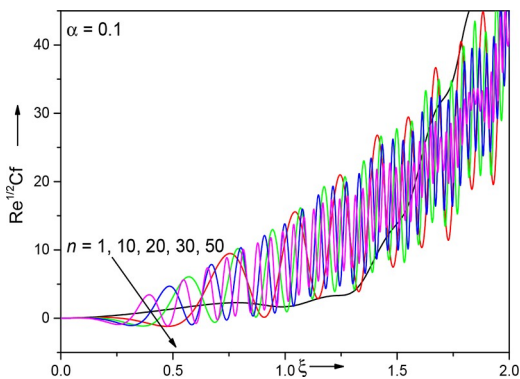


Figure 14. Impact of n on $Re^{1/2}Cf$ when $\alpha = 0.1$

The amplitude parameter tells how strong the geometric disruption is that each roughness cycle causes. For small amplitudes (e.g. $\alpha = 0.01$), the wall undulations are shallow, which means that the disturbances in the area near the wall are mild, causing oscillations of relatively modest size. As the values of α rise to 0.05 and 0.1 (Figure 13 and Figure 14), the peaks protrude farther into the flow and the troughs deepen, which changes the thickness of the boundary layer more. This makes the surface drag oscillate more, with a greater difference in shear values between peaks and troughs.

In short, the amplitude parameter α sets the size of the oscillatory changes, while the frequency parameter n sets their spatial frequency. Their combined effect fully explains why the oscillation occurs and why it becomes stronger as the surface becomes rougher.

5.4 Wall surface roughness on $Re^{-1/2}Nu$

A comprehensive investigation of the impact of surface roughness on the rate of heat transfer has been carried out, and the findings are presented in Figures 15, 16, 17 and 18, 19, 20 for the following parameter values: $Re = 10$, $Pr = 7.0$, $Ec = 0.1$, $Ri = 10$, $We = 0.5$, $\varepsilon = 0.2$, $M = 1.0$, $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 0.02$ and $m = 0.2$. Figures 15, 16 and 17 illustrate how the wall

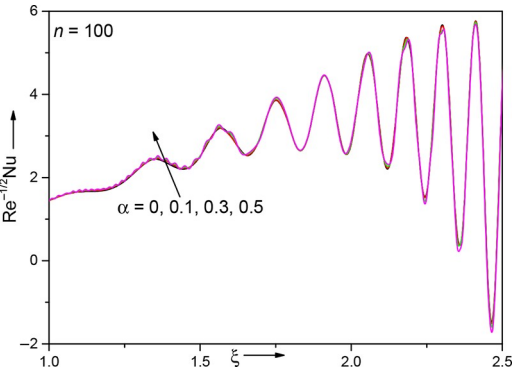


Figure 15. Impact of α on $Re^{-1/2}Nu$ when $n = 100$

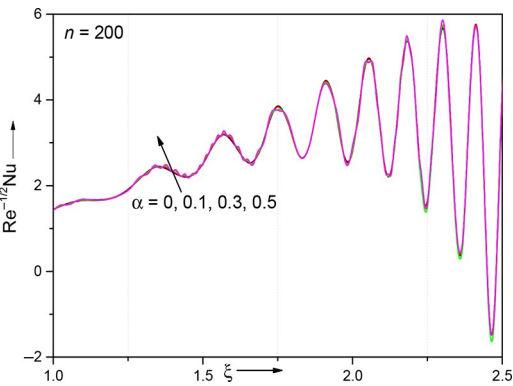


Figure 16. Impact of α on $Re^{-1/2}Nu$ when $n = 200$

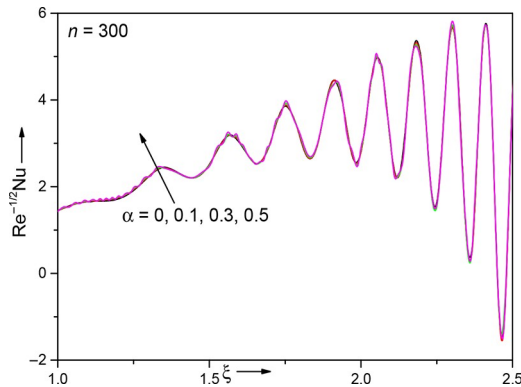


Figure 17. Impact of α on $Re^{-1/2}Nu$ when $n = 300$

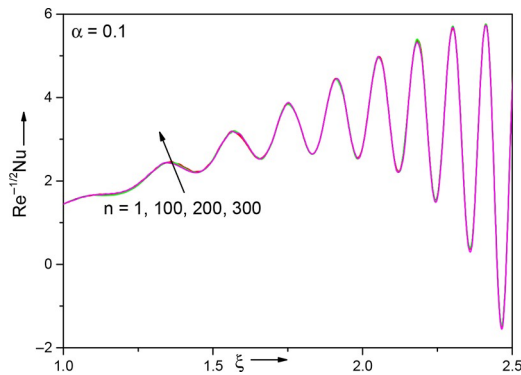


Figure 18. Impact of n on $Re^{-1/2}Nu$ when $\alpha = 0.1$

temperature gradient shifts throughout the wedge surface when the roughness amplitude α is altered ($\alpha = 0.01, 0.05$ and 0.1) while the roughness frequency parameter n remains unchanged. On the other hand, Figures 18, 19, and 20 illustrate how the roughness amplitude changes as the frequency parameter n is varied. In all cases, the temperature gradient profile exhibits distinctly oscillatory behaviour, with fluctuations superimposed on a mean trend that gradually decreases along the wedge length. In Figures 15, 16 and 17, the selected values $n = 100, 200$ and 300 correspond to increasingly rapid surface undulations. At these high frequencies, the near-wall thermal boundary layer becomes much less strongly modulated by the closely spaced peaks and valleys. As a result, the temperature gradient near the rough wall decreases due to enhanced local mixing and deformation of the thermal layer, rendering near-surface heat transfer less effective.

Nevertheless, the temperature gradient is slightly more pronounced farther from the wall due to the combined effects of flow disturbances induced by roughness and convective transport in the presence of the periodic magnetic field. As the roughness amplitude increases, the disparity in the efficiency with which a smooth surface and a rough surface transmit heat becomes more apparent. As it increases, the troughs become deeper and the

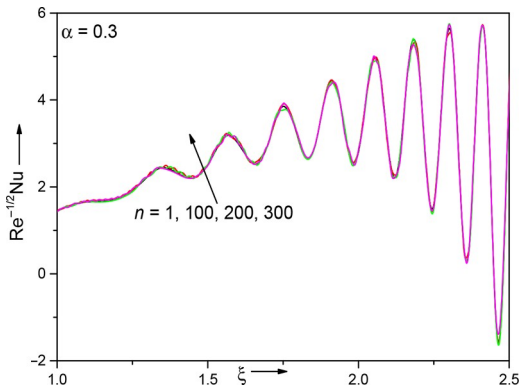


Figure 19. Impact of n on $Re^{-1/2}Nu$ when $\alpha = 0.4$

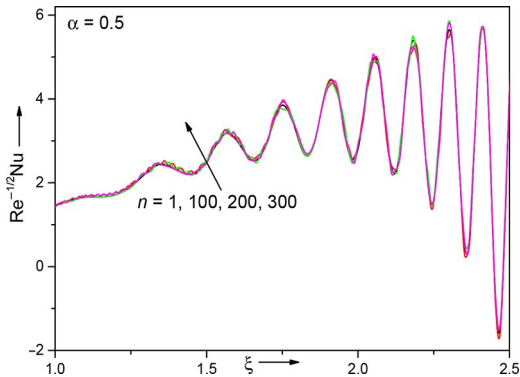


Figure 20. Impact of n on $Re^{-1/2}Nu$ when $\alpha = 0.8$

peaks become taller. This results in the thermal boundary layer becoming thinner and thicker more frequently. Due to the spherical nature of the nanoparticles, this does not result in a significant increase in the differential in wall heat flux. When both surface roughness and a periodic magnetic field are present simultaneously, the oscillatory nature of the heat transfer pattern becomes more apparent. The cyclical modulation of the wall-temperature gradient is strengthened by the interaction between the roughness-driven oscillatory boundary-layer deformation and the Lorentz force induced by the periodic magnetic field. Due to the combination of these two factors, the oscillations become more pronounced and exhibit greater contrast.

Figure 18, 19, 20 depict how the $Re^{-1/2}Nu$ changes for different roughness frequency parameter $n = 1, 100, 200$ and 300 while the roughness amplitude parameter α set at $\alpha = 0.1, 0.3$ and 0.5 , respectively. For all rough surface scenarios $\alpha > 0$, the $Re^{-1/2}Nu$ exhibits an oscillating pattern that moves over the wedge surface. The thermal boundary layer changes structure in a predictable manner as the flow encounters the rough wall's peaks and troughs, resulting in this oscillating motion. As the frequency parameter n increases, the oscillations become more closely spaced. Nevertheless, at the forefront of the wedge, the oscillations are

less distinct due to the boundary layer still developing and its inability to respond adequately to the rapidly changing wall geometry. As the flow progresses downstream, the boundary layer thickens and becomes increasingly responsive to the recurring alterations in shape, resulting in amplified oscillations. For substantial roughness amplitudes of $\alpha = 0.3$ and 0.5 , the slight increase in oscillation magnitude at elevated n remains minimal, indicating that the thermal boundary layer cannot completely adjust to rapid surface undulations.

The roughness of the surface generates small holes that act like “aspiration” zones for fluids. These zones pull cooler fluid towards the heated wall and push warmer fluid outward. This mechanism encourages better convective mixing and makes it easier for heat to go from the heated surface to the fluid around it. So, the temperature gradient along the wedge has large sinusoidal fluctuations for rough surfaces. This is very different from the case of a smooth wall $\alpha = 0$, where the temperature gradient goes down steadily without any oscillations.

5.5 Variations of the nanoparticle volume fraction

The variations of the nanoparticles volume fraction on $Re^{1/2}Cf$ and $Re^{-1/2}Nu$ is displayed in Figures 21 and 22, respectively, in comparison with mono, hybrid, ternary and tetra nanoparticles.

As can be seen in Figure 21, the $Re^{1/2}Cf$ variation is in a characteristic sinusoidal pattern all the way along the wedge surface. Increasing the number of nanoparticle components that are added to the base fluid (component 1) results in an increase in both the mean value and the oscillation amplitude. These components are 2, 3, 4 and 5. By incorporating additional nanoparticle types into the nanofluid, the effective thermal conductivity and rheological properties are enhanced, making the boundary layer more sensitive to rough surfaces. As a result, the roughness-induced disruptions become more evident, leading to a stronger, more pronounced oscillatory response along the wedge.

In Figure 22, the oscillatory behaviour is considerably clearer because the periodic magnetic field makes the thermal boundary layer deform more due to the surface roughness. The Lorentz force from the oscillating magnetic field continually changes the momentum near the wall. This causes the thermal layer to cycle through compression and relaxation, making the sinusoidal temperature gradient fluctuations more pronounced. When more nanoparticle elements are added to the base fluid, the average wall temperature gradient slowly goes down along the wedge and the oscillation amplitude goes down a little bit. This

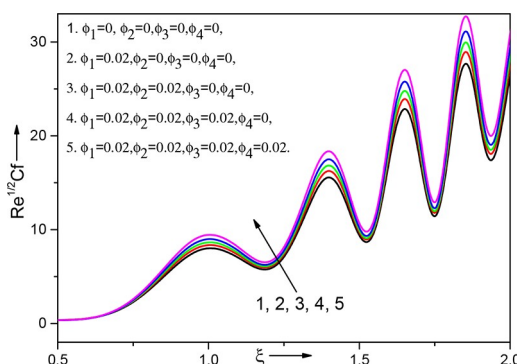


Figure 21. Variations of ϕ on $Re^{1/2}Cf$

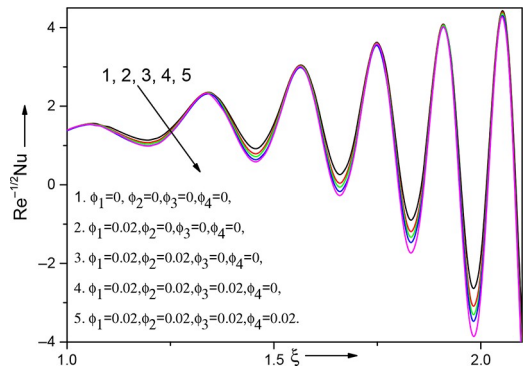


Figure 22. Variations of ϕ on $Re^{-1/2}Nu$

decrease occurs because multi-component nanofluids have a higher effective viscosity and stronger magnetic damping, both of which help prevent rapid thermal layer deformation. The rough wall and magnetic field still provide oscillatory forcing, but the thicker, more viscous effective fluid reduces the steepness of the wall-temperature gradient. This makes the thermal response pattern smoother, but it still oscillates.

5.6 Effects of nanoparticle sphericities on wall gradients

The influence of nanoparticle sphericity on the two wall quantities is illustrated in Figures 23 and 15 for sphericity values $S = 3, 3.7, 4.9, 5.7$ and 8.6 . In Figure 23, as S increases, the mean level of the sinusoidally varying skin-friction signal rises and continues to increase along the wedge. Physically, higher S (lower geometric sphericity, i.e. more aspherical/elongated plate- or blade-like particles) promotes anisotropic micro-structuring within the boundary layer, which elevates the effective viscosity and enhances momentum coupling between the fluid and the wall. Under roughness-induced peak–trough forcing, this makes the near-wall velocity gradient more responsive: peaks compress and thin the momentum layer more strongly, while troughs relax it less, so the oscillatory pattern rides on a higher mean shear that grows downstream.

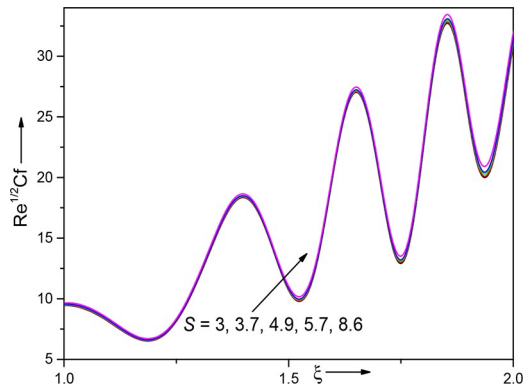


Figure 23. Variations of S on $Re^{1/2}Cf$

In Figure 24, the mean of the corresponding sinusoidal temperature-gradient $Re^{-1/2}Nu$ variations also increases with sphericity S , although the enhancement is more modest and grows gradually along the wedge length. Overall, the effects become more pronounced at higher sphericity parameter values, reflecting the stronger influence of particle shape on both shear and heat transfer characteristics.

5.7 The influence of nanoparticle shapes on $Re^{1/2}Cf$ and $Re^{-1/2}Nu$

The sphericity of a nanoparticle is inversely proportional to its shape factor S and this relationship $S = \frac{3}{\varpi}$ is used to quantify particle shapes, where sphericity is denoted by ϖ . As shown in Table 2, the value of $Re^{1/2}Cf$ increases with rising nanoparticle volume fractions for SWCNT, MWCNT, SiO_2 and TiO_2 across all considered shapes. Among the shapes examined, spherical nanoparticles consistently exhibit the lowest skin friction values, whereas blade-shaped nanoparticles demonstrate the highest. Specifically, $MWCNT(\phi_2)$ shows the lowest skin friction for sphere, brick, cylinder and platelet, while there is a slight increase in the case of blade-shaped nanoparticles. However, $TiO_2(\phi_4)$ displays the highest skin friction among the nanoparticles in all shapes. Additionally, as shown in Table 9, the skin friction enhancement (ECf) increases proportionally with increasing volume fraction of spherical, brick, cylindrical, platelet and blade-shaped nanoparticles. The enhanced skin friction is defined as follows:

$$ECf = \frac{Re^{1/2}Cf(Nanofluid) - Re^{1/2}Cf(Basefluid)}{Re^{1/2}Cf(Basefluid)} \times 100.$$

The values of $Re^{-1/2}Nu$ for sphere, brick, cylinder, platelet and blade-shaped nanoparticles are illustrated in Table 2. For each of the four nanoparticles (SWCNT, MWCNT, SiO_2 and TiO_2), the rate of heat transfer associated with spherical nanoparticles is negative and lower than the rate of heat transfer associated with the base fluid-water, as shown in Table 2. When the volume fraction (Φ_1 – Φ_4) grows from 0.01–0.05, there is a steady decrease in the heat transfer rate over time. It has been observed that the rate of heat transmission decreases for both spherical and brick-shaped SiO_2 nanoparticles. This contrasts with the remaining nanoparticle forms, which exhibit a consistent rise in heat transfer rate across the same volume fraction range. The $Re^{-1/2}Nu$ increases as the volume fraction of nanoparticles

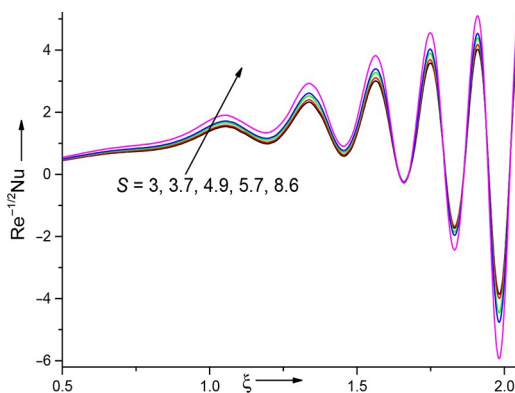


Figure 24. Variations of S on $Re^{-1/2}Nu$

Table 9. The values of $Re^{1/2}Cf$ and corresponding skin friction (ECf) for the variations in the nanoparticle volume fraction and particle shape are presented below at $\xi = 1.0$

$SWCNT(\phi_1)\backslash shape$	$Re^{1/2}Cf$			ECf (in %)						
	Sphere	Brick	Cylinder	Platelets	Blade	Sphere	Brick	Cylinder	Platelets	Blade
$SWCNT(\phi_1)$	8.01377	8.01377	8.01377	8.01377	8.01377	—	—	—	—	—
	8.18280	8.18515	8.19356	8.19646	8.21293	2.11	2.14	2.24	2.28	2.49
	8.35404	8.36210	8.37534	8.38365	8.41467	4.25	4.35	4.51	4.61	5.00
	8.52842	8.54059	8.56113	8.57447	8.61826	6.42	6.57	6.83	7.00	7.54
	8.70754	8.72426	8.75143	8.76757	8.82552	8.66	8.87	9.20	9.41	10.13
$MWCNT(\phi_2)$	8.89384	8.91192	8.94412	8.96513	9.03604	10.98	11.21	11.61	11.87	12.76
	8.15360	8.15917	8.16568	8.16913	8.18543	1.74	1.81	1.90	1.94	2.14
	8.29774	8.30540	8.31833	8.32730	8.35835	3.54	3.64	3.80	3.91	4.30
	8.44430	8.45646	8.47645	8.48880	8.53367	5.37	5.52	5.77	5.93	6.49
	8.59493	8.61154	8.63693	8.65358	8.71129	7.25	7.46	7.78	7.98	8.70
$SiO_2(\phi_3)$	8.75001	8.76942	8.80227	8.82217	8.89162	9.19	9.43	9.84	10.09	10.95
	8.17402	8.17435	8.17650	8.17908	8.18332	1.99	2.00	2.03	2.06	2.12
	8.33773	8.34070	8.34467	8.34804	8.35807	4.04	4.08	4.13	4.17	4.30
	8.50562	8.51012	8.51658	8.52071	8.53487	6.14	6.19	6.27	6.33	6.50
	8.67810	8.68322	8.69196	8.69763	8.71737	8.29	8.35	8.46	8.53	8.78
$TiO_2(\phi_4)$	8.85575	8.86172	8.87268	8.88004	8.90473	10.51	10.58	10.72	10.81	11.12
	8.22771	8.23106	8.23672	8.24044	8.25524	2.67	2.71	2.78	2.83	3.01
	8.44468	8.45169	8.46387	8.47194	8.49836	5.38	5.46	5.62	5.72	6.05
	8.66722	8.67851	8.69515	8.70618	8.74526	8.15	8.29	8.50	8.64	9.13
	8.89408	8.90802	8.93230	8.94591	8.99805	10.98	11.16	11.46	11.63	12.28
	9.12635	9.14282	9.17334	9.19082	9.25459	13.88	14.09	14.47	14.69	15.48

increases for all particle shapes, including spheres, bricks, cylinders, platelets and blades. Spherical nanoparticles have the lowest heat transfer rate of all shapes, while blade-shaped nanoparticles have the highest rate. Among the four nanoparticles, SiO₂ has the lowest overall heat transmission rate, whereas SWCNT shows the highest. The associated expression defines the improved heat transfer rate (ENu):

$$ENu = \frac{Re^{-1/2}Nu(Nanofluid) - Re^{-1/2}Nu(Basefluid)}{Re^{-1/2}Nu(Basefluid)} \times 100.$$

5.8 Impact of 5% volume fraction of nanoparticles

The numerical results for $Re^{1/2}Cf$ and the corresponding enhanced skin friction ECf for mono-, hybrid-, ternary- and tetra-nanofluids are compiled in Table 3. In contrast, the associated numerical results for $Re^{-1/2}Nu$ and the corresponding enhanced heat transfer rate ENu are presented in Table 4, compared with the base fluid with 5% volume fraction of each nanoparticle. The minus sign indicates a decrease in the values relative to the base fluid. In general, the tetra-hybrid nanofluid has a greater impact on velocity and temperature gradients than the mono-, hybrid- and ternary formulations. This indicates a greater likelihood that it will enhance the performance of thermal control systems.

The mono-, hybrid-, ternary- and tetra-hybrid nanoparticles with a 5% volume fraction of the considered nanoparticles have been analysed for enhanced skin friction (ECf), and the numerical details are given in Table 3. Suspensions of nanoparticles (SWCNT, MWCNT, SiO₂ and TiO₂), compared to water, with 5% volume fraction each, i.e., $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 5\%$ of different shapes, namely, spheres, bricks, cylinders, platelets and blades, yield enhanced skin friction (ECf) as observed from Table 3. Among the tested nanofluids, MWCNT exhibits the lowest skin friction (ECf) of 10.95%, followed by SiO₂ with 11.12%, SWCNT with 12.76% and TiO₂ with 15.48% for mono nanofluids of blade-shaped nanoparticles.

Adding more than one type of nanoparticle to the base fluid results in hybrid nanofluids, which have significantly greater skin friction values than mono nanofluids. For instance, among 5% volume fractions with six possible combinations, hybrid nanofluids of blade-shaped MWCNT + SiO₂ display the lowest enhanced skin friction ECf of 22.87%, followed by SWCNT + SiO₂ with 24.74%, SWCNT + MWCNT with 24.88%, MWCNT + TiO₂ with 27.53%, SiO₂ + TiO₂ with 27.67% and SWCNT + TiO₂ with 29.32%. The suspensions of ternary hybrid nanoparticles consist of three nanoparticles from the considered nanoparticles, forming four possible combinations: SWCNT + MWCNT + SiO₂, MWCNT + SiO₂ + TiO₂, SWCNT + SiO₂ + TiO₂, and SWCNT + MWCNT + TiO₂, each with a 5% volume fraction of blade-shaped nanoparticles. These ternary nanofluids obtain enhanced skin friction (ECf) of 37.72%, 40.58%, 42.45% and 34.52%, respectively. Among the combinations, the formation SWCNT + MWCNT + TiO₂ provides a lower enhanced skin friction (ECf), i.e., 34.52%. The tetra-hybrid nanoparticles comprising all four nanoparticles, SWCNT + MWCNT + SiO₂ + TiO₂, with a 5% volume fraction of blade-shaped nanoparticles, yield 48.03%. Overall, for blade-shaped nanoparticles with a 5% volume fraction, the mono, hybrid, ternary and tetra-hybrid formulations exhibit progressively higher ECf values, with the lowest enhancements as MWCNT with 10.95%, MWCNT + SiO₂ with 22.87%, SWCNT with 34.52% and SWCNT + MWCNT + SiO₂ + TiO₂ with 48.03%, respectively.

The mono-, hybrid-, ternary- and tetra-hybrid nanofluids formulated with a 5% volume fraction of nanoparticles have been examined for their enhanced heat transfer performance

(ENu), with the corresponding numerical results presented in [Table 4](#). When compared to water, suspensions of nanoparticles (SWCNT, MWCNT, SiO_2 and TiO_2) with a 5% volume fraction each, i.e. $\phi_1 = \phi_2 = \phi_3 = \phi_4 = 5\%$ of different shapes, namely, sphere, bricks, cylinders, platelets and blades, yield significantly enhanced heat transfer rates (ENu). Among the mono nanofluids verified, SWCNT exhibits the highest ENu of 17.92%, followed by MWCNT with 17.04%, TiO_2 with 17.91% and SiO_2 with 3.93%. For hybrid nanofluids with a 5% volume fraction, the six possible combinations of nanoparticles, SWCNT + MWCNT, display the highest enhanced heat transfer rate of 37.89%, followed by SWCNT + TiO_2 with 37.30%, MWCNT + TiO_2 with 36.43%, SiO_2 + TiO_2 with 21.89%, SWCNT + SiO_2 with 19.66% and MWCNT + SiO_2 with 18.84%. Compared to the base fluid, ternary hybrid nanofluids generated from three-component combinations yield four possible combinations: SWCNT + MWCNT + SiO_2 , MWCNT + SiO_2 + TiO_2 , SWCNT + SiO_2 + TiO_2 , and SWCNT + MWCNT + TiO_2 , each with a 5% volume fraction of blade-shaped nanoparticles. These ternary nanofluids achieve enhanced heat transfer rates ENu of 36.62%, 37.88%, 38.69% and 58.48%, respectively. The combination SWCNT + MWCNT + TiO_2 displays the highest enhanced heat transfer rate (ENu), i.e., 58.48%, among the four combinations. The tetra-hybrid nanofluid, combining all four nanoparticles SWCNT + MWCNT + SiO_2 + TiO_2 , considered with a 5% volume fraction for blade-shaped nanoparticles, yields an ENu of 56.48%. The highest values of ENu at 5% volume fraction, in the case of mono-, hybrid-, ternary- and tetra-hybrid nanoparticles, are SWCNT with 17.92%, SWCNT + MWCNT with 37.89%, SWCNT + MWCNT + TiO_2 with 58.48% and SWCNT + MWCNT + SiO_2 + TiO_2 with 56.48%, respectively.

Even though the tetra-hybrid formulation, SWCNT + MWCNT + SiO_2 + TiO_2 , is expected to have superior thermal performance, it is interesting that one of the ternary hybrid nanofluid formulations, SWCNT + MWCNT + TiO_2 , exhibits a higher increase in heat transfer rate (ENu) than the tetra-hybrid formulation. This variation arises from the inclusion of SiO_2 nanoparticles, which are recognised for diminishing the heat transfer rate when integrated into the mixture. [Tables 2](#) and [4](#) demonstrate that the ternary hybrid nanofluid exhibits a superior ENu value compared to the tetra-hybrid nanofluid when SiO_2 is present. Thus, the ternary formulation demonstrates superior heat transfer enhancement compared to its tetra-hybrid equivalent when SiO_2 is present.

5.9 Grid discretisation, grid number and grid independence test

Grid discretisation denotes the division of the computational domain into a finite number of grid points, typically characterised by the grid size. As the grid is refined, both the accuracy of the numerical solution and computational efficiency improve, up to an optimal level beyond which further refinement yields negligible gains. The computational domain, depicted in [Figure 25](#), is discretised using mesh spacings of $\Delta\xi$ and $\Delta\eta$ along the ξ - and η -directions, respectively. Each node in the domain is identified by an ordered pair (i, j) and the complete set of these nodes constitutes the grid system used in the numerical scheme.

A full grid independence test was performed to ensure the robustness and reliability of the numerical results. The results are summarised in [Table 5](#), which displays the changes in the skin friction coefficient and heat transfer rate for many updated grid resolutions, i.e., 100×100 , 200×200 , 300×300 , 400×400 , 500×500 , 600×600 , 700×700 and 800×800 . The results analysis reveals that the skin friction coefficient and the heat transfer rate exhibit significant changes at the coarser grid levels, but these quantities tend to stabilise with grid refinement. In particular, at a grid size of 400×400 and above, the oscillations of the obtained values decrease considerably, which means that the numerical solution has converged and is almost insensitive to further grid refining.

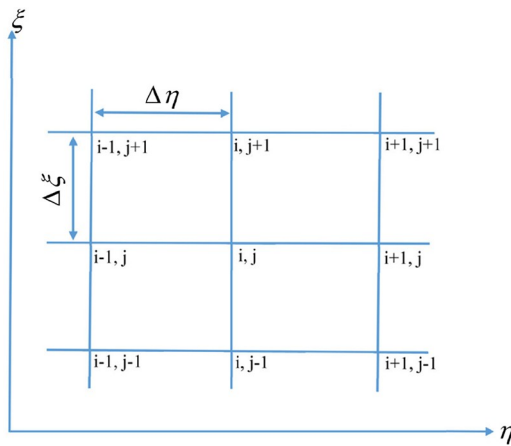


Figure 25. Grid discretisation

Based on this conclusion, we have chosen a grid resolution of 500×500 for our present computations, which gives the optimal trade-off between accuracy and computational cost. The grid gives enough resolution of the boundary layer and associated transport phenomena with a minimum of computationally unnecessary steps. For the analysis of the study, reliable and computationally efficient results are used. The relevant mesh spacings are taken as 0.01 in both tangential and normal orientations to the surface.

6. Conclusions

The present study presents a detailed investigation of the flow and heat transfer of a mixed-convective Williamson tetra-hybrid nanofluid under the impact of a periodic magnetic field and surface roughness. The proposed tetra-hybrid formulation is a synergistic combination of carbon-based nanoparticles [single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs)] and metal-oxide nanoparticles (SiO_2 and TiO_2) which provides better control over the thermophysical properties of the fluid. This well-designed nanoparticle formulation provides significant advantages over the traditional mono, hybrid and ternary nanofluids in terms of the enhanced thermal conductivity, stability and improved control over the rheological and transport features. These improvements lead to better heat transfer efficiency and optimised flow management in difficult physical settings, such as fluctuations in magnetic fields and surface defects. The numerical analysis performed at present leads to the following conclusions:

- Due to the roughness of the wedge surface, sinusoidal fluctuations are observed in the $\text{Re}^{1/2}Cf$ and the size of these variations, which increases with increasing values of Ri while decreasing for the Williamson fluid case ($We = 0.5$) when compared to the Newtonian fluid case ($We = 0$).
- The gradients $\text{Re}^{1/2}Cf$ and $\text{Re}^{-1/2}Nu$ are influenced more significantly by the magnetic field than that by velocity ratio ϵ . Further, $\text{Re}^{-1/2}Nu$ is impacted more significantly by the sinusoidal variations in the presence of a periodic magnetic field (M). The higher the value of M , the lower the value of $\text{Re}^{-1/2}Nu$.

- These oscillations are caused by periodic surface undulations, and the average amount of skin friction is always higher than it would be for a flat surface.
- For the wedge surface, a greater n shortens the roughness wavelength, bringing the peaks and valleys closer together and making the wall more heavily corrugated.
- The temperature-gradient distribution exhibits a distinct oscillatory component superimposed on a mean profile that decreases as it progresses along the wedge surface.
- At considerable roughness amplitudes of $\alpha = 0.3$ and 0.5 , the incremental rise in oscillation magnitude with increasing n is minimal, suggesting that the thermal boundary layer is unable to completely adjust to the rapid surface oscillations.
- Spherical nanoparticles consistently exhibit the lowest skin-friction coefficients, whereas blade-shaped nanoparticles display the highest. Skin friction remains minimal for spherical, brick, cylindrical and platelet shapes. Only blade-shaped nanoparticles exhibit a slight increase.
- At a volume fraction of 5%, blade-shaped nanoparticles exhibit increasing ECf values in mono-, hybrid-, ternary- and tetra-hybrid nanofluids. In terms of improvement, the MWCNT formulation is the smallest at 10.95%, followed by the hybrid MWCNT + SiO₂ at 22.87%, the ternary SWCNT + MWCNT + TiO₂ at 34.52% and finally the tetra-hybrid SWCNT + MWCNT + SiO₂ + TiO₂ at 48.03%.
- For all the types of particles tested, spherical, brick-, cylindrical, platelet and blade-shaped, the wall-heat-transfer rate increases as the volume percentage of nanoparticles increases. Spherical nanoparticles provide the minimal heat-transfer enhancement, in contrast to blade-shaped nanoparticles, which demonstrate the most enhancement.
- At a 5% volume fraction, the maximum enhanced heat-transfer rates (ENu) for the mono-, hybrid-, ternary- and tetra-hybrid nanofluids are SWCNT – 17.92%, SWCNT + MWCNT + TiO₂ – 37.89%, SWCNT + MWCNT + TiO₂ – 58.48% and SWCNT + MWCNT + SiO₂ + TiO₂ – 56.48%, respectively.
- One of the ternary hybrid formulations exhibits greater heat-transfer enhancement (ENu) than the tetra-hybrid nanofluid. This is due to the presence of SiO₂ in the mixture, which is known to reduce heat transfer efficiency.
- The current study gives a few new findings on the mixed convection flow and heat transfer properties of a Williamson tetra-hybrid nanofluid across a rough wedge in the presence of a periodic magnetic field. One of the major novelties of the present work is the simultaneous examination of the Williamson non-Newtonian rheology, surface roughness, mixed convection, nanoparticle sphericity and periodic magnetic field effects for mono-, hybrid-, ternary- and tetra-hybrid nanofluid configurations. Results reveal that the tetra-hybrid nanofluid comprising SWCNT, MWCNT, SiO₂ and TiO₂ nanoparticles has higher thermal performance than mono-, hybrid- and ternary-nanofluids. This gain is due to the synergistic interaction of highly conductive carbon-based nanoparticles and metal oxide nanoparticles, which increases the effective thermal conductivity and improves the heat transfer in the boundary layer.
- The findings of this study have an applied significance in modern thermal engineering systems where efficient heat transfer management is desired. The applicability of the proposed model is discussed in particular for electronic cooling systems, thermal

management of high-performance devices, magnetic field-assisted heat exchangers, industrial coating processes, polymer extrusion, lubrication systems and energy-efficient cooling technologies with rough or irregular surfaces. The presence of a periodic magnetic field also provides a mechanism for active regulation of fluid motion and heat transfer rates in electrically conducting nanofluids.

- Future extensions of this work may include experimental validation of the present numerical results, study of different nanoparticle shapes and volume fractions, consideration of different base fluids and addition of different physical effects such as thermal radiation, entropy generation, viscous dissipation, chemical reaction and slip boundary conditions. The model can also be expanded to unsteady three-dimensional geometries, porous media, rotating systems and other non-Newtonian fluid models to increase its applications in practical engineering and industrial operations.

References

- Akbar, T., Ahmed, K., Taseer, M. and Shahzad, M. (2025), "Physical characteristics of DuFour and Soret effects on MHD mixed convection flow of Williamson fluid past a nonlinear stretching porous curved surface", *Waves in Random and Complex Media*, Vol. 35 No. 1, doi: [10.1080/17455030.2021.2023233](https://doi.org/10.1080/17455030.2021.2023233).
- Amjad, M., Ahmed, K., Akbar, T., Muhammad, T., Ahmed, I. and Alshomrani, A.S. (2022), "Numerical investigation of double diffusion heat flux model in Williamson nanofluid over an exponentially stretching surface with variable thermal conductivity", *Case Studies in Thermal Engineering*, Vol. 36, p. 102231, doi: [10.1016/j.csite.2022.102231](https://doi.org/10.1016/j.csite.2022.102231).
- Amudhini, M. and De, P. (2024), "Comparative study of hybrid, tri-hybrid, and tetra-hybrid nanoparticles in MHD unsteady flow with chemical reaction, activation energy, Soret-Dufour effect, and sensitivity analysis over non-Darcy porous stretching cylinder", *Heliyon*, Vol. 10 No. 15, p. e35731.
- Amudhini, M. and De, P. (2025), "Irreversibility analysis on tetra-hybrid non-Newtonian fluids with thermal radiation and Soret-Dufour effects over porous inclined stretching sheet", *Results in Engineering*, Vol. 26, p. 104786.
- Chang, L. (1995), "Deterministic modelling and numerical simulation of lubrication between rough surfaces – a review of recent developments", *Wear*, Vol. 184 No. 2, p. 155e60.
- Dinesh Kumar, M., Díaz Palencia, J.L., Dharmiah, G., Wakif, A., Noeiaghdam, S., Fernandez-Gamiz, U. and Dinarvand, S. (2025), "ANFIS-PSO analysis on axisymmetric tetra-hybrid nanofluid flow of Cu-CNT-Graphene-TiO₂ with WEG-Blood under linear thermal radiation and inclined magnetic field: a bio-medicine application", *Heliyon*, Vol. 11 No. 1, p. e41429.
- Farayola, P.I., Aselebe, L.O., Salaudeen, K.A., Ogundiran, S.D. and Asiru, T.M. (2024), "Influence of temperature-dependent thermophysical properties of TiO₂-SiO₂-ZnO-Fe₃O₄/PAO tetra-hybrid nanofluid along a vertical porous surface with suction", *Open Journal of Fluid Dynamics*, Vol. 14 No. 03, pp. 123-146.
- Guedri, K., Faridi, A.A., Ahmad, H., Ali, K., Jamshed, W., Abd-Elmonem, A., Abdalla, N.S.E., and Almaliki, A.H. (2026), "Thermal analysis of ethylene glycol-based tetra-hybrid nanofluid flow over a stretchable permeable surface under the influence of induced magnetic field", *Journal of Molecular Liquids*, Vol. 437, doi: [10.1016/j.molliq.2025.128485](https://doi.org/10.1016/j.molliq.2025.128485).
- Hamzeh Taha, A. and Muhammad Khairul Anuar, M. (2023), "MHD (SWCNTS + MWCNTS)/H₂O-Based Williamson Hybrid Nanofluids Flow Past an Exponential Shrinking Sheet in a Porous Medium", *Frontiers in Heat and Mass Transfer*, Vol. 21, pp. 265-279.

- Inouye, K. and Tate, A. (1974), "Finite difference version of quasilinearisation applied to boundary layer equations", *AIAAJ*, Vol. 12, pp. 558-560.
- Jain, R., Mehta, R., Bhatnagar, A., Ahmad, H., Khan, Z.A. and Ismail, G.M. (2024), "Numerical study of heat and mass transfer of Williamson hybrid nanofluid (CuO/CNT's-water) past a permeable stretching/shrinking surface with mixed convective boundary condition", *Case Studies in Thermal Engineering*, Vol. 59, p. 104313.
- Kumari, M., Takhar, H.S. and Nath, G. (2001), "Mixed convection flow over a vertical wedge in a highly porous medium", *Heat and Mass Transfer*, Vol. 37 Nos 2-3, pp. 139-146.
- Mahmood, Z., *et al.* (2025), "Deep learning investigation of water-based tetra-hybrid nanofluid across a shrinking cylinder for variable electrical conductivity with thermal radiation", *Journal of Radiation Research and Applied Sciences*, Vol. 18 No. 1, p. 101213, doi: [10.1016/j.jrras.2024.101213](https://doi.org/10.1016/j.jrras.2024.101213).
- Mohana, C.M., Kumar, B.R., Nagarathnam, S. and Shivakumara, I.S. (2024), "Thermal performance analysis of magnetohydrodynamic Al₂O₃-SiO₂-TiO₂/water ternary hybrid nanofluid in converging and diverging channels with nanoparticle shape effects", *Case Studies in Thermal Engineering*, Vol. 64, p. 105429.
- Muhammad, A., Iftikhar, A., Kamran, A., Marei, S., Alqarni, T., Akbar and Taseer, M. (2022), "Numerical solution of magnetised Williamson nanofluid flow over an exponentially stretching permeable surface with temperature dependent viscosity and thermal conductivity", *Nanomaterials*, Vol. 12 No. 20, p. 3661, doi: [10.3390/nano12203661](https://doi.org/10.3390/nano12203661).
- Nisha, S.S. and De, P. (2025), "Comparative analysis of di-hybrid, tri-hybrid, tetra-hybrid on bioconvective Sisko nanofluid over porous vertical cone/plate with thermal radiation, heat generation, viscous dissipation, and chemical reaction", *Eur. Phys. J. Spec. Top*, doi: [10.1140/epjs/s11734-025-01743-4](https://doi.org/10.1140/epjs/s11734-025-01743-4).
- Patil, P.M. and Roy, S. (2010), (2010), "Unsteady mixed convection flow from a moving vertical plate in a parallel free stream: Influence of heat generation or absorption", *International Journal of Heat and Mass Transfer*, Vol. 53 Nos 21-22, pp. 4749-4756.
- Patil, P.M. (2012), "Effects of surface mass transfer on steady mixed convection flow from vertical stretching sheet with variable wall temperature and concentration", *International Journal of Numerical Methods for Heat and Fluid Flow*, Vol. 22 No. 3, pp. 287-305.
- Patil, P.M., Momoniat, E. and Roy, S. (2014), "Influence of convective boundary condition on double-diffusive mixed convection from a permeable vertical surface", *International Journal of Heat and Mass Transfer*, Vol. 70, pp. 313-321, doi: [10.1016/j.ijheatmasstransfer.2013.11.021](https://doi.org/10.1016/j.ijheatmasstransfer.2013.11.021).
- Patil, P.M., Anilkumar, D. and Roy, S. (2013a), "Unsteady thermal radiation mixed convection flow from a moving vertical plate in a parallel free stream: effect of Newtonian heating", *International Journal of Heat and Mass Transfer*, Vol. 62, pp. 534-540, doi: [10.1016/j.ijheatmasstransfer.2013.03.027](https://doi.org/10.1016/j.ijheatmasstransfer.2013.03.027).
- Patil, P.M., Roy, S. and Pop, I. (2013b), "Chemical reaction effects on unsteady mixed convection boundary layer flow past a permeable slender vertical cylinder due to a nonlinearly stretching velocity", *Chemical Engineering Communications*, Vol. 200 No. 3, pp. 398-417, doi: [10.1080/00986445.2011.614978](https://doi.org/10.1080/00986445.2011.614978).
- Patil, P.M. and Goudar, B. (2023), "Single and multiple walled CNTs-TiO₂ ternary hybrid nanofluid flow of Williamson fluid in an unsteady combined convective regime: an entropy analysis", *Numerical Heat Transfer, Part A: Applications*, Vol. 84 No. 10, pp. 1216-1237, doi: [10.1080/10407782.2023.2174222](https://doi.org/10.1080/10407782.2023.2174222).
- Patil, P.M. and Shankar, H.F. (2023), "Analysis of nonlinear thermal radiation and entropy on combined convective ternary (SWCNT-MWCNT-Fe₃O₄) Eyring-Powell nanoliquid flow over a slender cylinder", *Numerical Heat Transfer, Part A: Applications*, doi: [10.1080/10407782.2023.2195131](https://doi.org/10.1080/10407782.2023.2195131).

- Paul, P. and Das, S. (2024), "Electro-pumping paradigm of non-Newtonian blood with tetra-hybrid nanoparticles infusion in a ciliated artery", *Chinese Journal of Physics*, Vol. 87, pp. 195-231.
- Priyadharshini, P. and Vanitha Archana, M. (2024), "Collective impact of radiation on tetra-hybrid nanofluid flow over a stretching sheet: Solar-powered charging station", *Numer. Heat Transf. Part A Appl*, pp. 1-18.
- Sajid, T., Jamshed, W., Eid, M.R., Altamirano, G.C., Aslam, F., Alanzi, A.M. and Abd-Elmonem, A. (2023), "Magnetised cross tetra-hybrid nanofluid passed a stenosed artery with nonuniform heat source (sink) and thermal radiation: novel tetra-hybrid Tiwari and Das nanofluid model", *Journal of Magnetism and Magnetic Materials*, Vol. 569, p. 170443.
- Sajida, T., Jamsheda, W., Algarnib, S., Alqahtanib, T., Eid, M.R., Irshade, K., Altamiranof, G.C., El Din, S.M. and Nahar Tajer, K.W. (2023), "Catalysis reaction influence on 3D tetra-hybrid nanofluid flow via oil rig solar panel sheet, case study towards oil extraction", *Case Studies in Thermal Engineering*, Vol. 49, p. 103261.
- Sakkaravarthi, K., Reddy, P.B.A. and Sakthi, I. (2024), "Entropy optimisation in Casson tetra-hybrid nanofluid flow over a rotating disc with nonlinear thermal radiation: a Levenberg–Marquardt neural network approach", *Journal of Computational Design and Engineering*, Vol. 11 No. 5, pp. 333-354.
- Schlichting, H. and Gersten, K. (2000), *Boundary Layer Theory*, Springer, New York, NY.
- Sherani, M.A., Muhammad, S., Mahariq, I. and Abbas, S.T. (2025), "MHD flow of Williamson nanofluid using effective similarity variable considering viscous dissipation and thermal radiation over a nonlinear stretching surface via OHAM", *Partial Differential Equations in Applied Mathematics*, Vol. 13, p. 101088.
- Singh, P.J., Roy, S. and Ravindran, R. (2009), "Unsteady mixed convection flow over a vertical wedge", *International Journal of Heat and Mass Transfer*, Vol. 52 Nos 1-2, pp. 415-421.
- Tamanna, T. and Sarder Firoz, A. (2025), "Computational modelling of radiative Williamson nanofluid flow under the impression of periodic magnetic force", *International Journal of Thermofluids*, Vol. 26, p. 101065.
- Varga, R.S. (2000), *Matrix Iterative Analysis*, Prentice-Hall, USA, pp. 219-223.
- Yusuf, T.A., *et al* (2020), "Entropy generation in MHD Williamson nanofluid over a convectively heated stretching plate with chemical reaction", *Heat Transfer*, Vol. 49 No. 4, pp. 1982-1999, doi: [10.1002/htj.21703](https://doi.org/10.1002/htj.21703).

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