A benchmark modelling for participation-based tax increment financing

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Abstract

Purpose – This paper aims to present the theoretical and conceptual framework of a new method in public finance called "participation based tax increment financing (P-TIF)" by combining conventional tax increment financing (TIF) within the Sharī'ah-compliance structure.

Design/methodology/approach – This study develops a benchmark model for P-TIF, which offers a participative contract between both lender and borrower. With the help of this model, a financing schema in P-TIF is established by incorporating stochastic modelling. Possible implications and alternative options of application are also explored with a discussion of challenges.

Findings – The results mainly indicate that P-TIF promises lenders to be a part of increment from tax earnings, in return for a reduced interest rate. They show how a rise in participation of the lender in a given contract lowers the interest rate. Under the base case scenario, the interest rate is reduced to zero when the participation of the lender in tax increment is set at 50%.

Practical implications – With the feature of being interest-free, P-TIF can be implied also within the Sharī'ah-compliance framework, thanks to the model it is based on. Additionally, as the model in this paper is parametric, it can be applicable to various cases in Islamic finance.

Originality/value – To the best of our knowledge, this is the first paper in the literature in the sense that it provides a conceptual idea and respective model for TIF method within a Sharī'ah-compliance framework.

Keywords Public finance, Tax increment financing, Islamic finance and banking, Stochastic modelling

Paper type Conceptual paper

Introduction

In the current world, particularly after the start of the modern era, the transformations led by industrialization have had some significant effects on various phenomena in the economy. One of the most prominent effects is urbanization. Urbanization worldwide is reported at around 54% as of 2018; this is anticipated to increase to 68% by 2050 (United Nations, 2019). Because of increasing urbanization, today municipalities have to manage a complex and competitive urban setup. They have to manage their development and

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consumption expenditure from an increasingly limited local government revenue share. Because of fiscal constraints, municipalities face a trade-off between consumption and development/redevelopment expenditure. To augment public infrastructure development or to upgrade decaying infrastructure, municipalities have to look for alternative financing mechanisms. Tax increment financing (TIF) is one such mechanism used by local government authorities to combat fiscal constraints and upgrade local infrastructure.

TIF is one of public finance practices for economic development used by local governments and municipalities to finance new development and renovation projects in tax increment districts (TIDs). In TIF, future tax revenues of the designated district are sold or assigned to a third party in exchange for a cash payment. A traditional TIF works on the principle that infrastructure development in a TID stimulates economic activities by attracting private investment, i.e. business setups and employment opportunities, hence resulting in an increase in property values and property taxes. This financing tool allows the capture of incremental tax value from the increased property value within a TID that can then be used to finance the capital costs of development plans (Carroll, 2008).

TIF and similar methods have been used particularly in the USA for over 60 years where it has generally been deployed within the scope of fixed income financing such as project-specific TID bonds and some other public sector finance methods for urban infrastructure developments and urban renovation projects (McIntosh *et al.*, 2015). TIF began in 1952 with the creation of the legal framework in California and has been spread to other states in time. Outside of the USA, the most distinct usage of TIF can be seen in Canada. While the UK, New Zealand and Australia have been attempting pilot applications or trials of TIF, it can be seen that similar instruments aiming to finance urban renewal and infrastructure projects have been considered in mostly European regions (Squires and Hutchison, 2014).

Although the origin of TIF traces back to the 1950s when the State Government of California passed TIF legislation as an urban renewal strategy, other versions were used as traditional borrowing method by the Ottoman Empire and in many European countries as well (Man and Rosentraub, 1998; Schumann, 2017). This long-term borrowing was called the *esham* system in the Ottoman Empire (Pamuk, 2009). While the modern practice of TIF includes development/redevelopment projects that are financed by future tax revenues, the *esham* system in the Ottoman Empire involved the auctioning of tax revenue of a specific region to a large number of prospective buyers. Successful bidders acquired the right to collect taxes throughout their lives.

The TIF process starts when a municipality initiates a development plan by designating an area as a TID. At an area so designated, the property tax rates are frozen at a base assessment value (Smith, 2009). During the life of the TID, the tax authorities continue to collect taxes at a base rate. Meanwhile, the municipality issues revenue rights or bonds to fund development/redevelopment projects in the TID. As development occurs in the TID, the assessed value of land and properties increases which leads to an incremental increase in property taxes over and above the base tax rate. These incremental tax revenues are used by the municipality to repay the bonds that were issued to fund developmental projects (Merriman *et al.*, 2011). At the expiration of the TIF, all debts are paid off and the new higher-level property taxes (base plus incremental) are collected by the tax authorities (Byrne, 2010).

This study proposes participation-based tax increment financing (P-TIF) as a viable solution for local governments to cater for the needs of community development in both emerging and Muslim countries. This kind of financing might be popular when local authorities such as municipalities face a fall in central government tax collections, grant transfers or lack sufficient financial resources. Compared to other income-generating models

such as sale of municipal land or services, P-TIF may provide a long-term solution to the shortage of municipal budgets.

This study introduces a conceptual framework for P-TIF by applying the TIF method within a Sharī'ah-compliant structure. Under the P-TIF financing method, a benchmark model is established with the benefit of stochastic modelling. This model offers a participation-based contract between the lender and the borrower in a given TID. In return for a reduced interest rate (which is reduced to zero in our model), P-TIF promises lenders to gain an excess increment from tax earnings. P-TIF can also be applied within the framework of Islamic finance, thanks to the model it is based on. This paper contributes to the Islamic finance literature by combining TIF and the participation framework together. Finally, possible implications and application fields are investigated with a discussion of challenges.

The remainder of the paper is organised as follows: the next section provides a literature review; the following section then introduces benchmark modelling for P-TIF; next, the possible implications and challenges for the use of P-TIF are discussed; the main findings and concluding remarks are summarized in the final section.

Literature review

The literature generally discusses TIF as a tool of economic development and its relation with property values. Practically, TIF has been largely adopted by municipalities. In his study, Anderson (1990) uses a structural probit model to analyse some American cities that have adopted TIF plans and their impact on property values. The author finds that cities adopting TIF plans experience greater property values compared to cities not adopting TIF. This positive association between TIF adoption and property values does not, however, directly imply that TIF causes growth in property values. Similarly, Man and Rosentraub (1998) analyse the impact of TIF plans on property values by using panel data of Indiana cities. Using a first-difference model, they find that TIF plans increased median housing value by 11%.

In contrast to the previous two studies, Dye and Merriman (2000) provide a different result in their paper. Based on data from the Chicago metropolitan area, they show that cities adopting TIF grow more slowly than those which do not. They also examine why local governments offer economic development incentives (EDIs). Blighted areas where numerous buildings are dilapidated or abandoned, market failures, bidding wars and intergovernmental revenue shifting (e.g. where a state government may share a portion of its tax receipts with the county and municipal governments within its boundaries) are four key reasons why local authorities might offer EDIs.

TIF programs help municipalities which use it. TIF plans aim to support urban development/redevelopment programs and they require no outside money or financial resources. Briffault (2010) evaluates TIF plans as a potential source of conflict between the private and public sectors, as the expenditure of taxes should legally require public purpose. How the public authorities support private activities or how the public–private partnership is established is a legally sensitive issue. Another potential problem related to TIF is tax restrictions based on the fact that tax rates and assessments should be uniform throughout the taxing jurisdiction. Briffault (2010) claims that TIF practices violate uniformity, as the income generated within TIF districts is used for the development within the district, whereas income generated in the whole city is returned to the city including the TIF district. Briffault (2010) also emphasizes the cost-benefit analysis of TIF plans. He asks which municipalities choose it and whether the economic benefits of TIF compensate for its costs. Decentralization, interlocal conflicts and municipals' debt limits are other issues related to TIF that the author discusses. Despite its potential problems, TIF presents a significant alternative financing method for public and local authorities. Local and public authorities are able to modify TIF programmes and define new rules and criteria to develop this alternative financing method.

Pamuk (2009) has described in some detail the quasi-TIF practice known as *esham* in the Ottoman Empire. Potential buyers would bid for the right to collect taxes in a specific district. The amount that buyers of the *esham* certificates used to pay was six to seven times the annual tax revenue of the government. The main motivation of the *esham* system was to broaden the government's borrowing.

In the *iltizam* system, which is another institution of tax-farming and which appeared in the 15th century in the Ottoman Empire, the empire or state would organize a public auction and the sources of public revenue would come from applicants in the auction. The buyer of the *iltizam*'s right would have his own agents for collecting taxes or would sell his right to a third party. The average duration of the *iltizam* tax collection was three years. In contrast to the *iltizam* system, the *malikane* system entailed a life-long practice where rights could last over the purchaser's lifetime and could also be transferred to future generations and it created more cash for empires or states (Agoston and Masters, 2010). The *iltizam* and *malikane* systems were often abused by the holders or buyers of these certificates. The *esham* system was introduced to eliminate the problems of the previous two methods.

In relation to alternative financing methods for local development projects, Hummel and Goud (2017) investigated the *esham-ijārah* structure (a revenue-sharing arrangement based on the *esham* structure combined with *ijārah ṣukūk*), a type of Islamic borrowing product, in the USA. Unlike TIF, the traditional approach for local development investment, the authors find that when a crowdsourced option is incorporated along with an *ijārah* and *esham* structure, the investment returns are higher than when adopting the conventional approach. They also add that the *esham-ijārah* approach includes higher risks but that the returns of this alternative approach are also higher. The higher returns might increase the incentive for lenders and borrowers to invest in this option.

A recent study about *ijārah* (lease) financing by Adeinat *et al.* (2019) assesses customers' perception of Islamic banks and finds that there exists a high and positive association between customers' perception and satisfaction with Islamic banking services. Finally, as this study consists of a risk-sharing framework, it is worth mentioning risk management in Sharīʾah-based financial products. There are some empirical studies that compare conventional financial products with Sharīʾah-based financial products in terms of risk. Risk management of new Sharīʾah-based products directly affects the spread and acceptance of these new products by investors. One of these current studies by Al Rahahleh *et al.* (2019) explores the fundamental features of risks associated with Islamic finance products. Their findings reveal that Islamic finance products are more risk-sensitive because of the nature of their products, contract structure, legal costing, governance practices and liquidity infrastructure. In a very recent empirical study, Gün (2020) assesses the risks related to lease certificates in the Turkish capital markets and calculates the minimum expected loss and risks of various *ṣukūk* portfolios. The findings of this study could provide some insights into risk management and volatility of the lease and new Sharīʾah-based products. The findings of these studies need to be taken into account for the development of new Islamic finance products.

To the best of the authors' knowledge, this study is the first paper in the Islamic finance literature that provides a conceptual idea and respective mathematical model of the TIF method within a participation-based finance framework. This study will contribute to the Islamic finance literature as follows: the P-TIF model provides an alternative financing method within a Sharīʾah-compliant structure. The new conceptual model in this study offers a participation-based contract between the lender and the borrower in a given TIF, in return for having a reduced interest rate (which decreases to zero in our model). Moreover,

TIF and the participation framework are combined together under P-TIF, which introduces a new financial instrument in the literature.

Model

The studies mentioned in the previous section have not sufficiently investigated the technical background of TIF-based methods and alternatives. In this study, while the participation of both the lender and the borrower is included in a given TIF contract, called the P-TIF, a benchmark model for P-TIF is also introduced by incorporating stochastic modelling. In this direction, a financing model in P-TIF is established with the benefit of various stochastic processes.

Initially, to define the values of real estate properties in a given developing district, the properties in the given district are assumed as identical for the sake of simplicity. Then, following Ebrahim *et al.* (2011) and Varli and Yildirim (2015), property value of any real estate in the district, H_t, is generated with a stochastic process such as:

$$dH_t = (r_t - \delta_H)H_t dt \; + \; \sigma_H H_t dZ_t^H \tag{1}$$

where r_t is the expected return of property (i.e. risk-adjusted yield) and δ_H represents rental rate, which can be seen as rental income of the property and can also be calculated by a ratio of the property value. In the literature, δ_H is also denoted as a service flow coming from the use of property over time (Kau *et al.*, 1992). Deviation of the value of property from its mean is represented by the volatility σ_H in the process. Finally, Z_t^H is used as a standard risk-neutral Brownian motion.

We assume that the cost of redevelopment for a given property is a constant proportion L of the initial value of the property H_0 and that all properties are the same, so the initial loan Q_0 required for the redevelopment of the district equals the sum of redevelopment costs of all properties in a given district; i.e. $Q_0 = \mathrm{n} \times L \times H_0$, where n is the total number of properties. The loan consists of periodic payments a_t for $\forall \mathrm{t}\; t \in [0, T]$, where T is the maturity time of the loan and the terminal payment (or balloon payment) B_T at T. Then, the outstanding loan balance (OLB), Q_s, at any time $s \in [0, T]$ equals the sum of the future periodic payments' discounted expected value and the balance at the terminal such that:

$$Q_s = \int_s^T e^{-r_s(t)(t-s)} E_s[a_t]dt + \; e^{-r_s(T)(T-s)} E_s[B_T] \tag{2}$$

where r_s (t) denotes the term structure of risk-adjusted yield. In the loan, non-amortizing (i.e. interest-only) schema is used in which the entire amount of the loan principal is made as balloon payment at maturity. Therefore, for each period OLB is equal to the loan principal, implying $Q_t = Q_0 = B_T$ for $\forall t \in [0, T]$. Furthermore, periodic payments are constructed as a proportion i of OLB and so $a_t = iQ_t = iQ_0$ for $\forall t \in [0, T]$, where the interest rate, i, represents the cost of using the loan. If the probability of payments risk such as prepayment and default is zero, then the interest rate cannot be higher than the risk-free rate of interest.

P-TIF, as compared to the conventional TIF, constructs a participation-based contract between the borrower and the lender of a given TIF. With its participatory framework, P-TIF enables lenders to gain a part of excess tax increments in exchange for a reduced level of interest rate. That is, a lower level of interest rate is now compensated from the borrower by giving up a share of tax increments that occur after the urban renovation, through the commitment of future tax revenues from incremental increases in the property values [i.e. $(H_t - H_0)^+$] to the lender. Here, H_0 denotes the initial value of the property at a fixed

threshold point. Therefore, the share of tax increment becomes conditional, i.e. if there is an excess tax earning.

This fixed point is adjustable in the contract between both sides (i.e. the borrower and lender) and the share for continuous tax increment flow is binding as well. To calculate the amount of the increment in each period, a specialized index for the property value, such as house price index, can be formed and used. Therefore, periodic payments a_t and OLB at maturity B_t in P-TIF become:

$$a_t = iQ_t + \underbrace{\theta \pi n (H_t - H_0)^+}_{\text{Share of Excess Tax Increment}} \tag{3}$$

$$B_T = Q_T \tag{4}$$

where θ is the participation rate for tax increment, π is the tax rate determined by the local authority and n is the number of properties in the district.

The short term rate's term structure is embedded into the model to discount the payments in the future (Bakshi *et al.*, 1997; Deng, 1997). The risk-free short-term rate process is used to discount the future payments of the borrower and is defined following Vasicek (1977) as:

$$dr_t = \alpha(\mu - r_t)dt \ + \ \sigma_r dZ_t^r \tag{5}$$

where α represents the speed of mean reversion, μ is the mean rate in the long run and σ_r stands for volatility. The correlation between the property value mentioned earlier and the short rate is defined as:

$$E\left[dZ_t^H dZ_t^r\right] = \rho^{H,r} dt. \tag{6}$$

Then the OLB at time s, Q_s, is formulated as:

$$Q_s = \int_s^T e^{-r_s(t)(t-s)} E_s[iQ_t]dt + e^{-r_s(T)(T-s)} Q_T + \theta \pi n \int_s^T e^{-r_s(t)(t-s)} E_s\left[(H_t - H_0)^+\right]dt \tag{7}$$

Assuming the loan is non-amortizing, that is $Q_t = Q_0 = Q_T$, the OLB becomes:

$$Q_s = \int_s^T e^{-r_s(t)(t-s)} iQ_0 dt + e^{-r_s(T)(T-s)} Q_0 + \theta \pi n \int_s^T c\left(H_s, H_0, T, r_s(T)\right)dt \tag{8}$$

where $c\,[H_s, H_0, T, r_s\,(\mathrm{T})]$ stands for European call option pricing. Details of the valuation of this call option are derived in the Appendix.

Interest rate

At the time of loan origination (i.e. $s = 0$), by using equation (8), the formula for the interest rate, i, can be obtained as follows:

$$i = \underbrace{\left[\frac{1 - e^{-r_0(T)T}}{\int_0^T e^{-r_0(t)t}dt}\right]}_{\text{Conventional Rate}} - \underbrace{\left[\frac{\theta\,\pi n \int_0^T c\left(H_0, H_0, T, r_0(T)\right)dt}{Q_0 \int_0^T e^{-r_0(t)t}dt}\right]}_{\text{Reduction in P-TIF Structure}} \tag{9}$$

where the first part on the right-hand side shows the conventional interest rate, whereas the second part indicates the reduction in interest rate in the case of P-TIF against the conventional TIF. Basically, to have a lower level of interest rate, the borrower has to determine the proportion of the tax increment (i.e. participation rate, θ) sharing with the lender. It is obvious that an increase in θ lowers the interest rate.

The parameters used for calculating the interest rate in P-TIF are given in Table I. For the selection of these values, benefit is drawn from previous studies in the literature (Varli and Yildirim, 2015). Using the base case parameters presented in Table I, the interest rate, i, in equation (9) is calculated via:

$$i = \underbrace{\left[\frac{1 - e^{-0.075(30)\times 30}}{\int_0^{30} e^{-0.075(t)\times t}dt}\right]}_{\text{Conventional Rate}} - \underbrace{\left[\frac{0.5 \times 0.1 \times 100 \int_0^{30} c\left(100, 100, 30, 0.075(30)\right)dt}{8000 \int_0^{30} e^{-0.075(t)\times t}dt}\right]}_{\text{Reduction in P-TIF Structure}} \approx 0 \tag{10}$$

where 8000 in the denominator of the second fraction refers to the initial loan amount, $Q_0 = \text{n} \times L \times H_0 = 100 \times 0.8 \times 100 = 8000$. Other values of the parameters come from Table I

Parameter	Definition	Value
H_0	Initial value of property	100
r_0	Initial value of short term rate	7.5%
δ_H	Service flow rate	3%
σ_H	Volatility of property value	10%
σ_r	Volatility of short term rate	1%
A	Speed of mean reversion	5%
M	Long run mean of short-term rates	7.5%
$\rho^{H,r}$	Correlation between property value and short term rate	−0.5
T	Life time of the loan	30
L	Cost of redevelopment ratio	80%
θ	Participation rate for tax increment	50%
n	Number of properties in the district	100
π	Tax rate	10%

Note: These are the values for the parameters to calculate the interest rate in P-TIF

Table I. Value of parameters in the calculation of interest rate

and the call option in equation (10) as c [100, 100, 30, 0.075(30)] for $t \in [0, 30]$ is calculated with respect to equations and calculations derived in the Appendix. Therefore, it is found that while the interest rate in conventional TIF (i.e. $\theta = 0\%$) is 8.26%, the rate drops to nearly zero in P-TIF if the participation rate of tax increment is 50% (i.e. $\theta = 50\%$).

Possible implications and challenges

The P-TIF is easily applicable to urban transformation projects to create new urban settlement areas. Old cities may host risky building stocks in some districts where illegal housing or unpermitted buildings were tolerated and eventually recorded as legal. Current owners of housings and buildings pay regular property tax to municipalities, but the annual total amount of property tax that local authorities collect is insufficient to compensate for rehabilitation and regeneration of such areas. Instead, local municipals (the borrower in our model) and construction companies (the lender in our model) can make an agreement based on our suggested model. It is significant to note that property tax on buildings in any district tends to increase after urban transformation, as many positive externalities emerge in new urban settlements; for example, new and broad green areas, social reinforcement areas, planned infrastructure, earthquake-resistant designs and so on. The expected higher future property tax influences the participation rate for tax increment in our model. Besides urban transformation, P-TIF is also used for funding infrastructure development projects of commercial or industrial districts where some portion of the future income of these districts is transferred to the lenders that will undertake the infrastructure development.

In addition to urban transformation and infrastructure development, local authorities can also apply P-TIF to finance their public services such as intra-city or intra-district transportation. Local authorities such as municipalities might create a licence that includes the right to provide public transportation. Some portion of the expected future income of this public transportation right (licence) is sold to lenders in an auction so that lenders invest in transportation for stronger and quality infrastructure and services. The possible application areas of the TIF and P-TIF can be extended to any public authority that provides any service or commercial product via a licence. P-TIF provides alternative financing instruments to almost all public authorities which aim to solve their long-term borrowing problems for funding various projects.

As an alternative instrument in Islamic finance, P-TIF might function as a kind of capital partnership (known as *mushārakah*) that does not permit interest in any joint partnership or enterprise. In our baseline model, the higher participation rates in the tax increment would reduce the interest rate to zero, which attracts the applicability of our alternative instrument in Islamic finance. Compared to the classical *mushārakah* instrument that entails profit and loss based on a partnership ratio and investment risk, our P-TIF model entails tax increment and participation rate. By adding some properties, our model can be transformed into various P-TIF instrument types such as a diminishing participation rate over the term. As the tax revenue increases over the term, the parties might set a new participation rate to share the extra tax revenue.

The P-TIF model also provides an alternative instrument in *muzāraʿah* (agricultural) and *muḍārabah* (capital/labour) types of risk-sharing agreements in Islamic finance. Borrowers, municipalities, local authorities or public bodies, could directly provide labour or machinery support to lenders to increase their participation rate, as they have enough workforce and physical capital. This kind of support would also be advantageous to lenders, as they would incur less fixed and variable investment costs. Therefore, besides the tax increment and participation rate, there are other financial incentives for both lenders and borrowers that are available in P-TIF type of instruments in Islamic finance.

Additionally, the P-TIF model might gain an important place, as an alternative, in the non-banking financial system, as it eliminates brokerage between the lenders and borrowers. The elimination of brokerage would directly reduce fees and transaction costs associated with P-TIF financing. With required standardization and audit, public authorities should develop and disseminate this instrument through their local bodies. Compared to other instruments or partnership agreements in Islamic finance, P-TIF includes less risk. Assuming all economic and financial variables are constant, the participation rate and tax increment level would influence the P-TIF model and public authorities such as municipals would make a guaranteed tax payment to lenders. Other risks may arise related to the quality of construction or infrastructure projects or the end time of projects. Strict inspection and monetary penalties would reduce these other risks and increase the applicability of the P-TIF model.

Some challenges and troubles might emerge related to the implications of the P-TIF instrument as an alternative financing tool. One of the significant parts of the P-TIF model relates to which mechanism will be used to sell this alternative financial instrument. In practice, an auction mechanism is considered that might involve at least two stages to adjust the participation rate, tax sharing and maturity. Therefore, design of the auction mechanism is one of the most critical issues facing the applicability of the P-TIF model. The designed auction should take into account some risky aspects such as valuation, maturity, tax increment level, participation rate, audit and others. Moreover, public authorities should attract more lenders to participate in P-TIF auctions to create maximum welfare for the well-being of the district and the local economy.

Another challenge related to the P-TIF instrument is that real valuation of the planned project directly affects the parameters of the model. Therefore, the question of which institutions will calculate the real value of the project gains critical importance in this alternative system. Last but not least, our P-TIF instrument includes complex mathematical calculation and stochastic modelling that make it difficult for lenders and borrowers to understand. A simpler version of the model would help market players understand how the P-TIF instrument works.

Concluding remarks

TIF is a public finance instrument in which future tax revenues of the renovated district are sold or assigned to a third party in exchange for a cash payment to cover the cost of designation. This type of financing method might come at the forefront, particularly when local authorities such as municipalities lack sufficient funding sources. Compared to other income-generating models such as sale of municipal land or services, TIF provides a long-term solution to the shortage in municipal budgets. Although various modern implications of TIF practices have been widely observed in developed countries such as the USA and Canada, its different versions have been used as a traditional borrowing method by the Ottoman Empire and in many European countries as well.

In this paper, a conceptual and technical idea is introduced and developed for the P-TIF method by combining TIF and a participation-based traditional public financing method. In line with this concept, a benchmark model for P-TIF is established by incorporating various stochastic processes. The results mainly show that a rise in participation rate would lower the interest rate. Under the base case scenario in this paper, the interest rate is reduced to zero when the participation rate of the tax increment is set as 50%. With this feature, it is argued that P-TIF can be also applied within the Sharīʾah-compliance framework, thanks to the model shown in this study. Therefore, P-TIF can be used as an alternative financing

instrument by almost all public authorities who aim to solve their long-term borrowing problem for funding various projects.

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Appendix

In equation (8), c (.) stands for the formula for pricing of a European call option. For any time of maturity $t \in [s, T]$, this call option's value at time s with strike H_0 is:

$$c\left(H_s, H_0, T, r_s(T)\right) = H_s e^{-\delta_H(t-s)} N\left(d_1(s,t)\right) + H_0 B_{s,t} N\left(d_0(s,t)\right) \tag{11}$$

where N represents the function of standard normal cumulative distribution. The input values for N in equation (10) are provided by:

$$d_\lambda(s,t) = \frac{\ln(H_s) - \ln(H_0) - \ln\left(B_{s,t}\right) - \delta_H(t-s) + \left(\lambda - \frac{1}{2}\right)v_{s,t}^2}{v_{s,t}} \tag{12}$$

for $\lambda \in [0, 1]$, where $v_{s,t}^2$ satisfies:

$$\begin{aligned} v_{s,t}^2 = {} & \frac{\sigma_r}{\alpha^2}\left((t-s) + \frac{2}{\alpha}e^{-\alpha(t-s)} - \frac{1}{2\alpha}e^{-2\alpha(t-s)} - \frac{3}{2\alpha}\right) + \sigma_H^2(t-s) \\ & + \frac{2\rho^{H,r}\sigma_r\sigma_H}{\alpha}\left((t-s) - D_{s,t}\right) \end{aligned} \tag{13}$$

Furthermore, $B_{s,t}$ denotes the price of a zero-coupon bond and the formula of the bond in this structure is:

$$B_{s,t} = e^{\left[D_{s,t}(k-r_s) - (t-s)k - \left(\frac{\sigma_r D_{s,t}}{2\sqrt{\alpha}}\right)^2\right.} \tag{14}$$

where $D_{s,t} = \frac{1-e^{-\alpha(t-s)}}{\alpha} D_{s,t} = \frac{1-e^{-\alpha(t-s)}}{\alpha} D_{s,t} = \frac{1-e^{-\alpha(t-s)}}{\alpha}$ and $k = \mu + \frac{\sigma_r q}{\alpha} - \frac{\sigma_r^2}{2\alpha^2}$ $k = \mu + \frac{\sigma_r q}{\alpha} - \frac{\sigma_r^2}{2\alpha^2}$. And q represents the market price of risk and is given as 0.25 (see Hull *et al.*, 2014). The description of the interest rate's term structure r_s (t) is given by:

$$r_s(t) = -\frac{\ln(B_{s,t})}{t-s} = \frac{\left[-D_{s,t}(k-r_s) + (t-s)k + \left(\frac{\sigma_r D_{s,t}}{2\sqrt{\alpha}}\right)^2\right]}{t-s} \quad (15)$$

From equations (11) to (15), the findings are compatible with the previous studies in the literature such as Merton (1973) and Brigo and Mercurio (2007).

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