

# Traffic flow routing and scheduling in a food supply network

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## Abstract

**Purpose** – The purpose of this paper is to focus on the reference model of a grid-like supply network that enables formulation of delivery routing and scheduling problems in the context of the periodic vehicle routing problem.

**Design/methodology/approach** – The conditions for seamless (collision-free) synchronization of periodically executed local transport processes presented in this paper guarantee cyclic execution of supply processes, thereby preventing traffic flow congestion.

**Findings** – Systems that satisfy this characteristic, cyclic deliveries executed along supply chains are given and what is sought is the number of vehicles needed to operate the local transport processes in order to ensure delivery from and to specific loading/unloading points on given dates. Determination of sufficient conditions guaranteeing the existence of feasible solutions that satisfy these constraints makes it possible to solve the considered class of problems online.

**Practical implications** – The computer experiments reported in this paper show the possibilities of practical application of the proposed approach in the construction of decision support systems for food supply chain management.

**Originality/value** – The aim of the present work is to develop a methodology for the synthesis of regularly structured supply networks that would ensure fixed cyclic execution of local transport processes. The proposed methodology, which implements sufficient conditions for the synchronization of local cyclic processes, allows one to develop a method for rapid prototyping of supply processes that satisfies the time windows constraints given.

**Keywords** Declarative model, Discrete event dynamic system, Grid-like structure, Periodic vehicle routing problem, Traffic flow

**Paper type** Research paper

## 1. Introduction

The rapid development of goods, energy, and data-distribution systems, as well as systems of passenger transport fosters economic development and the associated development of the global market for industrial goods and services. Assuming a supply system has a structure and behaviour (Bocewicz and Banaszak, 2013), one can distinguish elements of the



structure of such a system (including transport roads, goods transfer facilities, etc.) and processes which determine its behaviour (e.g. the frequency and timeliness of deliveries, costs, etc.). This distinction is reflected in the perspective scientists take on supply systems, with some of them accentuating the context of analysis of behaviour reachable in systems with arbitrarily given structures, and others focussing on the synthesis of structures that enable the processes executed in those structures to run in a desired way.

From the perspective of such a division, it is easy to see that most of the work in the field of logistics of goods supply is devoted to the analysis of potential modes of organising transport processes. The main drawback of this approach is that it does not guarantee the existence of a feasible solution, or the solution is obtained within a time limit by which the decisions become outdated. An example of the former situation is the control of traffic lights in urban traffic management systems. The only type of structure that guarantees a green wave in each direction of each two-way road is a Manhattan-type road network (Bocewicz *et al.*, 2009).

A similar shortcoming characterises the approach that consists of searching for a system structure which guarantees the execution of the given behaviour. It is obvious that not every vehicle fleet operating in the available transport road structures will ensure timely service to and from the given goods transfer facilities. Both approaches lead to the class of NP-hard problems. The approach proposed in this paper assumes the possibility of decomposing the above-discussed problems of synthesis and analysis. Such decomposition would entail separate treatment of flow of means of transport and flow of materials transported by those means. An analogy to the case considered can be found when one considers examples of planning a trip in a public transport network or expanding (e.g. modernising) a transport network to meet the needs of passenger transport. In the first case, the problem boils down to answering the question of whether within a given public transport structure in which vehicles run to a given timetable, there is a route which ensures the passage between designated stops within the given time horizon? The relevant question in the second case is: Can one change the timetables of the public means of transport so that travel time along the given route between two selected stops does not exceed the given time horizon?

In addition to the paradigm of “layered decomposition” of supply problems, this work also adopts the paradigm of “regularity of route structure”, which assumes that routes together with the local transport processes executed along those routes have a regular grid-like or fractal-like topology. Such an assumption facilitates the design of supply structures composed of repeating fragments (grids). It also provides the possibility of predicting certain behaviours of the system (e.g. cyclic, collision-free, etc.) based on an analysis of the behaviour of a repeating part of its structure. The paradigm of regularity of a supply system’s structure meets the expectations associated with the simplification of design procedures as it entails that it is better to design regular structures than to look for this kind of behaviour in arbitrarily given structures, running the risk of failing to find a feasible solution.

The problem being considered is simultaneous supply of different deliveries/goods from suppliers to recipients, who are all located at various points of the transport network. Knowing the parameters of the local transport companies, one looks for delivery routes that, for example, would minimise transport time or satisfy the constraints regarding the number of vehicles that can be simultaneously loaded/unloaded at a transshipment point. On the other hand, in solving a synthesis problem subject to the same assumptions and constraints, one builds on knowledge of routes and transport times to find the parameters of local transport companies which would guarantee the delivery of goods within a given time window.

The aim of the present work is to develop a methodology for the synthesis of regularly structured supply networks that would ensure fixed cyclic execution of local transport processes.

The proposed methodology, which implements sufficient conditions for the synchronization of local cyclic processes, allows one to develop a method for rapid prototyping of supply processes that satisfies the time windows constraints given.

The paper is divided into seven sections. Section 2 presents a detailed review of the most important trends in research devoted to the vehicle routing problem. Section 3 introduces the reader into the subject of construction and management of grid-like structure supply networks, focussing particularly on the problems of modelling of time-driven and event-driven versions of system of concurrent cyclic processes (SCCP) systems. Section 4 formulates the research problem, the more specific aspects of which, namely, the problems of time window constrained delivery routing and scheduling are discussed in Section 5. Section 6 illustrates the possibilities of practical application of the proposed method by presenting the results of computational experiments conducted on a food supply chain system. Final remarks and the scope of future work are presented in Section 7.

## 2. Related work

The vehicle routing problem (VRP) is an optimisation problem commonly encountered in logistics and transport, which applies to supply chain management in the physical delivery of goods and services. The VRP can be defined as the problem of designing the least cost delivery routes from a depot to a set of geographically dispersed locations subject to a set of constraints. VRP is classified as an NP-hard problem (Kumar and Panneerselvam, 2012) and is one of the most widely studied topics in operations research.

There are several variants of VRP that incorporates real-life complexities while taking into account the nature of the transported goods, the quality of service required, and the characteristics of the customers and vehicles. Among a variety of possible extensions, the most popular are the heterogeneous fleet VRP which assumes that goods are delivered by a fleet of vehicles with dissimilar capacities, the VRP with time windows, assuming that deliveries to a given customer must occur in a certain time interval and the multi depot VRP assuming that multiple depots are geographically spread among customers.

A special role among VRPs is played by the periodic VRP (PVRP), in which delivery routes are constructed over a period of time (Francis and Smilowitz, 2006; Coene and Arnout, 2010; Angelelli and Speranza, 2002; Drummond *et al.*, 2001). The objective of both the VRP and the PVRP is to minimise travel costs or the total distance which must be travelled to visit all customers during the planning horizon. The difference is that, in PVRP the frequency of visits to customers within a given time horizon varies from customer to customer. Besides the assumptions traditionally accepted by the VRP, the PVRP has to take into account a time horizon, usually subdivided into regular periods, as well as a customer visit frequency stating how often within a particular period a customer must be visited. A solution to the PVRP consists of sets of routes which jointly satisfy demand constraints and frequency constraints. The objective is to minimise the sum of the costs of all routes over the planning horizon (Coene and Arnout, 2010). Many methods are used to find the optimal solution, among them can be highlighted meta-heuristic, tabu search (Angelelli and Speranza, 2002), computer simulation techniques (Villarreal *et al.*, 2016) or many kinds of genetic algorithms (Drummond *et al.*, 2001).

Similarly, there are several variants to the PVRP, among which the PVRP with service choice plays a pivotal role resulting in more efficient vehicle tours and/or greater service benefit to customers. The goal is to find an assignment of nodes to service schedules and a set of vehicle routes for each period of the given time horizon that minimises the total routing cost incurred net of the service benefit accrued (Francis and Smilowitz, 2006).

Since the design of an optimal route for a fleet of vehicles to service a set of customers is the main objective of the VRP, most study efforts are focussed on the problems of management of different modes of transport services rather than on the routing of delivery

processes within a given transportation network. Most of the research concentrates basically either on scheduling available transport modes so that they can service customers in given time windows, or on designing supply networks taking into account the size and capacity of the planned fleet and the topology and traffic capacity of routes. Examples of such research are works devoted to the synchronization of public transport services (Gaska *et al.*, 2015) and urban network analysis (Sevtsuk and Mekonnen, 2012).

Relatively few studies are devoted to the study of delivery-flow routing and scheduling in the existent supply networks; examples of this type of solutions are courier, express and parcel services, the rail-road freight services (Crisalli *et al.*, 2013), and many other door-to-door combined (multimodal) supply chains. The vast majority of studies consider the supply of goods. Besides vehicle routing, other media routings are considered such as message routing (Socievole *et al.*, 2015), energy routing, data routing (Gurakan *et al.*, 2016), cargo routing (Cargo Movement, Defense Transportation Regulation), and courier routing (Lan *et al.*, 2007).

VRPs are solved using exact algorithms (e.g. direct tree search methods, dynamic programming, integer linear programming) or approximated algorithms (i.e. those employing artificial intelligence heuristics) (Laporte, 1992). Among approximated methods worth mentioning are meta-heuristics such as tabu search, simulation annealing, and evolutionary algorithms (e.g. ant colony and particle swarm) (Lan *et al.*, 2007).

An alternative approach to traffic flow modelling which takes into account most of the aforementioned assumptions as well as the specific character of passenger traffic is found in the work of Gao and Wu (2011). The concept of multimodal transport processes presented by those authors assumes that the supply processes are executed in an environment of cyclic transport processes. An example of a serial multimodal process is the movement of passengers travelling by different metro lines along a route of arbitrarily selected stations. A characteristic feature is that it offers the possibility of distinguishing levels that determine traffic flows of transportation means and traffic flows of multi-commodity flows.

### 3. Grid-like supply networks

The numerous road network patterns deployed in public and/or freight transport systems range from the tightly structured fractal network with perpendicular roads in a regular raster pattern to the hierarchical network with sprawling secondary and tertiary roads feeding into arterial roads in a branch like system (Courtat, 2012; Kelly and McCabe, 2006; Levinson and Huang, 2012). The throughput of multimodal processes of passenger/delivery flows depends on geometrical and operational characteristics of public transportation and/or cargo supply networks (Nakandala *et al.*, 2016; Ahmed, 2009).

Multimodal transportation processes involve the movement of objects using different modes of transport in a single, integrated transport chain on a given route (Bahrehdar and Moghaddam, 2014). Examples of such processes follow from daily commuting (bus – tram – metro), courier services (e.g. DHL), etc. In that context, the main advantages following the regular structure of supply network layout are the flexibility and robustness that are vital to the improvement of robustness of supply networks (Haghani and Oh, 1996; Susan, 2014). More exhaustive studies, respectively, concerning mesh-like or grid-like as well as fractal-like structures of urban transportation networks, can be found in the papers (Buhl *et al.*, 2006; Xie and Levinson, 2009; Czerkauer-Yamu, 2012). Among many others the following findings showing “that grids’ greater connectivity offers shorter trips, trip frequency may be expected” (Stead and Marshall, 2001) and that “fractal geometry could be of interest for reducing negative impacts of urban sprawl” motivate this work, too.

The main advantage of the approach implemented is that it allows easy estimation of passenger travel schedules while taking into account the cyclic behaviour of both

local transportation modes and the whole transportation network. This is because, since the transportation processes executed by particular lines are usually cyclic, the multimodal transport processes supported by them also have a periodic character.

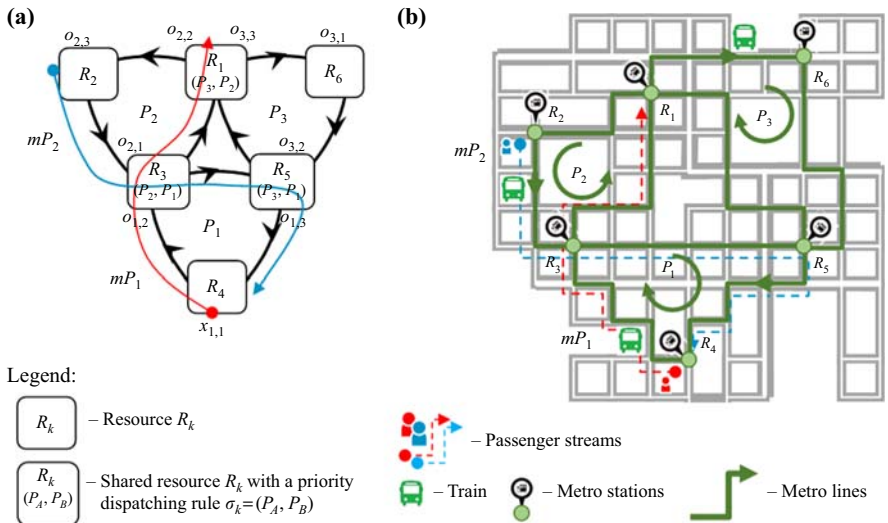
### 3.1 Modelling framework

A SCCP (see Figure 1) (Bocewicz and Banaszak, 2013) consists of a set of local processes  $P = \{P_i | i = 1 \dots \ln\}$  which execute operations periodically, based on a set of shared resources  $R = \{R_k | k = 1 \dots \text{lk}\}$ . Each process  $P_i$  is characterised by a sequence of operations  $O_i = (o_{i,1}, o_{i,2}, \dots, o_{i,j}, \dots, o_{i, \text{lr}(i)})$ , where  $\text{lr}(i)$  stands for the number of process operations  $P_i$ . Operation  $o_{i,j}$  of process  $P_i$  is executed based on the resources specified by process route  $M_i = (r_{i,1}, \dots, r_{i,j}, \dots, r_{i, \text{lr}(i)})$ , where  $r_{i,j} \in R$  stands for the resource used to execute operation  $o_{i,j}$ . Local processes are executed in a cyclic manner, which means that after all operations  $O_i$  have been executed, the sequence is started again.

Figure 1 shows an example of an SCCP which incorporates three local processes  $P = \{P_1, P_2, P_3\}$ , whose operations are executed along the following routes:  $M_1 = (R_4, R_3, R_5)$ ,  $M_2 = (R_2, R_3, R_1)$ . The elements shown make up structure SC of the SCCP, which is an ordered pair:

$$SC = (R, SL) \quad (1)$$

where  $R = \{R_k | k = 1, \dots, \text{lk}\}$  is a set of resources;  $\text{lk}$  the number of system resources of known availability  $C(R_k)$ ;  $C(R_k)$  the number of processes that can simultaneously use a resource  $R_k$ .  $SL = (P, U, O, \Omega)$  is the structure of local processes, where  $P = \{P_i | i = 1 \dots \ln\}$  is a set of local processes;  $U = \{p_i = (p_{i,1}, \dots, p_{i,j}, \dots, p_{i, \text{lr}(i)}) | i = 1 \dots \ln\}$  a set of routes of local processes;  $O = \{O_i = (o_{i,1}, \dots, o_{i,j}, \dots, o_{i, \text{lr}(i)}) | i = 1 \dots \ln\}$  a set of operation sequences; and  $\Omega = (\omega_1, \dots, \omega_b, \dots, \omega_m)$  a sequence of a certain number of local process streams, where  $\omega_i \in \mathbb{N}^+$  denotes the number of process streams  $P_i$ ; successive process streams are marked with a superscript  $h$ :  $P_i^1, \dots, P_i^h, \dots, P_i^{\omega_i}$  (when a process  $P_i$  has only one stream  $P_i^1$ , a superscript is not used).



**Figure 1.**  
The system of concurrent cyclic processes and its representation in an urban transport environment

**Notes:** (a) A schematic of the structure of a system of concurrent cyclic processes; (b) representation of the SCCP in an urban transport environment

Priority dispatching rules  $\sigma_k$  settle resource conflicts related to the order in which processes/streams can access shared resources.  $R_k$  (e.g.  $\sigma_1 = (P_3, P_2)$  from Figure 1) is a sequence which determines the order of processes supported by resource  $R_1$ : ...,  $P_3, P_2, P_3, P_2, P_3, P_2, \dots$ . These rules implement the process synchronization protocol which guarantees non-preemptive scheduling of resources, and the so-called mutual exclusion condition. Let  $\mathbb{S}$  stand for a set of admissible system states, i.e. states which specify the allocation of processes to resources. It is assumed that set  $\mathbb{S}' \subseteq \mathbb{S}$  is a set of states reachable from the initial state  $S^0 \in \mathbb{S}'$  if there is a sequence of states  $(S^0, S^1, \dots, S^g, S^d, S^f, S^k, \dots, S^w, S^r)$  of set  $\mathbb{S}'$  wherein each successive state is reachable, in accordance with  $\Theta$ , from a state immediately preceding it, i.e.  $\exists S^0, S^1, S^g, \dots, S^w, S^r \in \mathbb{S}'$  such that  $S^1$  is reachable from  $S^0$  and  $S^g$  is reachable from  $S^1$  and ... and  $S^r$  is reachable from  $S^w$ .

In this context, the behaviour of the SCCP can be defined as an ordered triple:

$$B = (\Theta, S^0, \mathbb{S}'), \quad (2)$$

where  $\Theta = \{\sigma_k = (s_{k,1}, \dots, s_{k,d}, \dots, s_{k,lh(k)}) | k = 1 \dots lk\}$  is a set of dispatching rules, where  $s_{k,d} \in P$  is a process/stream sharing resource  $R_k$ .  $\mathbb{S}' = \{S^r | S^r = (A^r, Z^r), S^r \text{ is reachable from } S^0 \text{ in accordance with } \Theta, S^r \in \mathbb{S}'\}$ ;  $S^0$  the initial state of the SCCP (initial allocation of local processes);  $S^r$  the  $r$ th state of the SCCP:

$$S^r = (A^r, Z^r), \quad (3)$$

where  $A^r = (a_1^r, a_2^r, \dots, a_k^r, \dots, a_{lk}^r)$ ,  $A^r$  is the allocation of local processes in the  $r$ th state,  $a_k^r \in P \cup \{\Delta\}$ ;  $a_k^r = P_i$  an allocation of a process which means that the  $k$ th resource  $R_k$  is occupied by process  $P_i$ ; and  $a_k^r = \Delta$  means that the  $k$ th resource  $R_k$  is free.  $Z^r = (z_1^r, z_2^r, \dots, z_k^r, \dots, z_m^r)$  is a sequence of semaphores of the  $r$ th state which specifies the current indication of dispatching rules (in state  $S^r$ ), where  $z_k^r \in P$  is a semaphore which specifies what process/stream has the right of access to resource  $R_k$  in the next step; i.e.  $z_k^r = P_i$  means that, in the  $r$ th state, process  $P_i$  has the right to access resource  $R_k$  next.

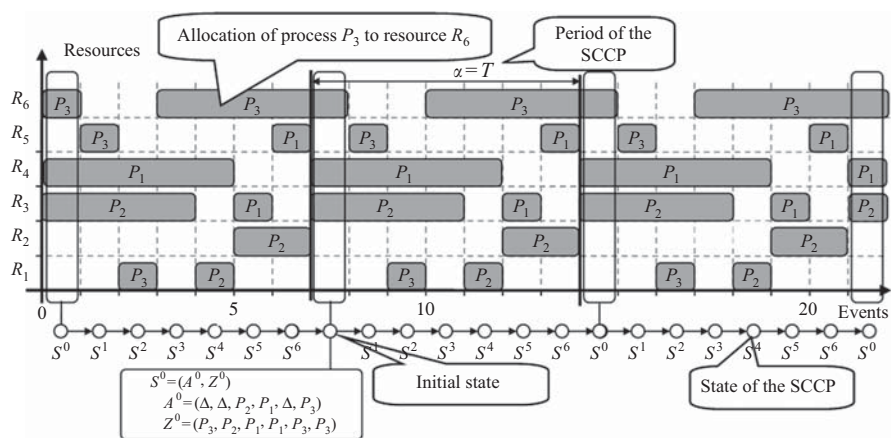
Fixed cyclic execution of a SCCP is reachable either directly from an initial state or as a consequence of reaching a state belonging to such a sequence. In other words, behaviour  $B$  described by fixed cyclic execution occurs when:

- it has a finite number of states  $\mathbb{S}'$ ; and
- $\forall S^i \in \mathbb{S}'$ , state  $S^i$  is reachable from  $S^{i-1}$  and state  $S^0$  is reachable from  $S^i \in \mathbb{S}'$  in accordance with the dispatching rules adopted and the principle of simultaneous local allocation.

Therefore, SCCPs belong to the class of discrete event dynamic systems. It is easy to observe that by introducing into the structure of local processes  $SL = (P, U, O, \Omega)$  a set of execution time sequences for operation  $T = \{T_i = (t_{i,1}, \dots, t_{i,j}, \dots, t_{i,lr(i)}) | i = 1 \dots ln\}$ , where  $t_{i,j}$  is the execution time of operation  $o_{i,j}$ , one can switch from event-driven to time-driven representation of the structure of local processes  $SL = (P, U, O, \Omega, T)$ . This means that by assigning to each operation  $o_{i,j}$  an associated execution time  $t_{i,j} \in \mathbb{N}^+$  and the starting time in each  $k$ th execution  $x_{i,j}(k) \in \mathbb{N}$ , one can naturally move from qualitative to quantitative assessment of the performance of the SCCP.

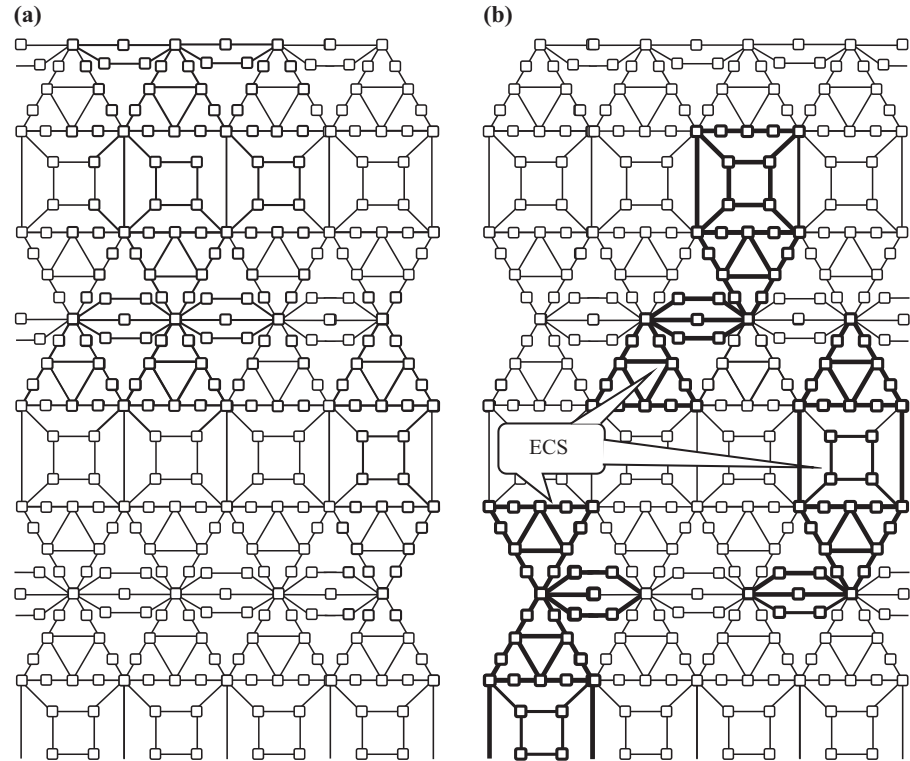
The cyclic behaviour of the SCCP from Figure 1 is illustrated by the diagram in Figure 2, which shows states and events which connects them. An event is understood here as a transition between two successive admissible process allocations. Of course, the topology of the structure of local processes may, but need not, coincide with the layout of routes which allow actual implementation of local processes. Figure 1(b) illustrates a situation in which

**Figure 2.**  
Examples of process  
allocations in the  
SCCP shown in  
Figure 1



the graph of execution of transport processes from Figure 1(a) is a subgraph of a certain public transport structure.

In the discussion to follow, we will focus on regular-structure SCCPs with grid-like topologies such as those encountered in crystalline structures and mosaics. An example of such a structure (Figure 3(a)) with a highlighted repeating pattern is shown in Figure 3(b).



**Figure 3.**  
Schematics of  
structures

**Notes:** (a) A regular structure; (b) selected examples of ECSs

In the light of the above considerations, a question arises whether there exists an ordered pair: (initial process allocation, a set of dispatching rules) which ensures a cyclic behaviour of an SCCP, i.e. one that guarantees a return to the initial allocation, once a sequence of transition allocations has been completed.

The problem of whether such a pair exists is a computationally hard problem, which limits the analysis to small-scale cases, much smaller than those encountered in practice. This, in turn, means that the problem of congestion avoidance in networks modelled by SCCP is also computationally hard.

### 3.2 Solution methodology

The approach proposed in the present study assumes a solution methodology based on:

- the paradigm of regular (e.g. grid-like) topology of a SCCP structure, which posits that the structures in question are composed of a finite set of identical repeating substructures;
- the scale decomposition paradigm, which posits that the behaviour of a given SCCP structure can be predicted on the basis of the behaviour of its substructure. Examples of such structures, called elementary covering structures (ECS), are shown in Figure 3(b); and
- the problem decomposition paradigm, which posits that a supply problem can be divided into a layer of problems of synthesis and analysis of transport mode infrastructure of an SCCP and a layer of problems of synthesis and analysis of goods transport processes of an SCCMP executed in that infrastructure, defined, respectively, by (1)/(2) and (4)/(5).

An ECS is a digraph comprised of elementary substructures (modelling local cyclic processes) which occur in the regular structure of an SCCP such that:

- the graph is composed of elementary cyclic digraphs which model local processes; and
- it can be used to tessellate a given regular structure.

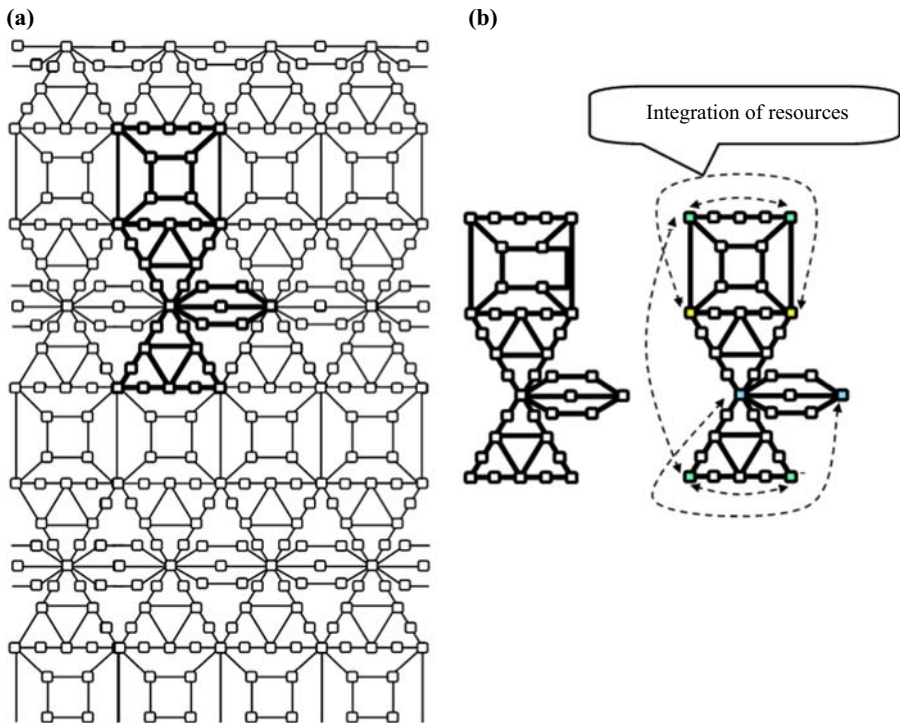
The images do not exhaust all possible cases of ECSs which may differ in shape and in scale (i.e. in the number of elementary structures they contain). However, their common feature is that they can be tessellated (like a mosaic) to form a pattern that recreates a given regular structure.

Each ECS has a corresponding covered form ECS that is formed by gluing together the vertices of the ECS that in the regular structure of the SCCP are joined with the corresponding fragments of the elementary structures of an adjacent ECS. An ECS extracted from the elementary structure is shown in Figure 4(a) and the relationships between the selected ECS and its covered form is presented in Figure 4(b).

The consequence of adopting the first two paradigms is the following theorem that relates the behaviour of a given regular structure to an elementary structure extracted from it.

**Theorem:** given are a regular-structure SCCP and its ECS. A cyclic behaviour of the covered form of the ECS entails a cyclic behaviour of the SCCP.

**Proof.** If, in accordance with the principle of decomposition of a regular structure, one replicates the same initial process allocation  $A^0$  and the same rule semaphores  $Z^0$  in all the remaining ECSs, the structure (see Figure 5(b)) will be free of collisions between processes that use the same resources. In summary, an initial allocation of local processes in a regular network will be followed in each individual ECS by a next allocation of local processes in compliance with the same priority selection rules. The new allocation in the



**Figure 4.**  
The regular structure

**Notes:** (a) An example of an ECS picked out from the regular structure; (b) The relationship between the ECS and its covered form

regular structure concerns the processes and resources that are “copies” of the processes and resources of the contemplated ECS. This type of situation is illustrated in Figure 5.

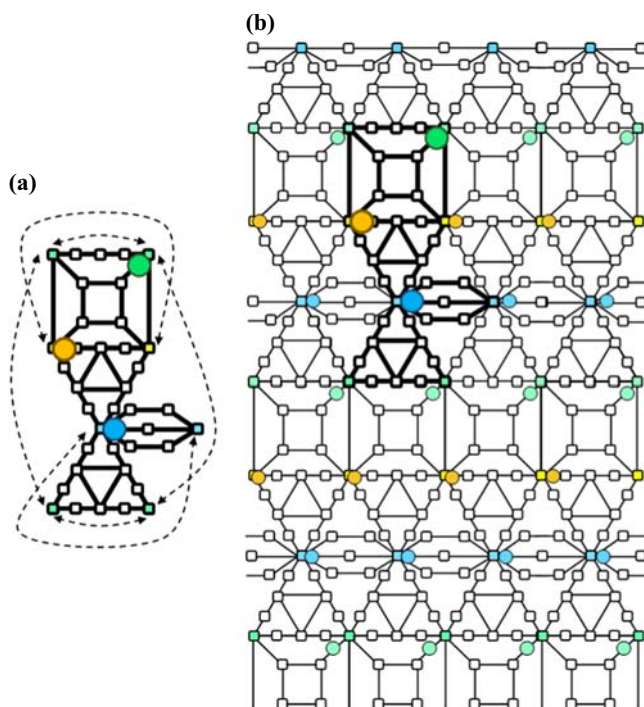
This means that the successive process allocations that occur in cycles after each initial allocation in the ECS will have their counterparts in the overall regular structure – allocations of all the local processes of the structure. Thus, period  $\alpha$  of the processes executed in the regular structure is equal to the period of processes executed in the ECS. This means that cyclic behaviour of the ECS implies cyclic behaviour of the regular-structure SCCP.

It follows from the theorem above that by solving a small-scale computationally hard problem (associated with an ECS), one can solve, online, a large-scale problem associated with a corresponding regular-structure SCCP. It is worth noting that the cycle period specified for the ECS determines an identical cycle period for the entire structure of the SCCP.

## 4. Traffic flow routing and scheduling

### 4.1 Multimodal processes

The concept of a multimodal process (Bocewicz *et al.*, 2014) refers to a process that is executed using different modalities, e.g. associated with different transmission media enabling radio, fibre optics, or wire communication. Processes are multimodal when operations require for their execution, beside resources  $R$ , also other processes. A pertinent example is the route of a passenger who travels to his destination by various metro lines (see Figure 1(b)).



**Notes:** (a) In an ECS; (b) in a corresponding regular structure

**Figure 5.**  
Schematics of initial  
allocation of processes

In the same way as for SCCP, one can also introduce a SCCP-driven system of concurrent cyclic multimodal processes (SCCMP). Accordingly, structure  $MSC$  of an SCCMP is an ordered pair:

$$MSC = (\bar{R}, MSL) \quad (4)$$

where  $\bar{R} \subseteq R$  is a set of resources of an SCCP of known availability  $mC(R_k)$  used by multimodal processes;  $mC(R_k)$  the number of processes that can simultaneously use a resource  $R_k$ .  $MSL = (MP, MU, MO, M\Omega)$  is the structure of multimodal processes, where  $MP = \{mP_i | i = 1 \dots lm\}$  is a set of multimodal processes;  $MU = \{mp_i = (mp_{i,1}, \dots, mp_{i,j}, \dots, mp_{i,lm(i)}) | mp_{i,j} \in R, i = 1 \dots lm\}$  a set of routes of multimodal processes;  $mP_i = (mp_{i,1}, \dots, mp_{i,j}, \dots, mp_{i,lm(i)})$ ,  $mp_{i,j} \in R$  stands for the route of process  $mP_i$  which is a combination of selected fragments of local process routes (Bocewicz *et al.*, 2015);  $MO = \{mO_i = (mo_{i,1}, \dots, mo_{i,j}, \dots, mo_{i,lm(i)}) | i = 1 \dots ln\}$  a set of operation sequences, where  $mO_i = (mo_{i,1}, \dots, mo_{i,j}, \dots, mo_{i,lm(i)})$  designates a sequence of operations  $mo_{i,j}$ ;  $lm(i)$  the number of operations  $mP_i$ . Each operation  $mo_{i,j}$  is assigned a local process necessary to execute operation  $m\mu_{i,j} \in P$ ;  $M\Omega = (m\omega_1, \dots, m\omega_i, \dots, m\omega_{lm})$  a sequence of a certain number of multimodal process streams, where  $m\omega_i \in \mathbb{N}^+$  denotes the number of process streams  $mP_i$ ; successive process streams are marked with a superscript  $h$ :  $mP_i^1, \dots, mP_i^{m\omega_i}, \dots, mP_i^{m\omega_i}$ .

Similarly to SCCPs, the behaviour of an SCCMP can be defined as an ordered triple:

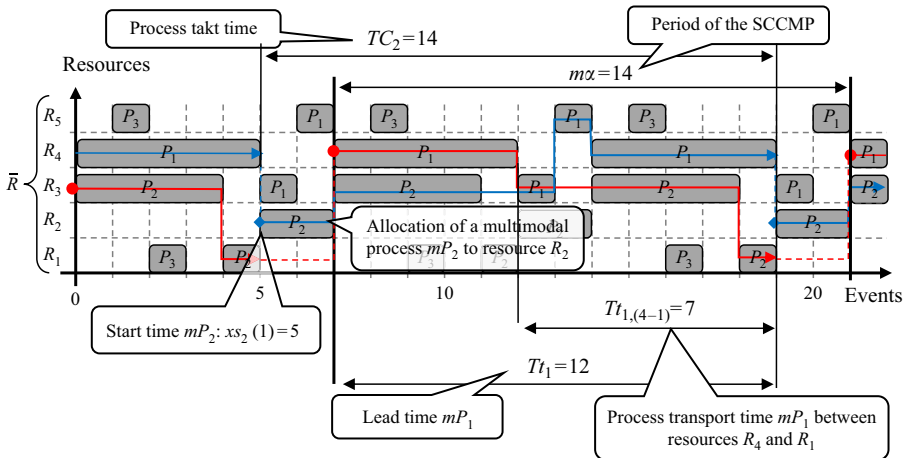
$$MB = (\bar{\Theta}, \bar{S}^0, \bar{S}^1), \quad (5)$$

where  $\overline{\Theta}$ ,  $\overline{S^0}$ ,  $\overline{S^r}$  are limited to subset  $\overline{R}$  and are defined analogously as in the case of an SCCP, i.e:  $\overline{\Theta} = \{\overline{\sigma}_k = (\overline{s}_{k,1}, \dots, \overline{s}_{k,d}, \dots, \overline{s}_{k,lh(k)}) \mid k = 1..lk\}$  is a set of dispatching rules governing the access of multimodal processes/streams to resources of set  $\overline{R}$  ( $R_k \in \overline{R}$ ), where  $s_{k,d} \in mP$  is a resource-sharing process/stream  $R_k$ .  $S^0$  is the initial state of the SCCMP (corresponding to the initial allocation of multimodal processes in the set of resources  $\overline{R}$ ).  $\overline{S^r} = \{\overline{S^r} \mid \overline{S^r} = (\overline{A^r}, \overline{Z^r}), \overline{S^r}$  is reachable from  $\overline{S^0}$  in accordance with  $\overline{\Theta}, \overline{S^r} \in \overline{S^r}\}$  is a set of states reachable from state  $S^0$ .  $\overline{S^r} = (\overline{A^r}, \overline{Z^r})$  is the  $r$ th state of the SCCMP (corresponding to the allocation of multimodal processes  $\overline{A^r}$  and semaphores  $\overline{Z^r}$  of the dispatching rules in resource set  $\overline{R}$ ).

Consequently, an SCCMP belongs to the class of discrete event dynamic systems. It is easy to observe that by introducing into the structure of multimodal processes  $MSL = (MP, MU, MO, M\Omega)$  a set of sequences of execution times of operation  $MT = \{MT_i = (mt_{i,1}, \dots, mt_{i,j}, \dots, mt_{i,lr(i)}) \mid i = 1 \dots lm\}$ , where  $mt_{i,j}$  is the execution time of operation  $mo_{i,j}$ , one can switch from event-driven to time-driven representation of the structure of local processes  $MSL = (MP, MU, MO, M\Omega, MT)$ . This means that by assigning to each operation  $mo_{i,j}$  an associated execution time  $mt_{i,j} \in \mathbb{N}^+$  and the starting time for each  $k$ th execution  $mx_{i,j}(k) \in \mathbb{N}$ , one can naturally move from qualitative to quantitative assessment of the performance of a SCCMP, e.g. from the simple conclusion that the behaviour is periodic to an estimation of the specific value of the period of cyclically repeating events.

Cyclic behaviour of the SCCMP from Figure 1 is illustrated by the diagram in Figure 6. It is easy to demonstrate that cyclic behaviour of an SCCP implicates cyclic behaviour of the multimodal processes executed in it. Assuming that a SCCMP has a multi-level structure, an SCCMP based on an SCCP is indexed as a first-level SCCMP (SCCMP<sup>(1)</sup>), a next system based on SCCMP<sup>(1)</sup> is indexed as SCCMP<sup>(2)</sup>, and so on. This also means that the behaviour of the successive layers SCCMP<sup>(i)</sup> will also be cyclic. Due to the wide array of practical applications of multimodal processes (Bocewicz *et al.*, 2014, 2015), these processes are characterised by many additional variables. In the context of the experiments conducted in this study (see Section 6), the following variables are found to be especially salient:

- takt time  $TC_i$  of process  $mP_i$  understood as the time between the moments of completion of successive executions of stream/process  $mP_i$ ;



**Figure 6.**  
A Gantt chart  
of execution of  
multimodal processes  
in the system shown  
in Figure 1(a)

- lead time  $Tt_i$  of process  $mP_i$  understood as the time between the start of the first operation and completion of the last operation of this process;
- transport time  $Tt_{i,(a-b)}$  between resources  $R_a, R_b \in \bar{R}$  of process  $mP_i$  understood as the time between the moments of completion of operations on resources  $R_a, R_b$ ; and
- start time  $xs_i(k)$  of process  $mP_i$  defined as a function determining the starting moment of the first operation of process  $mP_i$  in the  $k$ th cycle.

The values of these variables for the processes executed in the system shown in Figure 1(a) are presented in Figure 6.

#### 4.2 Problem formulation

The problems of cyclic scheduling in the domain of integer variables can be discussed in the context of Diophantine equations (Bocewicz *et al.*, 2009). This means that the problem of cyclic scheduling can be reduced to the problem of solvability of corresponding Diophantine equations, i.e. the problem of finding whether there exist solutions of these equations.

Because there are no general methods for solving such problems, each case requires an individualised approach taking into account the specific characteristic of the problem. For example, scheduling of SCCPs synchronized by the mutual exclusion protocol may result in deadlocks and starvation. In practice, this means that there are two problems to be solved. The first one is to determine the conditions which guarantee the existence of a non-empty set of feasible solutions. The second problem is to find in the set of feasible solutions, the optimum solution. It is worth noting that in a general case, both of these problems are NP-complete.

Cyclic scheduling problems, considered in the context of SCCP and SCCMP, are basically analysis and synthesis problems. Finding a solution to an analysis problem boils down to answering the question of whether in a given structure of a supply system, there is a way to transfer goods between selected points in a given period of time? On the other hand, an example of a synthesis problem can be reduced to the question of whether there exists a structure of the supply system which enables the transfer of goods between selected points in a selected way, in a given period of time? An example of synthesis problem is considered in Ahmed (2009), authors of which proposed a logistic plan design following customer requirements regarding a way of water distribution.

More formally speaking, in an analysis problem, given the structure of the SCCP SC (Equation (1)) which encompasses sets of process resources  $R$  and routes  $U$ , sets of operations  $O$ , as well as the number of streams  $\Omega$  of processes executed along the individual routes in a fixed cyclic sequence  $B$  (Equation (2)), one seeks SCCMPs with structure  $MSC$  (Equation (4)) (encompassing a set of resources  $\bar{R}$ , routes of processes  $MU$  connecting the given points of origin to their destinations, sets of operations  $MO$ , as well as the number of streams  $M\Omega$ ) which guarantee cyclic  $MB$  (Equation (5)) deliveries in specific time windows.

In a synthesis problem, on the other hand, given the structure  $MSC$  (Equation (4)) of an SCCMP, which guarantees cyclic  $MB$  (Equation (5)) deliveries in specific time windows, one seeks to find SCCPs with structure  $SC$  (Equation (1)) encompassing a set of resources  $R$  and routes of processes  $U$ , sets of operations  $O$  as well as the quantity of streams  $\Omega$  of processes executed along the individual routes in a fixed cyclic sequence  $B$  (Equation (2)) which ensures cyclic  $MB$  (Equation (5)) supplies in specific time windows.

### 5. Time window constrained delivery routing and scheduling

The Diophantine character of the cyclic scheduling problems described above, their diversity associated with the specific nature of the adopted assumptions, and constraints naturally imply their implementation in declarative programming environments.

The uniform representation of the problems of analysis and synthesis adopted in this study, treated here as a constraint satisfaction problem (CSP), reduces these problems to an ordered triple: a set of decision variables  $V$ , integer domains  $D$  which specify their values, and constraints  $C$  which describe the relationships among them. Finding a solution to a CSP boils down to determining such values of decision variables  $V$  for which all constraints from set  $C$  are satisfied.

### 5.1 Declarative modelling

The formulations of the problems of analysis and synthesis introduced earlier in this text, which make a connection between SCCPs and SCCMPs, also find their reflection in the declarative model. This model encompasses declarative models of the structure and behaviour of SCCPs and SCCMPs. In particular, in the context of the CSP, this model includes sets of decision variables  $V$ , domains  $D$  which specify their values, and constraints  $C$  which relate them within the SCCPs and SCCMPs considered separately, as well as a set of constraints  $C^*$  which relate variables and domains between those systems.

The set of decision variables found in this model includes:

- sets of resources  $R$  and shared resources found in an SCCP and sets of resources  $\bar{R}$  and shared resources present in an SCCMP;
- structures of local processes  $SL$  present in the SCCP and structures of multimodal processes  $MSL$  running in the SCCMP;
- a set of dispatching rules  $\Theta$  governing the access of local processes/streams to shared resources of the SCCP and a set of dispatching rules  $\bar{\Theta}$  governing the access of multimodal processes/streams to the resources of set  $\bar{R}$  in the SCCMP; and
- initial states:  $S^0$  in the SCCP (corresponding to the initial allocation of local processes) and  $\bar{S}^0$  in the SCCMP (corresponding to the initial allocation of multimodal processes to  $\bar{R}$ ).

Sets of decision domains corresponding to the above-mentioned decision variables determine the integer values of resource availability, integer operation execution times, integer vectors specifying the number of local process streams (vehicles), permutation vectors of local processes sharing the resources, and binary-valued vectors that specify the values of initial states.

Sets of constraints imposing regularity of structure of an SCCP and the principle of mutual exclusion of local processes as well as constraints enforcing the principle of mutual exclusion of multimodal processes in SCCMP structures (Bocewicz *et al.*, 2015) also include constraints which treat as identical resources of these structures, for example,  $\bar{R} \subset R$ , and times of goods transport operations with the travel times of the means of transport used for this purpose (Bocewicz *et al.*, 2014). Additionally, it is assumed that there are constraints which relate routes of multimodal processes  $mp_i$  with fragments of routes of local processes (see:  $mp_i$  expression (4)).

A declarative model of the integrated supply model is represented as an ordered triple:

$$(MD_{SCCP}, MD_{SCCMP}, C_{SCCP-SCCMP}) \quad (6)$$

where  $MD_{SCCP} = (V_{SCCP}, D_{SCCP}, C_{SCCP})$  is a declarative model of the SCCP, where  $V_{SCCP}$  is a set of decision variables of the SCCP;  $D_{SCCP}$  a set of domains of decision variables of the SCCP;  $C_{SCCP}$  a set of constraints relating the decision variables of the SCCMP.  $MD_{SCCMP} = (V_{SCCMP}, D_{SCCMP}, C_{SCCMP})$  is a declarative model of the SCCMP, where  $V_{SCCMP}$  is a set of decision variables of the SCCMP;  $D_{SCCMP}$  a set of domains of decision variables of the SCCMP;  $C_{SCCMP}$  a set of constraints relating the decision variables of the SCCMP.  $C_{SCCP-SCCMP}$  is a set of constraints relating variables  $V_{SCCP}$  and  $V_{SCCMP}$ .

The method of formal implementation of this model in the CSP is discussed below.

## 5.2 CSP

Depending on the class of problem considered (analysis or synthesis), the declarative model proposed (6), expressed using CSP terminology, is simplified:

- in the case of the analysis problem to the form:

$$CS_A = (V_{SCCMP}, D_{SCCMP}, C_{SCCMP} \cup C_{SCCP-SCCMP}), \quad (7)$$

according to which, what is being sought are the values of decision variables  $V_{SCCMP}$  characterizing an SCCMP (a set of resources  $\bar{R}$ , routes of processes  $MU$  connecting the given points of origin to their destinations, sets of operations  $MO$ , the number of streams  $M\Omega$  and behaviour  $MB$  (5)) which satisfy constraints  $C_{SCCP-SCCMP}$  characterizing a given structure of an SCCP (constraints entailed by structure  $SC$  (1)) and constraints on multimodal processes  $C_{SCCMP}$ .

- And in the case of the synthesis problem to the form:

$$CS_S = (V_{SCCP}, D_{SCCP}, C_{SCCP} \cup C_{SCCP-SCCMP}), \quad (8)$$

according to which, what is being sought are the values of decision variables  $V_{SCCP}$  characterizing an SCCMP (a set of resources  $R$ , process routes  $U$ , sets of operations  $O$ , number of streams  $\Omega$ , and behaviour  $B$  (2)) which satisfy constraints  $C_{SCCP-SCCMP}$  characterizing the given SCCMP (constraints entailed by structure  $MSC$  (4)) and constraints on local processes  $C_{SCCP}$ .

Among the constraints (7) and (8), special attention should be paid to those that guarantee deadlock-free and collision-free execution of local and multimodal processes. Bocewicz and Banaszak (2013) and Bocewicz *et al.* (2014) discuss constraints of this type in detail.

## 6. Computational experiments

### 6.1 Food supply grids network

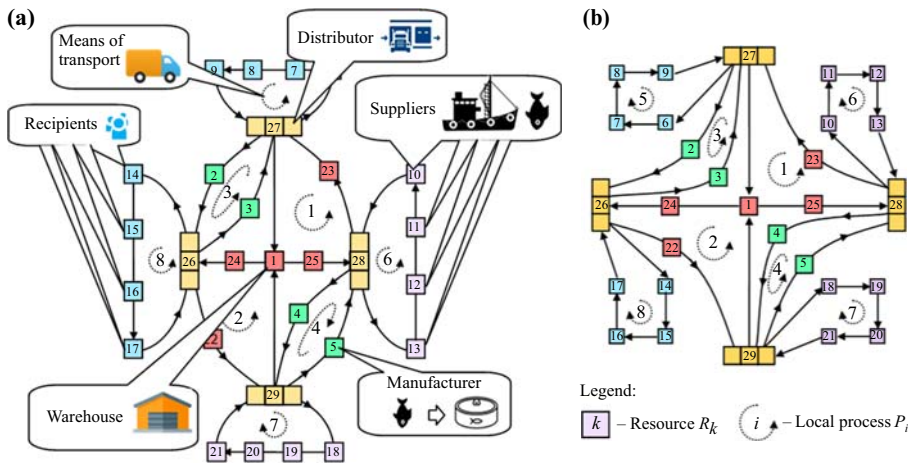
To illustrate the proposed approach, the transport network model shown in Figure 7(a) is considered. This is part of a food supply network shown in Figure 8, which is composed of autonomous grids, i.e. ECSs (see Figure 7(b)). In each ECS, there are two groups of recipients (resources  $R_6-R_9$  and  $R_{14}-R_{17}$ ), two groups of suppliers ( $R_{10}-R_{13}$  and  $R_{18}-R_{21}$ ), two groups of manufacturers (resources  $R_2, R_3$  and  $R_4, R_5$ ), and sets of warehouses ( $R_1, R_{22}-R_{25}$ ) and distributors (resources  $R_{26}-R_{29}$ ). Goods are delivered from suppliers to recipients by eight means of transport. The structure shown in Figure 7(a) and its ECS counterpart from Figure 7(b) corresponds to typical goods supply.

The ECS from Figure 7(b), marked out in the network of Figure 8, is characterised by the following variables:

- a set of resources  $R = \{R_1, R_2, \dots, R_{39}\}$  with unit availability  $C(R_k) = 1$  for  $R_k \in R$ ;
- a set of single-stream local processes  $P = \{P_1, P_2, \dots, P_8\}$ ; and
- a set of routes of local processes.

$p_1 = (R_1, R_{25}, R_{28}, R_{23}, R_{27})$ ,  $p_2 = (R_1, R_{24}, R_{26}, R_{22}, R_{29})$ ,  $p_3 = (R_2, R_{26}, R_3, R_{27})$ ,  $p_4 = (R_4, R_{29}, R_5, R_{28})$ ,  $p_5 = (R_6, R_7, R_8, R_9, R_{27})$ ,  $p_6 = (R_{10}, R_{11}, R_{12}, R_{13}, R_{28})$ ,  $p_7 = (R_{18}, R_{19}, R_{20}, R_{21}, R_{29})$ ,  $p_8 = (R_{17}, R_{18}, R_{19}, R_{20}, R_{26})$ , number sequence of streams of local process  $\Omega = (1, 1, 1, 1, 1, 1, 1, 1)$ . The resources of set  $R$  are non-preemptive, and the execution of local processes  $P$  is coordinated by the principle of mutual exclusion.

**Figure 7.**  
The a goods supply  
network and its SCCP  
representation



**Notes:** (a) A schematic of a goods supply network; (b) its counterpart described in terms of the SCCP

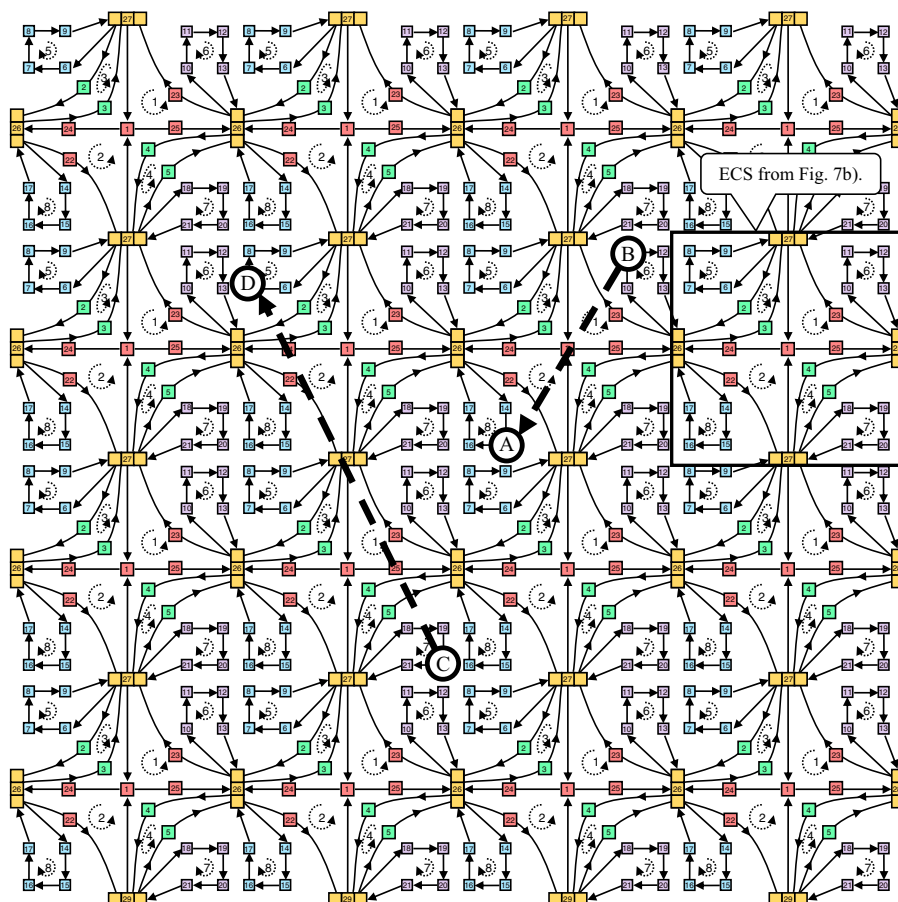
### 6.2 Delivery routing and scheduling

Computational experiments were conducted for the food supply network shown in Figure 8 to illustrate the proposed approach in the context of:

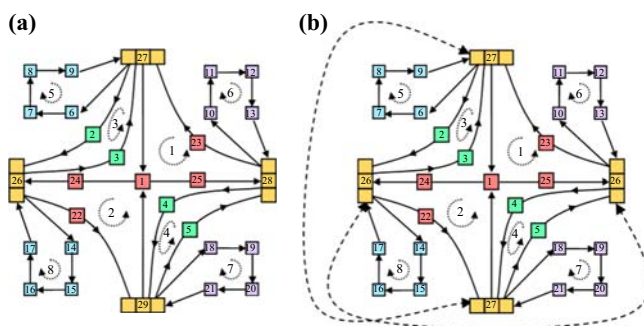
- the problems of analysis, in which delivery routes are being sought which can ensure the fulfilment of given customer expectations; and
- the problems of synthesis which seek to find an organisation of available transport modes which guarantees food deliveries in accordance with given customer expectations.

*The analysis problem.* Given is the network in Figure 8. In the network, two recipients A and D (resources  $R_{15}$ ,  $R_{16}$ ) and two food suppliers B and C (resources  $R_{11}$  and  $R_{13}$ ) are marked with circles. Deliveries should be executed in a cyclic mode every 11 s.u.t., in  $\pm 2$  s.u.t. time windows. In other words, the takt time of the multimodal processes representing potential food supply routes is  $TC_i \in (9, 13)$  s.u.t. Because of the need to maintain an adequate quality of the goods supplied (freshness), an additional requirement is that the time between delivery from manufacturer (resources  $R_2-R_5$ ) to customer (resources  $R_{15}$ ,  $R_{16}$ ) not exceed  $Th_{i,(a-b)} \leq 30$  s.u.t. For such a network, what is being sought is an answer to the following question: What route using what local means of transport, guarantees servicing of deliveries between  $B \rightarrow A$  and  $C \rightarrow D$ , at the same time satisfying the constraints given?

The analysis problem being considered involves seeking for the structure of an SCCMP given the structure of an SCCP; this is tantamount to solving problem (Equation (7)). According to the paradigm of regular topology of the structure of an SCCP, to evaluate the behaviour of the ECS from Figure 9(a), it is enough to evaluate the behaviour of its covered representation – refer Figure 9(b). An appropriate instance of problem (Equation (7)) was implemented and solved in the constraint-programming environment OzMozart (CPU Intel Core 2 Duo 3GHz RAM 4 GB). It was assumed that all operation times for local processes were the same at  $t_{ij} = 1$  s.u.t. and the dispatching rules on shared resources were in the form:  $\sigma_1 = (P_1, P_2)$ ,  $\sigma_{26} = (P_2, P_1, P_6, P_4, P_8, P_3)$ ,  $\sigma_{27} = (P_2, P_1, P_5, P_3, P_7, P_4)$ . The first feasible solution was obtained after eight seconds. Figure 10 shows three routes of multimodal processes which satisfy the constraints given:  $mP_1$  and  $mP_2$  (marked in red and blue) for deliveries  $B \rightarrow A$  and  $mP_3$  (marked in purple)  $C \rightarrow D$ . An example



**Figure 8.**  
A food supply  
network with  
recipients A and D  
and suppliers B and C  
marked with circles

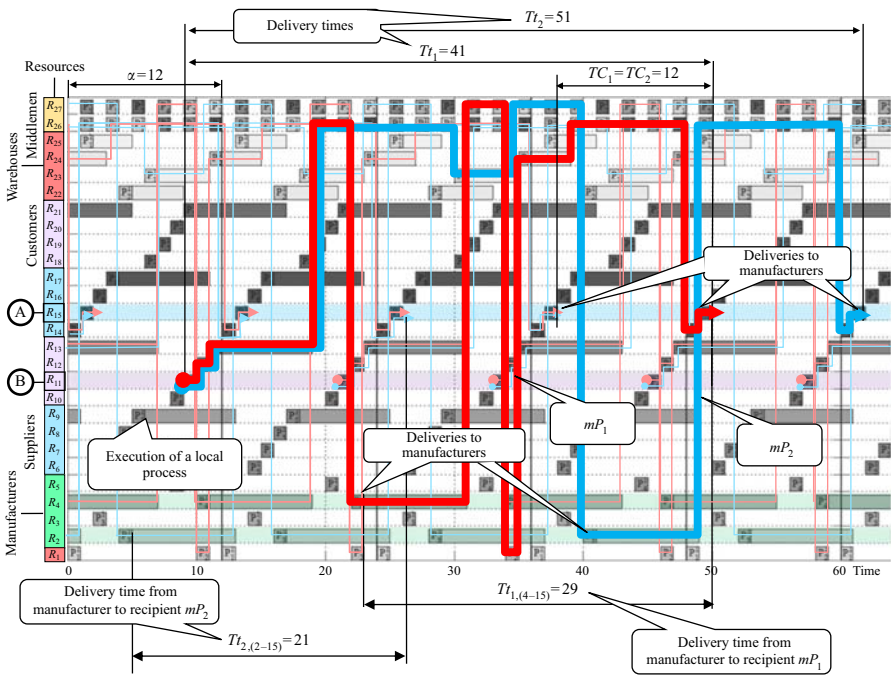
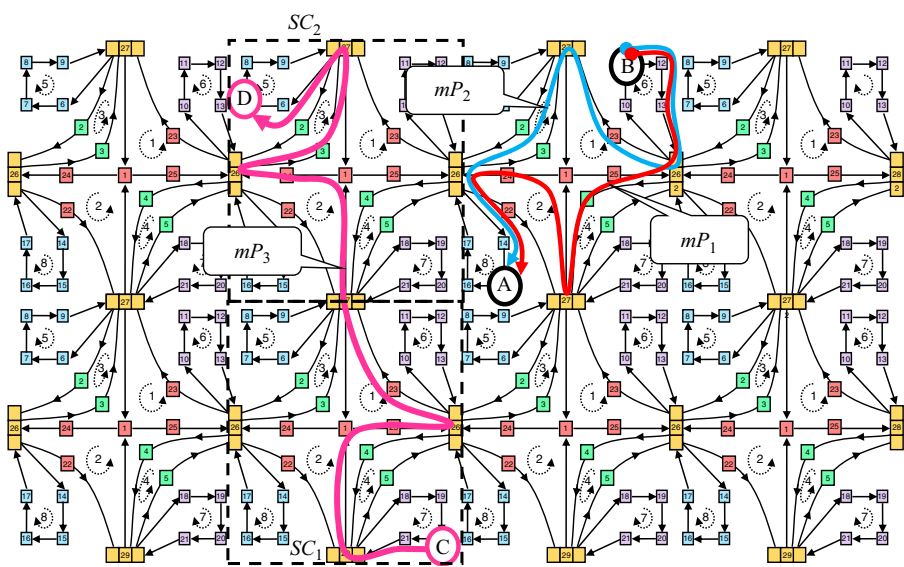


**Notes:** (a) ECS from Figure 7(b); (b) its covered representation

**Figure 9.**  
The covered  
representation of ECS

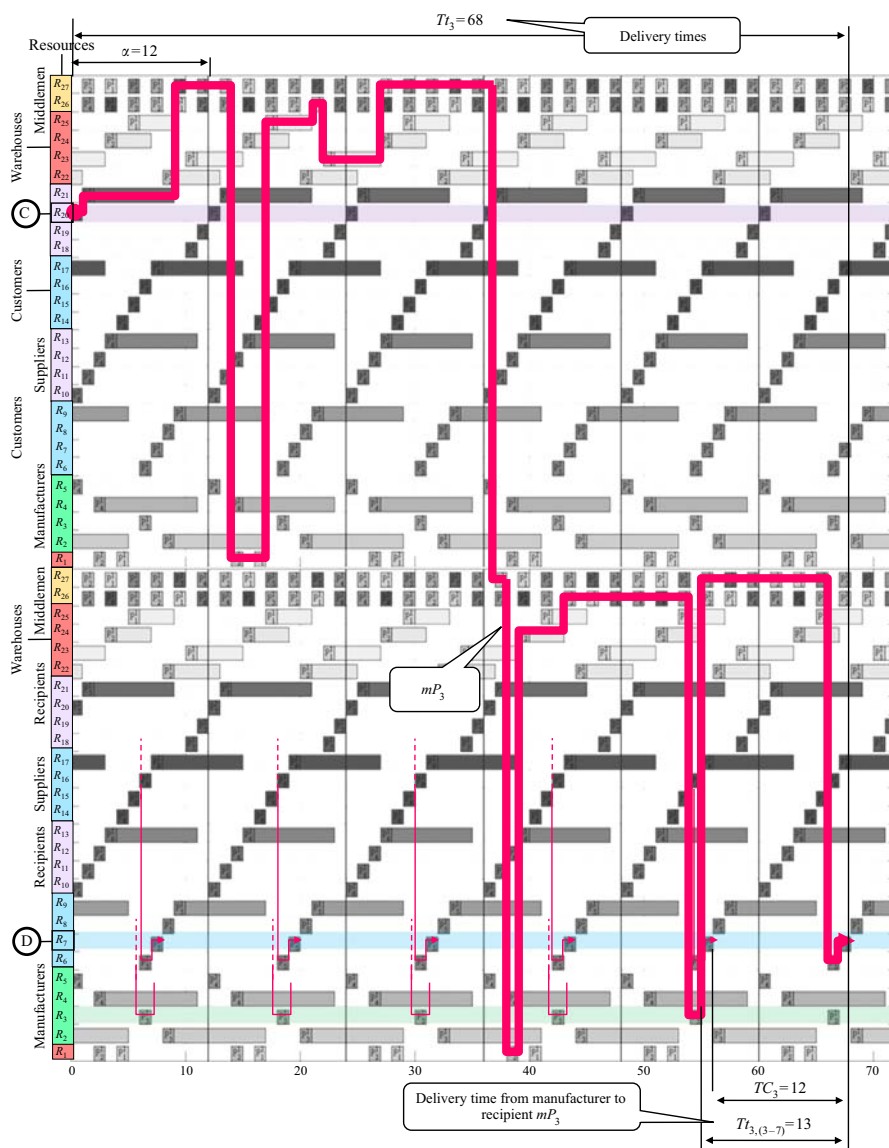
of a process schedule is shown in Figures 11 and 12, and the parameters of these processes are summarised in Table I. Note that for delivery  $B \rightarrow A$ , two alternative routes can be determined. Both of them satisfy the constraints given, but process  $mP_1$  guarantees a shorter delivery time  $T_{t_1} = 41$  s.u.t. (for process  $mP_2$ , this time is  $T_{t_2} = 51$  s.u.t.).

**Figure 10.**  
A goods supply network with  
calculated routes of  
supplier-recipient-type  
multimodal food  
transport processes



**Figure 11.**  
A schedule of  
processes  $mP_1$  and  
 $mP_2$  for the covered  
representation of  
Figure 9(b) by  
applying rules  
 $\sigma_1$ ,  $\sigma_{26}$ ,  $\sigma_{27}$

**Notes:**  $\sigma_1=(P_1, P_2)$ ;  $\sigma_{26}=(P_2, P_1, P_6, P_4, P_8, P_3)$ ;  $\sigma_{27}=(P_2, P_1, P_5, P_3, P_7, P_4)$



**Notes:**  $\sigma_1=(P_1, P_2)$ ;  $\sigma_{26}=(P_2, P_1, P_6, P_4, P_8, P_3)$ ;  $\sigma_{27}=(P_2, P_1, P_5, P_3, P_7, P_4)$

**Figure 12.**  
A schedule of process  $mP_3$  in structures  $SC_1$  and  $SC_2$  in Figure 10 by applying rules  $\sigma_1, \sigma_{26}, \sigma_{27}$

| Period of the SSCP $\alpha$ | Multimodal process | Takt time $TC_i$ | Delivery date $xs_i(k)$     | Delivery time $Tt_i$ | Time between the moment of release of goods by the manufacturer and their reception by the recipient $Tt_{i,(a-b)}$ |
|-----------------------------|--------------------|------------------|-----------------------------|----------------------|---------------------------------------------------------------------------------------------------------------------|
| 12                          | $mP_1$ (red)       | 12               | $2 + 12k, k \in \mathbb{N}$ | 41                   | 29                                                                                                                  |
|                             | $mP_2$ (blue)      | 12               | $2 + 12k, k \in \mathbb{N}$ | 51                   | 21                                                                                                                  |
|                             | $mP_3$ (purple)    | 12               | $12k, k \in \mathbb{N}$     | 68                   | 11                                                                                                                  |

**Table I.**  
Parameters of the multimodal processes of the schedule in Figures 10 and 11

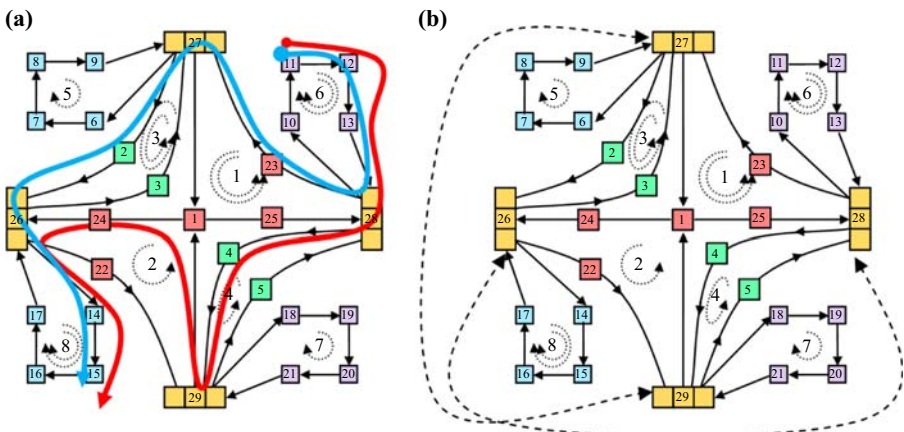
*The synthesis problem.* Consider a set of multimodal routing processes  $mP_1$ ,  $mP_2$  and  $mP_3$  (see Figure 10 and Table I) determined in the given structure of local processes SC in Figure 8. In the solution obtained, process  $mP_2$  has a longer delivery time  $Tt_2 = 51$  s.u.t. compared to  $mP_1$ . In this context, the question arises: Is there an organisation of local processes which guarantees servicing of deliveries along the route of a multimodal process  $mP_2$  (of deliveries  $\rightarrow A$ ) in a time not exceeding 50 s.u.t.?

Seeking to solve this problem that each local process responsible for the transport of goods along the route of process  $mP_2$  was a multi-flow process composed of two flows (e.g. process  $P_1$  consists of flows  $P_1^1$  and  $P_1^2$ ) is assumed. A multi-flow ECS with its covered representation is shown in Figure 13. In the version considered, the number sequence of streams of local processes was in the form  $\Omega = (2, 1, 2, 1, 1, 1, 1, 2)$ .

The problem considered boils down to determining the dispatching rules for shared resources:  $R_1$ ,  $R_{26}$ ,  $R_{27}$ ,  $R_{28}$ ,  $R_{29}$ , and hence to problem (Equation 8) in which resources  $R_{26}$  and  $R_{28}$  as well as  $R_{27}$  and  $R_{29}$  were merged in the covered form of the ECS from Figure 13. Problem (Equation (8)) was implemented and solved in the constraint-programming environment OzMozart. The first feasible solution is obtained after 32 s. The set of priority dispatching rules obtained was in the form:

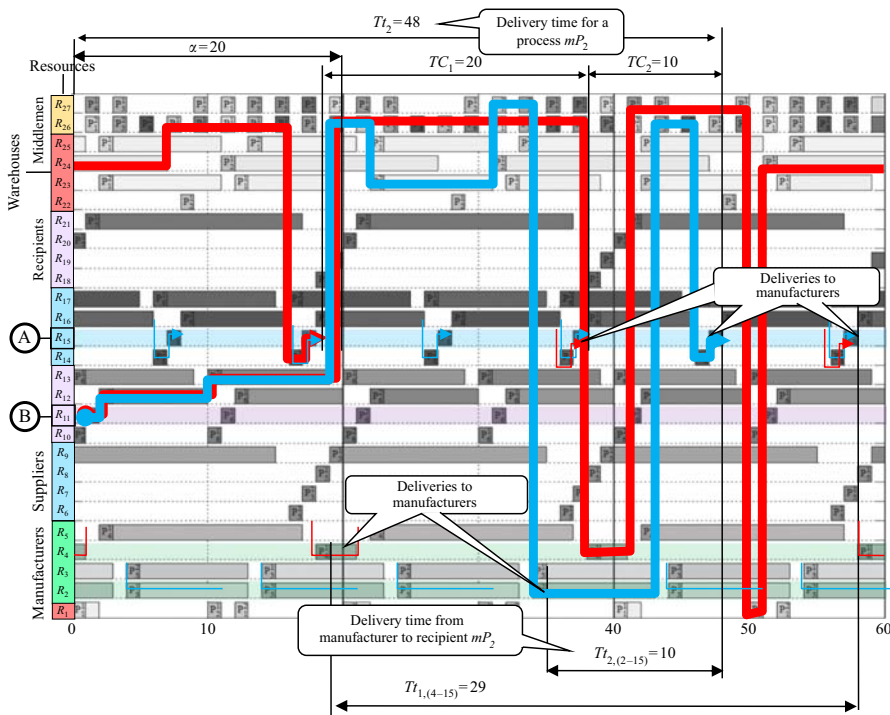
$$\sigma_1 = (P_1^1, P_2^1, P_2^2) \sigma_{26} = \sigma_{28} = (P_2^1, P_6^1, P_1^1, P_3^1, P_8^1, P_4^1, P_6^2, P_1^2, P_3^2, P_8^2),$$
$$\sigma_{27} = \sigma_{29} = (P_2^1, P_1^2, P_3^2, P_5^1, P_7^1, P_1^1, P_4^1, P_3^1).$$

A process schedule determined by such a set of dispatching rules is shown in Figure 14, and the parameters characterizing the processes are summarised in Table II. As it is easy to note, the multiplication of the modes of transport in the local processes running along process route  $mP_2$  resulted in the improvement of delivery parameters. Deliveries were executed at a takt time of 10 s.u.t.; delivery time was 48 s.u.t., and the time of delivery between manufacturer and recipient was 10 s.u.t. It should be noted that execution time of process  $mP_2$  was reduced at the expense of deterioration of the parameters of process  $mP_1$ , see Table II.



**Figure 13.**  
The covered  
representation of  
multi-flow ECS

**Notes:** (a) Multi-flow elementary covering structure for the network of Figure 7(b); (b) its covered structure representation



**Notes:** By applying rules  $\sigma_1=(P_1^1, P_2^1, P_2^2)$ ;  $\sigma_{26}=\sigma_{28}=(P_2^1, P_6^1, P_1^1, P_3^1, P_8^1, P_4^1, P_6^2, P_1^2, P_3^2, P_8^2)$ ;  $\sigma_{27}=\sigma_{29}=(P_2^1, P_1^2, P_3^2, P_5^1, P_7^1, P_1^1, P_4^1, P_3^1)$

**Figure 14.**  
A process schedule  
for the covered  
representation of the  
ECS from Figure 13

| Period of the<br>SCCP $\alpha$ | Process       | Takt<br>time $TC_i$ | Delivery<br>date $xs_i(k)$ | Delivery<br>time $Tt_i$     | Time between the moment of release of<br>goods by the manufacturer and their<br>reception by the recipient $Tt_{i,(a-b)}$ |
|--------------------------------|---------------|---------------------|----------------------------|-----------------------------|---------------------------------------------------------------------------------------------------------------------------|
| 20                             | $mP_1$ (red)  | 20                  | 78                         | $8 + 20k, k \in \mathbb{N}$ | 39                                                                                                                        |
|                                | $mP_2$ (blue) | 10                  | 48                         | $8 + 10k, k \in \mathbb{N}$ | 10                                                                                                                        |

**Table II.**  
Parameters of the  
multimodal processes  
of the schedule  
shown in Figure 13

Delivered solution employs observation that the behaviour of a given network can be predicted based on the behaviour of its ECSs. That means that by solving a small-scale computationally hard problem at low time cost, a large-scale problem associated with a whole network can be solved in online mode. Moreover, because different conflict resolutions of processes executed in a SCCP results in different periods  $\alpha$  of its cyclic steady state, different behaviours of the whole grid-like networks can also be assessed. Consequently, due to proposed methodology assuming an incremental, gradual (from simple to complex) inheritance of both structural and behavioural features of grid-like networks, each SCCP's cyclic steady state can be searched as driven by covered representation of arbitrary extracted ECS.

The majority of problems stated within a framework of grid-like networks can be seen as concerning its schedulability, and especially decidability of cyclic scheduling problems. Therefore, since the behaviour of a given network can be predicted based on its EPNs, hence

the relationships linking whole network's periodicity with possibly extracted, different ECSs play a crucial role. In turn, solution to this problem enables to consider a new approach to DSS design dedicated to online prototyping distributed control procedures aimed at real-life-size routing problems.

## 7. Conclusion

1992

The approach proposed in this paper, which assumes that an intermodal goods supply network has a regular structure composed of a set of cyclic transport processes, allows fast prototyping of deliveries that is very relevant in a food supply chain network. Computer implementation of the declarative model of the problem of supply that assumes a regular network structure of local processes enables a fast online search for routes of supply chains and re-scheduling of local transport processes (related, for instance, to a change in the number of vehicles involved in a given process). At the heart of the presented solutions are two paradigms: scale decomposition of the problem of seeking a cyclic SCCP and layer decomposition of a supply problem into the problems of analysis and synthesis of the SCCP and the SCCMP, respectively. These paradigms make it possible to quickly solve small NP-hard problems and to map (generalise) the results online onto large-scale problems. They also allow separate, phased solving of integrated supply/distribution problems by separating the problems of cyclic scheduling of the SCCP and the problems of routing and scheduling of multimodal processes of the SCCMP. To summarise, computer implementation of declarative models of these problems provides an alternative to solutions that use meta-heuristics and evolutionary methods.

The proposed declarative modelling-based methodology, which seems to be an attractive alternative to other methodologies driven by mathematical programming, computer simulation and/or OR-based meta-heuristics, allows us to formulate the PVRP problems stated either in direct (aimed at analysis) or reverse (aimed at synthesis) ways. So, the main advantage of proposed approach follows from the fact that any of the considered decision variables can be seen as indicators of the system's performance, and none of them require that relevant goal functions are set up.

Coupling both the grid-like-oriented methodology of transportation networks design and proposed declarative-model-driven approach may result in a new generation of DSS enabling online prototyping and refinement of admissible distributed control procedures aimed at real-life-size routing problems. The possibility of fast prototyping of feasible solutions creates promising prospects for further research. One potential direction of such research is to investigate how the proposed approach could be applied to determine the supply portfolios in regular-structure supply networks. It is quite clear that rapid determination of a set of alternative supply portfolios allows one to develop supply/distribution procedures robust to interference caused by failures of the means of transport and/or changes in order size. In particular, it allows fast determination of cyclic robust schedules for deliveries of goods in regular-structure supply networks.

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#### Further reading

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