Discussion.

The Chairman said that the absence of Dr. R. E. Stradling, Chairman of the Division, was due to medical advice.

The Author, in presenting the Paper, said that, whatever other objections to it might be offered, few would quarrel with the opening statement regarding the increasing interest in the subject of indeterminate structures.

With regard to single-storey structures questions of appearance probably accounted to some extent for that interest. For other structures there was very probably a strong desire to bring methods of design more into line with both the actual and the theoretical behaviour of structures, it being realized that nearly all the structures with which engineers had to deal were, in fact, indeterminate, although that might not be recognized in existing methods of design. That desire was probably intensified by the fact that recently the allowances for superimposed loading had been considerably decreased and the working stresses increased so that a building designed to-day contained much less structural material than it would have done twelve years ago. Many felt that those reductions could go still further, particularly if methods of design could be revised to give a more uniform factor of safety.

The increased interest was shown largely by articles and text-books dealing with methods of analysis. Two of those methods had been briefly described in the Paper—the method of moment distribution due to Professor Hardy Cross and a method of arithmetical integration. The former was becoming increasingly familiar and for many structures was comparatively easy to apply. An advantage was that it dealt with bending moments, right from the start, rather than with more abstract quantities of slopes and deflexions.

Methods of analysis, however, were only a small part of the designer's work. Before a structure could be analysed the sizes of its members must be settled and its joints arranged to reproduce the assumptions of analysis. Unfortunately much less published information was available to the designer on those matters. Without adequate knowledge of the extent to which a structure followed the assumptions made, an elaborate analysis might be a source of danger and was, in any case, a waste of the designer's time. Probably the most difficult problems of design were represented by steel and reinforced-concrete framed buildings.

The Paper discussed briefly some of the problems which arose in both types of structure. On the whole he felt that the present-day methods of design for reinforced-concrete structures represented a working compromise between the design of a theoretical frame and a reinforced-concrete structure.

As regards steel-framed structures the position was much less happy. The present methods of design, assuming free ends for beams and small eccentricities on the columns, were very far from representing what actually happened. On the other hand, the methods proposed by the Steel Structures Research Committee were not, in his opinion, workable for the reasons briefly discussed in the Paper.

Mr. Guthlac Wilson observed that the Author's inquiry as to the reason for the increased interest in indeterminate structures resembled the question as to which came first, the hen or the egg. The interest in steel-framed structures might have led to interest in those improved methods of design, or the outstanding advances made by Hardy Cross might have led engineers to feel that they could tackle such structures in applied practice. The Author had not mentioned the other tool with which Hardy Cross had provided them, namely, the column analogy—a useful tool in conjunction with moment distribution and, by itself, a simple means of dealing with single-cell structures. In the case of side sway, to which the Author had called attention, it eliminated the need for separate calculation.

Mr. G. P. Manning said that the use of the word " structures " in the title of the Paper had rather puzzled him. Possibly the Author had had in mind only the one problem of designing frame buildings. With regard to the Author's remark that " an analysis deals with a perfect structure with perfect members obeying rigid laws," various people had given some thought to what happened when a structure had passed its elastic limits and ceased to behave in accordance with elastic laws. A fair amount of work had been done on the ultimate strength of an isolated beam or column, and attempts had been made to work out the ultimate strength of a complete structure, though little had been published on the subject.

The Author had also referred to a widening gap between the theoretical analyst and the practical designer. Mr. Manning had never been able to see any necessity for such a gap. He considered that an author should not publish a method of analysis unless and until he had succeeded in reducing it to such a state that it could be dealt with in the drawing office. If he did not succeed in so reducing it he had failed in the most important and most difficult part of the analysis.

Mr. Manning had at various times inflicted on a long-suffering profession a large mass of theoretical analysis, but he was also a practising engineer. Therefore, being both a theoretical analyst and a practical designer, it would appear that there was a widening gap between himself and himself!

In tracing the history of methods of analysis the Author had not gone far enough. The ancient Egyptians, Greeks, and Romans must have had some method of analysis. Mr. Manning did not suppose that any building regulations were then imposed in the city of Rome. He imagined that designing was mostly done by eye, that was to say, the designers sketched out whatever they intended to build; and from the resulting structures it was quite clear that that was an efficient method. In Mr. Manning's opinion, it was still the best method of design.

Which were positive and negative bending moments? He had found the description in the Paper rather difficult to follow. The Author had used the positive sign for fixed end moments at the right hand end, whereas Mr. Manning thought that the right side of the joint was meant. He suggested that a sketch might be included in the Paper to show the exact meaning.

He believed that the formula for the principal horizontal thrust (p. 7) was true only if the two pins were at the same level. He did not quite see from that formula how the Author dealt with horizontal loading. Reference had been made to "numerous handbooks giving moments for various loadings on most types of frames." Those were extremely useful provided a personal check were made of the moment.

With regard to side sway in buildings, again it was questionable whether the allusion was to structures in general or to a frame building. Mr. Manning agreed that the applied horizontal loads in a frame building were very small as a rule and could be neglected. On the other hand, the same problem arose from shrinkage. As a rule heavy horizontal shrinkage occurred in the first suspended floor, and in water-towers and chimneys side sway was considerable. Both water-towers and chimneys presented certain problems which were statically indeterminate.

The formula on p. 12 was based upon the assumption that the far ends of the members were actually fixed, but allowance could be made for continuity in them by making the last term in the denominator $\frac{1}{2}K_b$ instead of K_b.

With regard to steel frame buildings, Mr. Manning's thoughts turned more to the question of welding and the particular type of welding used, and the statical analysis of the structure took second place to the practical problems.

He agreed with the Author's first conclusion, but not with the second, as he did not think it possible to say that there was any "most useful" method of office analysis. The best method could be decided only having regard to the actual type of the problem—a method which was quite good in one case would be wrong in another—and very largely to the individual's own outlook and possibly office training.

An analysis of Mr. Manning's own methods after many years' experience led to the following result:—

Approximate coefficients	90	per cent.
Straightforward applications of theorem of three moments	5	,,
His own displacement method	2	,,
For long columns, his own *L.U.D.* method	1	,,
His own ready-made influence lines	1	,,
Other methods	1	,,

To that he would add that energy methods, experiments on models, moment distribution, characteristic points, and graphical or semi-graphical methods he did not use at all. That was an entirely personal view, and he did not suggest that it was correct. A friend was very fond of using characteristic points, and had found them most useful; but to Mr. Manning they were of no use at all.

He would again advise Authors never to publish a method until it had been actually tried out in the drawing office. About 20 years ago he was dissatisfied with the method of designing long columns, and worked out an eccentric " diamond " method, which had the support of a set of attractive curves: but in use it never gave him the values he wanted, and he was glad that he had not rushed it into print.

The Paper dealt almost exclusively with framed buildings, but nasty problems of restraint arose in connexion with all watertight structures, whilst there was also the large question of arch design. He had been asked how he invented the various methods which had appeared in the technical press. The answer was that he had not invented them, but that they had been forced upon him by practice. If a problem arose in his drawing office he tried not only to solve it for the structure which he was designing, but also to evolve a more general design—and in course of time that developed. The displacement method, which he generally used for all kinds of frames, originated in 1914 with his first attempt to design an arch, and developed from job to job.

Mr. C. E. Reynolds observed that the main value of the Paper was that it emphasized the gap between the analyst and the designer. It was not possible to over-emphasize the existence of that gap between the designer's calculations and working details. If it were ever to be closed his design might become just a substitution of given data into an appropriate set of formulas; bridging the gap, as opposed to closure, constituted part of the fascination of the structural and civil engineering profession. It was very important to realize that methods of analysis were only instruments by which attempts were made to measure tendencies and perhaps the relative magnitudes of forces and bending moments to which structures were subjected. The wit and experience of the designer translated those indications into concrete construction. Of all the analytical instruments available to the designer the moment distribution method made the greatest appeal. It had a simple theoretical basis which could be grasped by junior as well as senior designers; it was essential that those fundamentals should be understood so that the respects in which a structure varied from the premises made by the analyst would be recognized.

Secondly, the moment distribution method was easy to remember and all results were more speedily obtainable than by the alternative methods. The usual method of presentation of moment distribution principles was by a chart of the nature of that given in *Fig. 1*; but the solution of the problems of continuous beams and certain simple frames could be further simplified by the substitution of a series of formulas for the chart. For example, in the case of a beam continuous over a number of supports—say $RSTUV$, the bending moment at T could be given by :—

$$M_T = \frac{D_{TU}}{2}[2M_{TS} - D_{ST}(M_{SR} - M_{ST})] - \frac{D_{TS}}{2}[2M_{TU} - D_{TU}(M_{UV} - M_{UT})]$$

where M_{TS}, etc., denoted the numerical values of the fixed end bending moments, and D_{TU}, etc., were distribution factors.

That formula was the result of two distributing operations, and to that extent it was an approximation, but the degree of accuracy was within 2 or 3 per cent. of the result furnished by the more precise classical methods, and was therefore sufficient for everyday use. In practice numerous simplifications could be introduced, depending upon the data of the particular problem to be solved, such as symmetrical loading, equal spans and uniform sections throughout the beam, and also various modifications according to whether the beams were fixed or freely supported at the end supports.

In presenting the formula in that form Mr. Reynolds had taken certain liberties with the conventional signs usually employed in the moment distribution method, and had adopted the system whereby a bending moment which gave tension in the top of the beam was considered negative and one which gave tension in the bottom of the beam was considered positive. By direct substitution of the span factors and the numerical values of the fixed bending moments, the sign of the moment over the support was automatically determined.

Professor A. J. Sutton Pippard observed that in his opinion antagonism between analysis and design had been overstated. It was perfectly true that an analyst might not be able to design; he was then an applied mathematician, and he might or might not become an engineer with training. But the designer who could not analyse was much more dangerous. Such a man was between two stools all the time; either he would waste material, which was uneconomical, or he would risk a collapse. Professor Pippard spoke with feeling because he had had to deal with collapses which had occurred because the designers were not analysts.

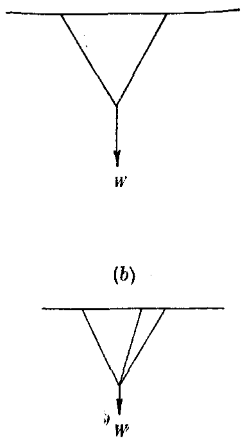
An incredible prejudice appeared to exist among British engineers in regard to statically indeterminate structures. A glance through a volume or two of the publications of British civil engineering bodies would reveal that it was considered not quite " nice " to suggest that a structure could be redundant, and still less so to propose anything in the nature of a mathematical solution. Many British engineers thought that every problem should be capable of solution by a slide-rule, and the answer should be obtainable within 3 minutes.

At the risk of being professorial, he wished to state simply the problem presented by the redundant structure.

The weight in *Figs. 4* was hung from two wires. A schoolboy could calculate the tensions, the only problem being to make them balance the loads. But if another wire were added (*Fig. 4* (*b*)) he did not know the answer although he had set the problem. He could ascribe to that load any value from zero—in which case he went back to the original problem

Figs. 4.

(*a*) (*b*)



—upwards, but now, in addition to making the loads balance the strains had to be compatible. and until the areas of the wires and the material of which they were made were specified it was impossible to obtain the solution. That, in elementary form was the essence of the statically indeterminate problem.

Several classical solutions had been presented. The first was the slope-deflexion method. The second was due to Castigliano. The third and most recent method, that of Hardy Cross, was a system of successive approximation until balance was obtained at the joints. These three methods were fundamental steps in the study of structures. All kinds of special theorems could be used to help the analysis, but they were in the nature of variations on the main theme. He agreed with Mr. Manning that there was no " best method," any more than there was one " best " drawing instrument. Mathematical results should be viewed as tools, and the best one chosen for a particular purpose. Some problems might be more readily solved by the Hardy Cross method and others by direct analysis.

Such arithmetical summation as was mentioned in the Paper was not a method in the sense of the others. Graphical or arithmetical summation could be used for any problem worked out on any particular lines. The equation for the arch which had been mentioned was a direct statement of the strain-energy equation and was not a special method.

Upon one matter Professor Pippard felt rather strongly. The Author had asserted that a great argument against elaborate calculations for reinforced concrete structures lay in the impossibility of assigning accurate values to either the modulus of elasticity (E) or the second moment of area (I); he then mentioned that the Code allowed the choice of three methods in calculating I. Since the second moment of area depended simply upon the geometrical shape of the section, beams of the same dimensions made in any materials would have the same value of I. E was a property of the material determined experimentally. In calculations on beams the terms did not appear separately, but always as a product, EI, which was known as the flexual rigidity. That should be thought of as a single term, and the absurdity of having alternative values for a purely geometrical property then disappeared.

Trouble arose in regard to struts for which the Code gave correcting factors for different values of $\frac{L}{k}$, to allow for buckling, where k denoted the radius of gyration. Since k would have alternative values consistent with the alternative values of I, there was doubt which $\frac{L}{k}$ ratio ought to be taken. The Code gave no light on that and until k was defined the correcting factors were useless.

Mr. W. A. Mitchell observed that in presenting the Paper the Author had obviously hoped to promote discussion. A conflict of opinion existed concerning the methods which might be used. Generally his practice was to use the simplest and quickest method.

The Paper had provided a good impression of the Hardy Cross method. He would have liked to see, in connexion with what was called the integration method, the inclusion of the expression for the moment of inertia, because in so many portal frames inertia varied throughout the length, and it was detrimental to disregard variants of that nature. He would also have liked to see some indication of how a change in temperature could be dealt with in two-pinned arches; and of the Author's opinion concerning solid web portals versus lattice portals. Further it would have been helpful if some guidance had been given on the treatment of fixed arches.

Reference had been made to the possible handicap that codes of practice might impose upon designers generally. Mr. Mitchell hoped that the codes of practice now being formulated would prove a handicap to people whom it was desirable to handicap and a help to others. Revision of loads, more in keeping with actual conditions, revision of stresses, and methods of design could all be included in a code of practice, but the danger was that favourable portions only would be taken by some people and that should be guarded against. Whether it was possible to encourage the use of rigid frame structures by permitting greater freedom to a qualified engineer, so that he would not be controlled entirely by by-laws, was a question which had distinct possibilities; it might involve the issue by the engineer of a certificate to be covered by insurance.

In his opinion the Steel Structures Research Committee's third report on new methods of design was doomed at birth owing to the limitation of stress put upon stanchions. The statement that stanchions as designed to-day were grossly over-stressed, was misleading, for high stressing could occur at the floor-levels, without harm. He hoped that from the Research Committee's work a method would be developed which would enable something practical to be applied to rigid design.

With regard to the development of that form of structure, he hoped that in the post-war world architects would become more "grid-minded," and also that the railway companies would continue to encourage that type of work, as they were a law unto themselves.

Dr. K. Hajnal-Kónyi observed that he agreed that high accuracy was a waste of time and might be dangerous if the designer failed to realize that such accuracy was only a matter of figures. He thought that a little unfairness had been exhibited towards the so-called classical methods in comparison with the method of the so-called arithmetical integration. In cases of a large number of loads it was fully justifiable to replace the system of point loads by a uniformly distributed system of loading.

If, in the second example given by the Author, the sum of the loads were:—

$$W = 2 \times 3{\cdot}41 = 6{\cdot}82 \text{ tons},$$

then
$$M_c = \frac{6{\cdot}82 \times 48}{8} = 40{\cdot}92 \text{ foot-tons},$$

instead of the value of 40·6 foot-tons given in the Paper.

The calculations for the horizontal thrust in a two-pinned portal (*Fig. 3*) were the following (see *Fig. 5*):—

$$M_b = 3{\cdot}41 \times 0{\cdot}75 = 2{\cdot}56 \text{ foot-tons}$$

$$\int y^2 ds = \frac{11^3}{3} + \frac{25}{3}(11^2 + 11 \times 20{\cdot}5 + 20{\cdot}5^2) = 444 + 6{,}390 = 6{,}834$$

$$\int M_F y ds = \frac{11^2}{3} \times 2{\cdot}56 + \frac{25 \times 2{\cdot}56}{6}(2\times 11+20{\cdot}5) + \frac{25\times 40{\cdot}92}{6}(20{\cdot}5+3 \times 15{\cdot}75)$$

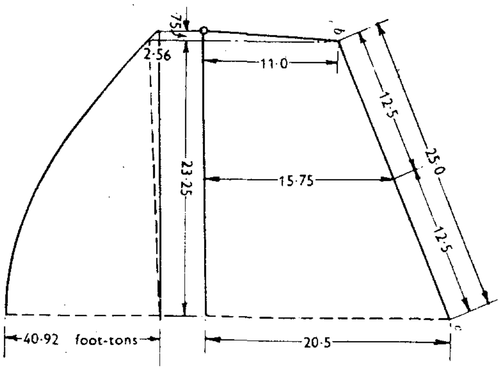
$$= 103 + 454 + 11{,}640 = 12{,}197$$

$$H = \frac{12{,}197}{6{,}834} = 1{\cdot}783 \text{ ton,*}$$

which accorded very well with the Author's value of $H = 1{\cdot}772$ ton.

The purpose of that demonstration was to show that in cases where the moment of inertia was constant over the whole frame there was no justification for using arithmetical summation, which was especially tedious since it necessitated the calculation of a large number of bending

Fig. 5.



moments. The method was necessary only if the moment of inertia varied throughout the whole frame, and even in such cases it was often possible to carry out the integration with an assumed average value.

In the first example, demonstrating the Hardy Cross method, the Author's assumption that no side sway occurred was correct; but it was rather dangerous to suggest that there were very few actual structures where side sway did not occur. In some structures side sway was very serious, and to neglect it gave completely erroneous results.

It was perhaps not quite right to condemn the classical methods as unsuitable for office practice: in fact they were mostly very suitable for office practice. The Hardy Cross method had advantages for certain systems, such as multi-storey frames with a large number of spans, but he would hesitate to accept Conclusion (2). He agreed with Mr. Manning and Professor Pippard that there was no method which was most suitable

* That was the complete calculation, comparable with Table II, p. 9, *ante*.

in general. Every case had to be judged on its merits, and for different cases different methods might be most convenient.

The Appendix was very useful, but for the purpose of simplifying the calculations he wished to suggest that the ratios $\frac{a}{L}, \frac{b}{L}$, etc., should be introduced throughout. As the formulas stood, it was necessary to work in feet and inches, whereas with the suggested method the calculation would be carried out in figures smaller than unity and independent of any units of measurement. The calculation was simpler and less likely to produce errors.

He did not understand the Author's statement that, "Actually it is extremely difficult for the designer to compare the relative costs of two structures. This is particularly the case where steel structures are concerned and a system of price control is in force."

The difficulty of office work could be overcome by the publication of reliable formulas, but such formulas should be used only by people who were able to develop them themselves.

*** **Dr. P. W. Abeles** observed that, in order to clarify the Author's question, why increasing attention was being devoted to indeterminate structures and rigid frames, he would refer to two statements, with which he fully agreed, taken from the introduction to "Analysis of Rigid Frames," by A. Amirikian [1]: (i) "With the increased use of welding, as now extensively utilized in a large variety of steel framings, few structures, built of steel or reinforced concrete, could justly be classified outside the scope of rigid framing." (ii) "A rigidly connected framing constitutes an elastic unity. For a satisfactory design, we must have a clear concept of its elastic behaviour under loading."

In the preface to that text book, Rear-Admiral B. Moreell, of the U.S. Navy, had stated: "In this national emergency, when conservation of all structural materials is a paramount necessity, the importance of modern [2] methods of analysis as a means of realizing appreciable savings in material, cannot be over-emphasized." Dr. Abeles considered that those three statements met the whole issue. He agreed with the views of several speakers in the discussion that there should be no gap between the designer and the analyst. The Author's conclusion No. 2 did not appear to be conclusive, as had been pointed out by Mr. Manning and Professor Sutton Pippard. Dr. Abeles agreed that for office design simple methods were

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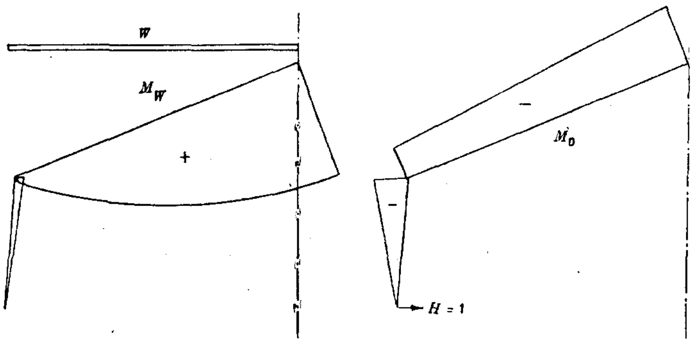
*** This and the following communications were submitted in writing. SEC. INST. C.E.

[1] United States Government Printing Office, Washington, 1942 (396 pages), price \$1·0.

[2] The text book relates to an "Application of the Slope Deflection," which is called "modern", as against "classical" by the Author.

needed, requiring no other means than the slide rule and avoiding complicated theorems or the derivation of formulas. However, simplicity alone was not sufficient; brevity and lucidity were also required, with the possibility of simple checking of the results. The method of arithmetical integration, advocated by the Author as most suitable for office work besides the Hardy Cross method, although simple, appeared to lack special shortness. The replacement of the multiplication of whole areas such as rectangles, triangles, trapezia, and parabolas, by an arithmetical integration of parts, did not seem to constitute a simplification. If, in the example, the point loads were replaced by a uniformly distributed load, as suggested by Dr. Hajnal-Kónyi, the method shown by him—apparently related to the strain-energy method with reference to formulas in text books—could be used, without a knowledge of any particular formulas. It was sufficient

Fig. 6.



to know only the basic formula for a deformation (1), considering also that the Author assumed knowledge of the formula for H,

$$d_{xy} = \int \frac{M_x . M_y}{E.I} ds \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

resulting in the special case in

$$H = -\frac{d_{wo}}{d_{oo}} = -\frac{\int M_w . M_o . ds}{\int M_o^2 . ds} \quad . \quad . \quad . \quad (2)$$

M_w and M_o denoted the bending moments due to the load w and due to $H = 1$ respectively, as shown in *Fig. 6*; the same values—as sums of products—were obtained, as shown by Dr. Hajnal-Kónyi.

With regard to side sway, horizontal loading was often of appreciable influence. A considerable displacement of the joints occurred also with unsymmetrical structures, if they were vertically loaded. The moment distribution method—most suitable for multiple frames—required, however, a rather lengthy calculation for the side sway. Dr. Abeles wished to draw attention to the four-moment theorem as a suitable method for office design. That theorem was first derived by the Austrian engineer, Dr. F. Bleich in 1918 [1] on a general line, suitable for any structure. Dr. Abeles had published a simplification of that method for symmetrical frames, suitable for office work, in 1924,[2] and further developments in 1929 [3] and 1942.[4] That method appeared to combine for the analysis of many frame structures both simplicity and brevity, allowing the results to be checked by special equations. The equations contained mostly finally only from two to four unknown quantities and could be solved by slide rule. It was even possible to carry out the analysis mechanically, when general rules regarding the setting up of the equations were taken into account. Any problem such as influence of temperature or displacement of supports, could easily be dealt with, since all that was already contained in the derived formulas of the theorem, which also allowed the deformations of the indeterminate structures to be calculated and influence lines for the unknown quantities to be found. He had solved many rather complicated systems by that method during the past 20 years, and had successfully taught the theorem to a number of structural engineers and students in a relatively short time.

For continuous beams, such as that shown in *Fig. 1*, the use of the three-moment theorem was likewise very suitable for office design. The three equations for that three times statically indeterminate system, in simplified general form were :—

$$2(k_1 + 1)M_B + M_C = k_1.N_{BA} + N_{BC} = N_1 \quad . \quad (3a)$$

$$M_B + 2(l + k_2)M_C + k_2M_D = N_{CB} + k_2N_{CD} = N_2 \quad . \quad . \quad (3b)$$

$$M_C + 2M_D = N_{DC} = N_3 \quad . \quad . \quad . \quad . \quad . \quad (3c)$$

In these equations N denoted the respective loading terms and k the ratios of the stiffness factors : for example N_{BC} and N_{CB} represented the influence of the loading upon the member BC to the points B and C respectively. In the 1942 publication [4] individual values for N were given in a Table, wherein twenty-eight different loading cases were considered. In *Fig. 1* the following values had to be taken into account :—

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[1] Dr. F. Bleich, " The Calculation of Statically Indeterminate Structures by the Four Moment Theorem ", Springer, Berlin, 1918 (2nd edition, 1925).

[2] Paul Abeles, " The Calculation of Symmetrical Frame Structures by the F.M.T.", *Beton und Eisen*, 1924, Nos. 9 and 10.

[3] Paul Abeles, " The Deflection of Statically Indeterminate Frame Structures," *Beton und Eisen*, 1929, No. 19.

[4] Paul Abeles, " The Four Moment Theorem, Simplification in Calculation of Frame Structures," *Concrete & Constr. Eng.*, April, May, and June, 1942.

$$k_1 = \frac{K_{BC}}{K_{AB}} = \frac{0{\cdot}25}{0{\cdot}20} = 1{\cdot}25,\ k_2 = \frac{K_{BC}}{K_{CD}} = \frac{0{\cdot}25}{0{\cdot}5} = 0{\cdot}5,\ N_{BA} = -\ 1{,}200\frac{20^2}{4}$$

$$= -\ 120{,}000,\ N_{BC} = N_{CB} = -\ 2{,}000\frac{12^2}{4} = -\ 72{,}000, \text{ and } N_{CD}$$

$$= N_{DC} = -\ 2{,}400 \times 9 \times \tfrac{2}{3} = -\ 14{,}400 \text{ ft.-lb.}$$

The equations could be solved by means of determinants, in which case each part of the equation (3*c*) might be multipled by k_2 in order to obtain a symmetrical denominator-determinant of the form

$$\begin{vmatrix} 4{\cdot}5 & 1 & 0 \\ 1 & 3 & 0{\cdot}5 \\ 0 & 0{\cdot}5 & 1 \end{vmatrix},$$

resulting in $4{\cdot}5 \times (3 - 0{\cdot}5^2) - 1 = 11{\cdot}375$. Also the individual numerator-determinants could easily be computed as shown in the following equations (with $N_3' = k_2 N_3 = 0{\cdot}5 N_{DC}$) :—

$$M_B = \frac{1}{11{\cdot}375}[(3 - 0{\cdot}5^2)N_1 - N_2 + 0{\cdot}5N_3']$$

$$M_C = \frac{1}{11{\cdot}375}[-\ N_1 + 4{\cdot}5N_2 - 4{\cdot}5 \times 0{\cdot}5N_3']$$

$$M_D = \frac{1}{11{\cdot}375}[0{\cdot}5N_1 - 4{\cdot}5 \times 0{\cdot}5N_2 + (4{\cdot}5 \times 3 - 1)N_3']$$

Another way was to transform equations (3*a*) and (3*c*) to eliminate M_B and M_D respectively by inserting their values in equation (3*b*) (substitution-method); thus the results $M_B = -\ 47{,}020$; $M_C = -\ 10{,}400$; and $M_D = -\ 2{,}000$ ft.-lb.; were obtained, which agreed very well with the published results. However, it was mostly necessary to consider the influence of dead weight and various live loads separately in order to obtain the maximum bending moment for each place. In that case it was possible to develop the equations (3*a*), (3*b*), and (3*c*) on a general line for all the loading terms, with the following results :—

$$M_B = +\ 0{\cdot}302N_{BA} + 0{\cdot}242N_{BC} - 0{\cdot}088N_{CB} - 0{\cdot}044N_{CD} + 0{\cdot}022N_{DC};$$
$$M_C = -\ 0{\cdot}110N_{BA} - 0{\cdot}088N_{BC} + 0{\cdot}396N_{CB} + 0{\cdot}198N_{CD} - 0{\cdot}099N_{DC};$$
$$M_D = +\ 0{\cdot}055N_{BA} + 0{\cdot}044N_{BC} - 0{\cdot}198N_{CB} - 0{\cdot}099N_{CD} + 0{\cdot}550N_{DC};$$

that allowed unsymmetrical loadings in the individual bays to be also taken into account.

The new theories of the plastic behaviour of materials [1] were of special importance for steel structures owing to the great superiority of rigid

[1] Enrico Voltera, "Results of Experiments on Metallic Beams Bent Beyond the Elastic Limit," Journal Inst. C.E., vol. 20 (1942–43), p. 1 (March 1943).

structures in view of the possibility of a stress re-distribution in the event of extraordinary loadings, thus ensuring increased carrying capacity.

Professor J. F. Baker observed that the Author might be interested to know that at least one building, the new Zoological Laboratory for the University of Bristol, had been designed in accordance with the Recommendations of the Steel Structures Research Committee. That method of design certainly took more time than was required by the normal method—what could take less ?—but the real reason why it had not proved acceptable was that, in common with most other more exact methods as they were framed at present, it gave little, if any, advantage in the form of economy. The Steel Structures Research Committee's Recommendations were, in fact, damned by the quite arbitrary choice of a load-factor of 2 on an assumed yield stress of 18 tons per square inch in the stanchions when the greatest possible stresses, bending and direct, due to the worst possible arrangement of loads, were acting. Under such an improbable arrangement of loads and rigidity of the beam-to-stanchion connexions there was good reason to believe that the total stress in many stanchion lengths of buildings existing to-day would be of the order of 13 tons per square inch. Therefore, if a load-factor of 1·5 instead of 2 had been chosen, the Committee's method would have resulted in considerable economy of material whilst giving an overall factor of safety equal to that provided by the normal design method.

Professor Baker was not sanguine about the Author's suggestion of reopening the whole question and hammering out a reasonably simple method, " more in accordance with the observed behaviour of steel structures." The only step that could be taken in that direction had been taken, as the Author had pointed out, on page 9 of the Report of the Joint Committee of the Institutions of Civil and Structural Engineers. Professor Baker considered that the Steel Structures Research Committee had gone as far as it could in simplifying its method. If the Committee had gone farther the method would have ceased to be rational ; it would not, in fact, have been in accordance with the observed behaviour of steel structures. But the position might not be hopeless even though, as Professor Baker suspected, nothing more complicated than the " normal " method would be acceptable—although it was difficult to understand why that should be so, when the steelwork designer individually was every bit as able and enterprising as other engineers. A frankly empirical method might be produced, identical with the " normal ", assuming pin-jointed beams applying reactions to the stanchions at some fictitious eccentricity, with stanchion-lengths designed as pin-ended but with some artificial " effective " length. The working stresses in beams and stanchions, the magnitudes of the eccentricities, and the " effective " lengths to be assumed, would be fixed by comparison with the study of a number of careful designs made by the available rational method, so that, whilst the empirical method did not pretend to represent the behaviour of the structure, it would result in structures being produced, in future, in reasonably close agreement with those which the more complicated rational method would produce.

The present time was auspicious for carrying out that work economically. Doubtless numbers of blocks of offices and flats were now under consideration for erection after the war. If the Ministry of Works would encourage the use of rational methods in the design of the steelwork for those buildings, economies could be effected and sufficient data would become available for the development of the empirical method.

Professor Baker agreed that the widening gap between the scientific investigator and the practical designer would have to be bridged before further progress in structural design was possible. The research worker alone would never be able to produce satisfactory design methods. His results should be used by the designer as they became available so that with close collaboration a satisfactory method could be evolved. The Codes which governed most building were so restrictive that little good could come of such collaboration at the present time.

It was quite possible, moreover, that designers had been following the wrong line in their preoccupation with stresses in the elastic range, which made analysis so complicated. Already for certain structures the failing load could be calculated much more easily and accurately than the load to produce a given permissible elastic stress. Given a simple specification, setting out that the structure should be designed to carry so many times the working load without collapse and with certain limits of deflexion under the working load, the competent man would be able, immediately, to pass on to the community some of the benefits made possible by recent advances in the theory of structures and, in the course of time, data would be amassed that would enable improved simple rules for every class of structure to be drawn up for the use of those less competent.

Mr. T. C. Grisenthwaite observed that the Author had very rightly emphasized the divergence between a theoretical and abstract analysis and the practical problems of design. He had apologized for the need for such structures and had expressed a degree of surprise at their increasing adoption. The arguments adduced for greater rigidity, lessened deflexion and stiffness were substantial. In addition, however, there was a likely decrease in constructional depth at the centre of a frame or continuous girder which was often of considerable importance.

Another important advantage of the indeterminate structure was its capability of transferring a load from a severely stressed section to another part which had not yet been so highly stressed.

Those who had followed the Continental press before the war, and particularly the proceedings of the International Association of Bridge and Structural Engineers, could not have failed to be struck by the attention given to that phase of plastic yield. Attention should also be drawn to the contributions of Dr. Bleich, Dr. Maier-Liebnitz, Dr. Melan, and others who had devoted much study to the problem of plastic yield, the results of which were based on both theory and experiments. Dr. Bleich had explained that as follows :—

> "If one part of a girder in which the maximum stress has been reached fails owing to the occurrence of permanent deformation, then a part of the girder which has, up to that time, not been completely utilized will be loaded more heavily owing to the new distribution of stresses."

Figs. 7 illustrated that theory as applied to a two-span continuous beam with a load applied centrally to one span and no load on the other. Assuming a yield stress of 16 tons per square inch on the section of the beam given, then a load of 9·14 tons would cause a bending moment of 222 inch-tons under the load and one of 103 inch-tons over the centre supports. Assuming plastic yield took place at 18·6 tons per square inch, then the bending moment in the span was increased to 259 inch-tons, whilst the moment over the support was still increased only to 120 inch-tons.

If the load were further increased, then owing to plastic yield having taken place under the load, the moment at that point could not be exceeded and it would act as a virtual hinge. The load could then be increased to nearly 13 tons before the moment over the support reached 259 inch-tons and plastic yield occurred.

That was a very elementary example, but it explained to some extent the preference shown by Continental engineers for the indeterminate structure. It would be realized that the figures given applied only to a beam of constant section throughout.

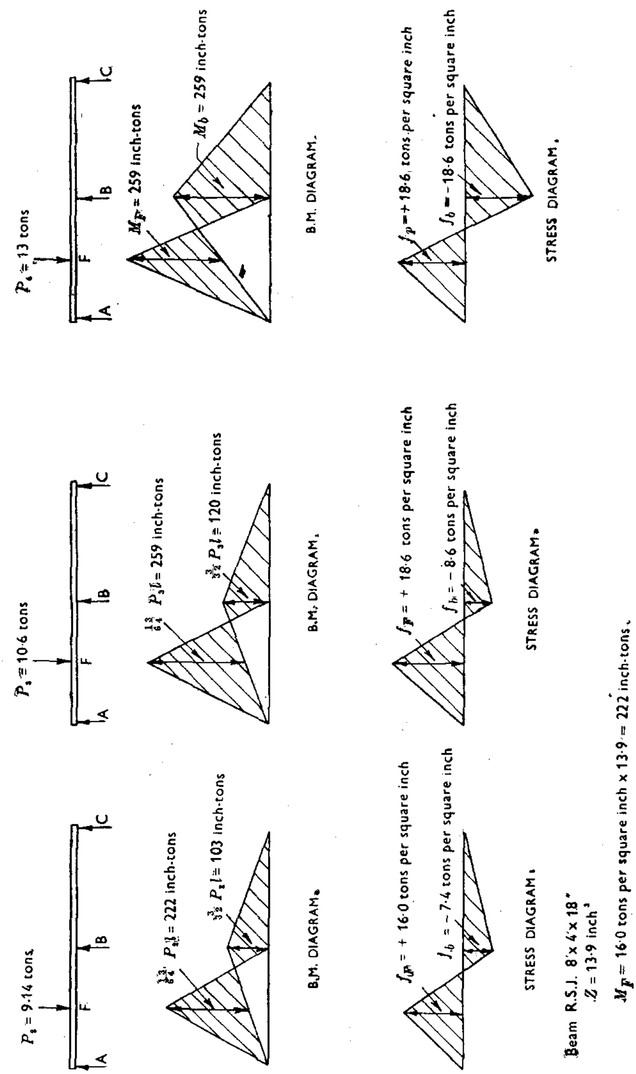
One objection which was often raised against a continuous beam was the possibility of severe overstress due to settlement of supports.

Further investigations into the plastic theory also showed that such settlements did not produce the severe stresses calculated by the usual methods.

Mr. H. V. Hill observed that, in his view the so-called gap between the theoretical analyst and the practical designer was not so wide as the Author believed. So long as the designer treated his mathematical knowledge as a tool (as had been suggested by Professor Pippard) there should be no danger of employing highly theoretical calculations which bore no relationship to the behaviour of the actual structure. But there were pitfalls for the unwary, one of which would appear to be that of assuming all continuous beams in a structure to be on rigid supports. Frequently that was not the case. For example, secondary beams passing over main beams could only be regarded as resting on rigid supports when a check had been made on the deflexions in the main beams to determine whether or not they could be neglected. Another error was to assume connexions to be 100 per cent. rigid when there was a possibility of angular change at the joint intersection point.

In the future the designer of reinforced concrete or of welded steelwork would require a wider knowledge of the theory of structures than had

Figs. 7.



Load of 9·14 tons causing yield stress of 16·0 tons per square inch under load.

Load of 10·6 tons causing plastic yield at F.

Load of 13·0 tons causing plastic yield at B and failure of beam.

previously been the case. The former method of construction had already taken its place in the building industry, whilst the latter (in spite of Mr. Manning's comment) was due for a rapid expansion after the war. The most urgent need in order to cope with the design of those two classes of construction was for methods of rapidly pre-determining the approximate relative stiffnesses of members in a redundant framework. At present redundant structures could be designed only by trial and error, but for rapid calculation that rather tedious process would eventually have to be eliminated by a simpler and more direct method. Moment distribution might be regarded as the first step towards that end.

Mr. Hill agreed with the Author as to the difficulty of comparing the costs of steel structures. Experience, however, both in Great Britain and, prior to the war, abroad, had proved that by adopting structures with rigid joints considerable economies in weight of steel could be effected. Belgian Engineers had shown that to be true for both riveted and welded steelwork; and also that the saving in weight of steel by employing welding was much greater than that obtained by riveting.

Mr. R. R. McIlmoyle observed that the Paper was somewhat incomplete, inasmuch as it was confined almost entirely to buildings.

The Author seemed to have had some difficulty in accounting for the increasing popularity of indeterminate structures and rigid frames, but there could be little doubt that the almost universal adoption of those types in the United States and the increasing interest in Great Britain were due entirely to their economy in both first and ultimate cost. The rigid frame, either in steel or concrete, was the only type of structure which offered a reasonable solution to many bridge problems. The reduced headroom obtained was reflected in the reduction in cost of the embankment and the height of the wing walls, to which should be added the very large saving in the quantity of material required for the abutment.

It was agreed that with that type of design the reduction of moment in the centre was accompanied by increase elsewhere, generally at the supports, but that enabled the structure to fit snugly to the clearance lines and actual experience had shown that such structures were, in fact, much more rigid (they approximated to the arch type) than those of more orthodox design.

A definite economy was evident in favour of the rigid structure when carried out in concrete, but it was not so apparent in the case of steel structures, owing to the unfortunate system of price control mentioned by the Author.

For buildings, in addition to the advantages indicated in the Paper, an undoubted saving was effected owing to the fact that the total height of the structure was reduced considerably, affecting the wind stresses and also enabling a considerable saving to be made when the building had to be heated. That was particularly apparent in the large span structures; with the more orthodox designs the roof trusses required to cover those areas had a much greater height, or depth, than the alternative rigid frame design. The solid web rigid frame was also cheaper to maintain than the usual truss.

The Author seemed to be unduly prejudiced against the slope deflexion method of analysis. Mr. McIlmoyle had always found that to be much the most flexible method of the many available, and whilst he agreed that it entailed the solution of simultaneous equations involving a difference of two nearly equal quantities, he had never had any trouble in easily obtaining accurate results. In one case where thirty-two unknowns were involved, the whole of the work of setting up the equations, solving them, checking and finding the corresponding moments, was performed on one sheet of paper 30 inches by 20 inches. The calculation, however, was not carried out with a slide rule, but with a calculating machine, and that method was essential for the maximum to be obtained from slope deflexion analysis. No engineer's office could be considered complete unless such a machine were available for use by the designers.

He was familiar with the Hardy Cross method, and agreed that it could be quickly and easily applied. He had, however, confined its use to finding approximate moments in tentative designs, and when approximate outlines of the structures had been obtained he had invariably found the final moments by means of the slope deflexion method.

He agreed that in most cases side sway could be neglected. The difference in final moments when that was done, where the loading was unbalanced or due to earth pressure, was so small as to justify the additional work only in cases where the structure was of considerable importance or subject to moving point loads.

He was unable to follow the Author's remarks with regard to the difficulty arising from lack of knowledge of accurate values of E and I. In most of the work it would be found that where E occurred in the primary formula it cancelled out, except in such cases as rib shortening and temperature. In most cases what was required was not a definite value of I, but some relative measure of the stiffness of the proposed sections, and for that purpose an assumption that I was proportional to d^3 or $\sqrt{d^5}$ (the latter value had some support from tests) was very easily applied and yielded satisfactory results. That, however, did not solve the difficulty arising from T-beams.

The Author had deplored the widening gap between the theoretical analyst and the practical designer, but suggested an increase in the number of handbooks giving moments for most types of frames. Those opinions appeared to be antagonistic.

The good designer considered his structure as a whole; it was alive and he knew exactly how it would respond to any change in conditions. That sense, feeling, or awareness, could be attained only if the designer had made a full methodical analysis of the structure by one of the so-called classical methods; the results then had meaning and he was in a position to produce the most appropriate sections for the members. On the other hand, a design arrived at by the mechanical substitution of numerical values in formulas taken from a handbook could never give a structure which embodied all the qualities comprised or inferred in the expression "good design."

Mr. E. S. Needham observed that the Author had used (p. 5) the convention of the positive sign when the fixed end moment tended to turn the joint in a clockwise direction. A much more logical conception of the fixed end moment was one that prevented rotation. In the mind of the ordinary engineer that which tended to rotate the joint was the exterior loading, and the resulting signs were just the opposite to those given in *Fig. 1.*

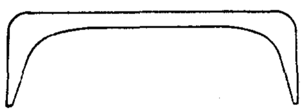
For example, considering the middle span of the figure carrying 2,000 lb. per foot run, that load tended to turn the left hand end (joint B) in a clockwise direction, and therefore the moment was positive, not negative as shown. Similarly at the right hand end (joint C) rotation tended to be

Figs. 8.

(*a*).

FIXED END MOMENT, SETTLEMENT OF SUPPORT AT B.

(*b*).



anti-clockwise, with the negative sign. Further examples were shown in *Fig. 8*(*a*).

Therefore in place of the academic convention based upon the direction of the resisting moment, Mr. Needham preferred the simpler convention based upon the direction of external loads, using the positive sign for moment at the end when the forces acting on the member tended to rotate the joint in a clockwise direction, and vice versa. In his view, that convention was much easier to visualize in a mechanical sense, and therefore the possibility of error in signs was less and confusion about joint rotation in indeterminate structures disappeared.

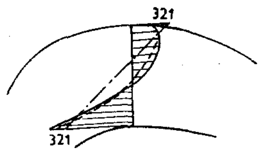
He endorsed the Author's tribute to Professor Hardy Cross for the moment distribution method of analysis of indeterminate frames. Professor Cross's Paper occupied only ten pages, whilst the written discussion occupied 145 pages. An enormous contribution to the solution of indeterminate structures had since appeared in the Transactions of the American Society of Civil Engineers, but the Cross method remained the basic tool for a rapid solution when it applied. Unfortunately it did not apply in the simple case presented in *Fig. 3*, of a two-pinned portal frame for a sloping roof, because of the change in the alignment of the roof beam between supports.

In the simple form presented by the Author, the method was further limited to frames whose members were of constant moment of inertia between joints. For tapered members or members of varying section the stiffness should be calculated from the basic principles of slope deflexion, or obtained from tables [1] or graphs. The carry-over factors were also variable, and unless one were familiar with the graphs and charts on the subject it was better to use the classic method which the Author had used for the two-pinned portal (Table II) for determining the value of H.

Computations for determining the dead load H for portal frame structures could be simplified by use of the " substitute I-curve " method of Weiskopf and Pickworth.[2]

The portal frame shown in *Fig. 8(b)* was an example where H was determined by the classic method, rather than by the Cross method, in view

Fig. 9.



of the complication involved in arriving at the stiffness and carry-over factors for the tapered members.

In rigid frames of the portal type special care should be taken in the design of the knee joint. The neutral axis or point of zero stress was not at the centre of gravity of the section, but was somewhere near the third point toward the inner flange. Mr. Needham knew of no rational method of calculating the concentrated stresses which accumulated at the inner flange, but had discussed the problem [3] using the following example, which was now repeated as a warning. In *Fig. 9*, lines 1–1 and 2–2 represented the distribution of normal stress calculated by the ordinary theory of flexure and by the formula for curved beams. The actual stress obtained by full-sized test [4] was much greater, and the distribution was probably

[1] Walter Ruppel, " Tables for Use in Designing Tapered Beams," Trans. Amer. Soc. Civ. Engrs., Vol. 90 (1927), p. 167.

[2] " Tapered Structural Members—An Analytical Treatment," Trans. Amer. Soc. Civ. Engrs., Vol. 102 (1937), p. 1.

[3] " Design of Knee Joints for Portal Frames," by E. S. Needham, *Welding Industry*, Vol. VIII, No. 6, July 1940, p. 175.

[4] " The Testing of Two Portal Frame Girders," by Leeming and Radshaw, *Structural Engineer*, Vol. XVII, No. 2, Feb. 1939.

as indicated by the shaded area bounded by line 3–3. There was also a cross-bending in the flange in a curved knee joint which further increased the concentration of stress.

The Vierendeel formula might not belong to the routine of office practice, but it was a modern computation method for the design of the open-web type of statically indeterminate structure, as developed by Professor Vierendeel for the design of bridge girders of the type which bore his name. A clear exposition of that formula was published in 1937 [1] in a Paper dealing also with the design of bents with sloping legs. Mr. Needham had found that formula useful in the design of the bent illustrated in *Fig. 10*, which was subject to a horizontal wind load in addition to vertical dead load.

He agreed with the Author's distinction between design and analysis, and had made use of models only as a check on design. Analysis by the

Fig. 10.

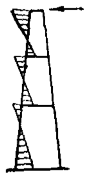
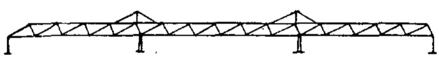


Fig. 11.



use of models required a technique which was not easy to acquire; moreover the time and expense involved with the model were unwarranted except for structures of unusual type or of outstanding importance. For the example illustrated in *Fig. 11* [2] a complete model in celluloid was made in order to check the four reactions by the indirect method described by Miss Chitty: that method made use of the fact that every deflexion curve was an influence line, the ordinates to which in that case determined the four reactions. Given the reactions the stresses in the girder were statically determinate.

Mr. Eric Shepley observed that the whole attention of a reinforced-

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[1] "Analysis of Vierendeel Trusses," by Dana Young, Trans. Amer. Soc. Civ. Engrs., Vol. 102 (1937), p. 869.

[2] E. S. Needham, "The Design and Construction of a Large All-Welded Factory", Trans. Inst. of Welding, Vol. 1, No. 3, July 1938.

concrete design office had always been directed to the continuous structure and, in regard to structural steelwork, the increase arising from the contemplation of all-welded construction was negligible.

If the Author were confusing the new interest with that given to the theory of indeterminate structures, Mr. Shepley agreed that a study of the current literature showed a remarkable growth of attention to methods of analysis of continuous structures. He attributed that directly to the belated introduction into Great Britain of Professor Cross's method of moment distribution. Prior to Mr. Shepley's own work [1] on the subject in 1939–1940, he could trace only four British writers on the subject, namely, Dr. Coleman (1932); L. N. Prismall, Assoc. M. Inst. C.E. (1934); Professor A. J. Sutton Pippard, M. Inst. C.E. (1938); and Dr. E. H. Salmon, M. Inst. C.E. (1938). In 1939, therefore, it appeared to him that a powerful method was likely to remain buried in the two text-books and three articles then existing.

His treatise dealing with the Cross method and with a "degree of fixity" method derived more formally from the "fixed-point" method, was prepared with the object of stimulating interest in the fruits of Cross's discovery. To quote Vauvenargues, "A new principle is an inexhaustible source of new views." That had been so, and the increase of attention to the theory dated from 1939–1940.

Interest had still to be awakened in the column analogy—also due to Cross. That was particularly suitable for the analysis of portal frames such as that illustrated in *Fig. 3*, or component members of continuous frameworks where the moment of inertia was variable within the span. Briefly, $\int y^2 ds$ in the formula for H for a portal was clearly identifiable with I, the moment of inertia of the outline of the frame about the hinges, and so the formula could be identified with the bending term in the better-known column formula My/I. The analogy had wide applications and would repay advanced investigation on the lines set out by Cross at the end of his Bulletin.

The column analogy was an ideal method for standardized drawing-office practice.

Mr. Shepley agreed that the application of the sway correction factors to the method of moment distribution added to the labour of designing; it had always been office practice in the past to allow for sway, maybe unwittingly, by using formulas for frames as given in the standard handbooks. It should be borne in mind that the formulas in the handbooks—for example, that of Kleinlogel—automatically included the sway effect, and therefore it became equally laborious to neglect sway when it was realized that it could not freely take place; (for example, in the case of live loading one of a gallery of building frames which could not sway freely without the others).

[1] "Continuous Beam Structures," Concrete Publications, Ltd.

Many frames had been designed in the past by the Marsh and Dunn or Kleinlogel formulas without full realization of that fact.

The modern methods afforded opportunity of making an allowance, either partially or fully, for sway effects. The value of the allowance was still a matter of experience or " structural sense "; but that could only be obtained by adopting the modern methods as standard office practice.

The column analogy method also included, and thereby hid, the sway component.

Mr. Shepley considered that side sway could rarely be neglected, and that when it was necessary to do so the simple moment distribution method was the safest way of leaving it out of the picture.

The Author's opinion should perhaps be modified as follows: " there are few actual structures in which side sway would never occur, but side sway can almost always be neglected in normal building construction where the loading is only vertical and the frames are fairly symmetrical and of rectangular pattern." It was clear that a sloping column on one side of a frame, such as occurred in some roof portals, had a high sway effect, even with symmetrical loading.

The " Code " formula for moments in outside columns,

$$M_e \frac{K_1}{K_1 + K_u + K_b},$$

had been presented by the Author with little comment as to its suitability and none as to its validity and limitations. If those moments had to be calculated—and Mr. Shepley could not see how they could ignore them, since he agreed with Dr. Oscar Faber, M. Inst. C.E., that " we cannot alter the laws of nature by the simple process of making assumptions,"—then a more satisfying method than the Code formula was required. No account was taken of the ratio of live to dead load, and the assumptions of fixity of the remote ends of the columns and beam bore no relationship to the usual conditions.

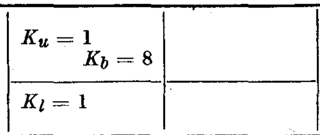
The moment distribution method gave a rapid and easy solution for either regular or irregular frameworks, and had been used to obtain the comparative results shown in Tables III and IV. A two-bay frame had been used for illustration and it would be seen that in Table III the precise results agreed with the " Code " in only two instances—dead loading with remote column ends fixed, and live load equal to twice dead load with remote column ends hinged.

For normal cases of live load equal to dead load, and taking account of the more than full fixity of the column ends (caused by loading on the storeys above and below) the " Code " gave beam end moments less than the precise moments by nearly 40 per cent. as shown in Table IV. Doubtless more extreme cases could be found with irregular frameworks.

TABLE III.

VALUES OF BEAM END MOMENT.

Regular 2 bay frame.



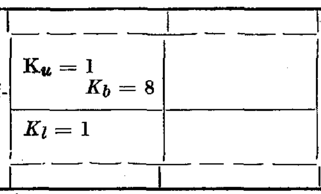
For Fixed End Moment = 100 due to symmetrical total load on span.

Load.	Column ends.	Code moment.	Precise moment.	Code error: per cent.
Dead only . . .	Fixed	20	20	0
	Hinged	20	15·8	+ 26·6
Live only . . .	Fixed	20	27·6	− 27·5
	Hinged	20	22·2	− 9·9
Live = Dead . .	Fixed	20	23·8	− 16·0
	Hinged	20	19·0	+ 5·3
Live = 2 × Dead.	Fixed	20	25·1	− 20·3
	Hinged	20	20·1	0

TABLE IV.

VALUES OF BEAM END MOMENT.

Regular 2-bay frame.
"Double curvature" stiffness for all columns.



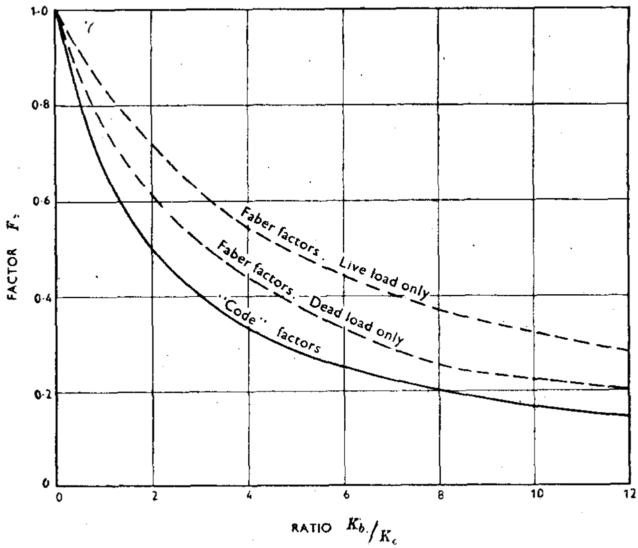
For Fixed End Moment = 100 due to symmetrical total load on span.

Load.	Code moment.	Precise moment.	Code error: per cent.
Dead only . . .	20	27·3	− 26·7
Live only	20	36·5	− 45·0
Live = Dead . .	20	31·9	− 37·3
Live = 2 × Dead .	20	33·4	− 40·2

Fig. 12 had been plotted from Dr. Faber's[1] factors for moments in outside columns monolithic with two or more spans, to show how much the " Code " values fell short of the precise values over the range of beam-column stiffness-ratios ranging from 1 to 10.

Mr. Shepley wished to suggest, therefore, that standard office practice should be based on " more exact methods " as allowed by the " Code ", and that those methods should be either the coefficient method of Dr. Faber, which made the minimum of assumption, or the moment distribution

Fig. 12.



Moment at end of beam framing into outside columns $[K_c = \frac{1}{2}(K_n + K_l)]$.
For intermediate storey beam if two or more nearby equal sides $M = F\left(\frac{WL}{12}\right)$.

method, which needed even fewer assumptions without much increase in the labour of computation.

Mr. J. McHardy Young observed that he welcomed the Paper, since the present tendency was towards indeterminate structures, from both the aesthetic and the economic points of view. It had the added virtue that the Author had avoided the use of higher mathematics in analysis, and had wisely differentiated between analysis, which was a purely mathematical process, and design, which was a combination of experience and engineering sense.

It was true that the slope deflexion method was not of great use to the

[1] Faber, " Reinforced Concrete Design," Vol. II, Arnold.

designer, but a modification known as the "Moment Balance" method [1] could be used in the solution of building frames. From a comparison of the slope-deflexion, the moment-balance, and the Hardy Cross methods, as applied to a simple frame, the following statements could be made :—

(1) the slope-deflexion method gave a number of simultaneous equations which could be solved by the method of "remainder distribution" (*Structural Engineer*, Nov. 1936).
(2) the moment balance method gave a more direct solution ;
(3) the Hardy Cross method also gave a solution.

Mr. Young regretted that the Author had confined himself to applying the Hardy Cross method to the simple case of a continuous beam. Except in the case of single portals, side sway could be neglected, even with unsymmetrical loading.

In their 1940 Specification the American Society of Civil Engineers recommended the use of the Hardy Cross method in dealing with reinforced concrete. That was comparatively simple in cases where there was repetition of members, as in building bents.

The disappointment felt by the Author with the method of design recommended in the Second Report of the Steel Structures Research Committee would be shared by many engineers. The fixing of arbitrary coefficients for bending moment was a retrograde step, and the whole frame could be investigated by the moment balance method. In Mr. Young's view, it could be said that no steel-framed structural member was ever freely supported, bearing in mind the fact that members were cased in concrete, and that fact should be borne in mind in view of the Author's remarks in regard to the adaptability of steel-framed structures.

The Author, in reply, observed that the Paper did not condemn classical methods in general. It simply stated that solutions by the slope deflexion method involved dealing with the differences between small quantities.

The differences of opinion between himself and Dr. Hajnal-Kónyi and other speakers all stressed the fact that there was no best method for the analysis of statically indeterminate structures, but that the choice of methods depended on the problem and also to a large extent on personal preferences.

The examples given in the Paper were deliberately simple so as to indicate the methods involved, and the Author agreed that there were various other equally simple solutions for the cases chosen. The method of arithmetical integration was of considerable use in cases of varying moment of inertia where the method shown by Dr. Hajnal-Kónyi could not be so easily applied.

Regarding side sway, in the Author's opinion there was a considerable

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[1] Trans. Amer. Soc. Civ. Engrs., Vol. 108, p. 677.

difference between calculated and actual values, since horizontal restraints were very often present but were neglected in calculations.

With regard to the difficulty of comparing prices of structures, particularly of steel structures, engineers were very much in the hands of the trade, who settled what price should be charged for various types of work: therefore it was very difficult to make a fair comparison of the economy of one design over another.

Professor Sutton Pippard, in his brilliant analysis, had dealt somewhat scathingly with the problem of E and I. The Author agreed that those two should always be used together as a product and that that product was a measure of the flexural rigidity. The Author still did not see that that simplified the problem of calculating that product in the cases he had mentioned, and its value could not be determined experimentally for all cases.

In reply to Mr. Manning, he thought that the matter of signs applied to moments was almost entirely one of convention. The notation adopted in the Paper would be seen in *Fig. 1.* The Author agreed that the formula given for the thrust in the two-pinned arch applied only when the supports were at the same level. It could, however, be used for horizontal loads.

Professor Baker's remarks were of great value, but the Author felt that neither he nor Professor Pippard had realized the designer's difficulties, particularly in the case of framed buildings where there were so many factors concerning the design over which normally the architect, and not the designer of the framework, had control.

In reply to Mr. Grisenthwaite, the Author did not remember that he had apologized for the need for indeterminate structures. He did, however, agree that one advantage of the indeterminate structure was its ability to transfer load from a highly stressed section to another part which had not yet been so highly stressed.

The Author could not agree with Mr. McIlmoyle's statement that "the almost universal adoption" of indeterminate structures in the United States was due to their economy in first and ultimate cost." He doubted whether there was, in fact, universal adoption, and he did not believe that economy in first cost was often achieved by the substitution of a rigid frame for more conventional designs.

He had indeed suggested that there would be an increase in handbooks giving moments for most types of frames, but he had not, as Mr. McIlmoyle seemed to think, expressed any opinion as to their desirability. Much of the information given in present-day handbooks was unnecessary to the designer and very dangerous in the hands of others than designers.

In conclusion, and in spite of Professor Baker's remarks, the Author was still of opinion that a reasonably simple method of design of steel-framed structures more in accordance with their observed behaviour could be hammered out with closer co-operation between the analysts and the designers.