M. Inst. C.E., and also to those for a proposed welded steel portal designed by Messrs. R. T. James and Partners, Aldwych, for the new Central Depot of the City of Westminster.

Professor Beggs has also applied the method to the solution of suspension bridges.[1] In an earlier Paper [2] he refers to the Grandfey viaduct near Fribourg, Switzerland, as having been designed by the use of models. Models were also used in the design of the steel frames for supporting signals and electric installations over the tracks of the Illinois Central Railroad, Chicago. In his original Paper [3] he gave partial results for a model of a three-storeyed building. It is hoped that more examples of the application of the method to storeyed buildings will be available shortly.

The Paper is accompanied by six sheets of diagrams, from which the Figures in the text have been prepared.

[1] "Suspension Bridge Stresses Determined by Model", G. E. Beggs, E. K. Timby, and B. Birdsall, *Engineering News-Record*, 9th June, 1932.

[2] Ref. 2, p. 4, *ante*. [3] Ref. 1, p. 4, *ante*.

Discussion.

The Chairman observed that the absence of Dr. Stradling, Chairman of the Division, was due to medical advice.

The Author introduced the Paper with the aid of lantern slides and demonstrated the methods described by means of models.

Professor A. J. Sutton Pippard said that the Author and he had worked together for many years on all kinds of problems, and the reception accorded to the Paper had afforded him great pleasure. In the recent discussion on Mr. Budgen's Paper [1] various speakers had stated that the methods of calculation described were not exclusive.

There was, however, a tendency to feel that methods were in some measure exclusive, and that they should not be mixed. Various experimental methods were available, including the direct method and the indirect method, and others to which the Author had not referred; for example, the analysis of plane stress by means of indiarubber sheet, which had been developed by the Author and himself.

Just as calculation and experiment were not mutually exclusive, so there was no reason why experiment should not be combined with calculation. In most cases a designer, especially if confronted with a new type

[1] W. E. J, Budgen, "Modern Computation Methods and Office Practice in the Design of Statically Indeterminate Structures." Structural Paper No. 6.

of problem, would be very glad to have some kind of check on his calculations. That could be done by handing them over to somebody else, but unless the checking was by a complete re-calculation, that was of little value. If a rapid means were available, such as the experimental device described by the Author, surely it was worth while for the designer himself, or perhaps a member of his staff, to make himself familiar with such methods. The necessary equipment would include celluloid, pins, and a fretsaw. The proverb about bringing horses to water would come to mind, and the Author had provided some sparkling drink. Plenty of "old horses" in the design world were present, and he hoped it would not be too difficult to persuade them to imbibe a pint or two.

He believed that those methods were commonly used in America, but, whether that was so or not, they were useful, and not just pretty amusements. He mentioned that with some feeling because he had heard them described as toys. At a certain conversazione model analysis and the game of "Tiddlywinks" had been put on the same plane!

A year or two before the war he had been invited to lecture at a large aircraft works, and had addressed an enthusiastic audience of designers and draughtsmen on the use of models. He was afterwards approached by the firm to know whether a special course of instruction could be given, with laboratory work, to train their people, and as a result a number of the drawing office staff were sent to the lectures, whilst specially chosen ones spent three or four afternoons at the Imperial College in work. The reaction of those people was excellent. They invented new devices for doing the job, and altogether it was a very pleasant experience. He wished to see that kind of thing extended. It was no use for methods to be developed and explained if they did not find application in service. If they were not used by the practical engineer then those who devised them were wasting their time in the university—and the fault was not theirs.

As the Author had explained, one could make a model and get the influence lines rapidly and accurately. The curves which she had shown were very good, and results like that could be obtained quite regularly. Moreover, models could serve a useful qualitative purpose in preliminary design, although they might be rough—or exist only in the imagination.

In the large deflexion method to which the Author had referred, a model was made, pinned down on a sheet of drawing-paper, and pushed about considerably, say $\frac{1}{4}$ inch or so on a 10-inch length. The idea, when it first originated, was that the displacement should be very small. The advantage of the large displacement was that small errors due to temperature-changes and other factors were eliminated. When working with small displacements it was possible to upset the results even by breathing on the specimen, whereas with larger ones those effects were trivial.

The Author had described the degree of accuracy which she obtained, but that was not altogether conclusive. It was hoped to make a statistical examination of the accuracy of some of those methods. For that purpose a large number of people would be asked to co-operate and to make measurements on certain specimens which would be provided. Different groups of people would assist, so that the results would indicate their consistency and how different types of workers varied, and he hoped that members who received requests to co-operate would do so, as their help might establish the utility of the method.

He desired to call attention to one other point. In demonstrating the Westminster structure the Author had given the calculated values and the experimental values, but she had not stated what the error was, because, frankly, that was not known. Was it more correct to do the work experimentally or by calculation? He did not doubt that the calculations were right, but certain factors were neglected of which the scale model took account to some extent. Therefore when a difference of 6 per cent. occurred between the model result and the calculated result the correct answer was probably somewhere between the two.

In the Paper the Author had referred to the analysis of a Vierendeel truss. It was actually a rib from a British bomber in its early stages. The question of stressing it had arisen, and as the Author and himself had a student who was anxious to do experimental work of that kind, some calculations were made and checked experimentally.

Mr. S. R. Sparkes observed that it might be helpful to elaborate one or two details connected with model methods of structural analysis.

No single method would solve all structural problems, and he would indicate the particular uses, and some of the difficulties, of the various methods referred to in the Paper.

The Author had given examples of the application of the indirect method using large displacements to the continuous beam. Unless it were a particularly difficult continuous beam, he did not think that many designers would use a model method to solve it, as the Hardy Cross and other methods were quite manageable. The indirect model method using large displacements was very useful in the case of portals and arches of variable section, but was not practicable for determining loads in internal members of an indeterminate structure. To obtain the horizontal thrust at the pin of a two-pinned arch or portal subjected to unit vertical point loading, the experimental procedure was simply to displace one pinned end of the model a distance $\triangle$ in the direction of the horizontal thrust and to measure the vertical displacement δ of each load point in turn. The required horizontal thrust was then $\frac{\Sigma\delta}{\triangle}$. That was all the experimental work involved, whether the arch or portal was of constant or variable section or of any shape. It might be said that measuring to 1/500th of an inch with the naked eye, as mentioned by the Author, was impossible. By using an ordinary engineer's scale divided $\frac{1}{4}$ inch to 1 foot, however, perfectly consistent results could be obtained by reading to

1/10th of one division, that was, to 1/480th of an inch. Errors were more likely to arise through displacing the pin of the model inacccurately than through mistakes in measuring displacements.

It was not practicable to apply large deformation to internal members of an indeterminate structure, and Beggs's apparatus was merely a device for applying some known displacements in one direction only. It consisted of two arms held by springs against carefully ground plugs. By changing the plugs a known displacement between the arms of the apparatus could be obtained. There was no difficulty in using Beggs's apparatus to determine the end reactions of an encastered arch. One arm of the apparatus was fixed to the table, and the abutment of the model was fixed to the other arm. A known displacement to the abutment could be applied by changing the plugs of the apparatus, and displacements of the points on the model corresponding to load points on the actual structure would be measured by a micrometer microscope. Using the same apparatus to find the load in an internal member of an indeterminate structure, the member in the model was cut and each side of the cut section was fixed to one arm of Beggs's apparatus. The apparatus itself, however, had to be mounted on rollers, so that each side of the cut section might be properly displaced, and it required considerable care and an experienced operator to change the plugs without moving the whole model.

The slope deflexion method—the direct method which had been developed to a great extent by Professor Baker—was another method in which a micrometer miscroscope was required. Considerable experience was called for, and the method was not readily available to a drawing-office staff.

Mr. Sparkes thought that the indirect model method using large displacements was generally the most useful method available for the drawing office. He was surprised at the way in which designers usually fought shy of model methods. It was not a great matter to purchase a few pins and a piece of cardboard or some celluloid, and surely a good design was worth that.

Dr. L. A. Beaufoy observed that the title of the Paper was ambitious for a short Paper and was hardly justified by its contents. No reference had been made to methods such as the hinge method of Sayed Abd El-Wayed, the moment indicator of Ruge and Schmidt, the Continostat of Gottschalk, or the Nupubest of Rieckhof, whilst only a sideways reference had been made to the most popular method—that using Beggs's deformeter. Therefore the Paper hardly covered the ground adequately.

The Author had stated that there was no inherent reason why the direct method should not be applied to a model in three dimensions. In fact, the method had been so applied, and, particularly in the case of arch dams, it had been used very considerably. For example, in 1926 the Stevenson Creek dam was investigated by three-dimensional models.

Two models were used, the first of concrete and the second of celluloid, both being mercury-loaded. That application was the forerunner of a whole series of investigations made upon the Gibson dam and other dams.

He was glad that, in her introductory remarks, the Author had corrected the impression that might have been drawn from the statement, on p. 4, *ante*, that " The scale for the lengths of members is chosen independently of the scale for their cross-sections. This in no way vitiates the results. . . ." To leave the statement in that form was dangerous; in fact, it was not true, particularly if the model represented a frame with stiff joints. Twelve or thirteen years ago Dr. Beaufoy had published some results[1] which referred specifically to that point, and in the particular case which he tested he found that differences of something like 10 per cent. in the values of stress components were obtainable simply by varying widths of model members relative to the general scale of the model. Those variations were not excessive ones; he thought that large widths in comparison with small widths were of the order of 4 to 1, and, even at the largest size, the width was still in reasonable relationship to the general dimensions of the model.

Another indication of the effect of the introduction in the model of the width of model members was given by the case of a five-story, two-bay building frame in which experimental thrusts at the bases were compared with the results of a rigorous mathematical analysis neglecting depths of members but allowing for rigidity of joints. There were differences between theoretical and experimental values, of about 5 per cent. at the centre and about 7 per cent. at the bases of the outer stanchions, which were rightly attributed to the stiffening effect of the portions of material at the joints of the model. The reduction in the free length of the beams and the local stiffening at the joints led to a redistribution of stress throughout the framework.

The Author had also stated that " Devices can be used for satisfying both conditions if essential "; that was to say, satisfying the condition that $\frac{td^3}{12}$ represented the second moment of area of the prototype section to the chosen scale, and the further condition, if the effects of direct compression were to be taken into account, that td should represent the cross-sectional area of the prototype to the same scale. One device for doing that was to use several thicknesses of celluloid built up in layers cemented together. Perhaps the Author would describe other devices.

Dr. Beaufoy considered that the Author's assertion that, for the direct method, celluloid was unsatisfactory, should not pass unchallenged, since it had been shown that the moment indicator applied to celluloid models could give results within 1 per cent. Certain advantages were obtained by

[1] " Fundamental Difficulties of Deformeter Analysis," *Engineering*, vol. 134, No. 3491.

using celluloid models, and, if a technique could be devised which would eliminate inaccuracies, something useful would have been achieved.

In what way could an internal stress component be determined by means of the method which was more specifically discussed in the Paper? The Author had been concerned in the Paper only with external redundants. Some of the models shown had indicated methods which might be used, but it would be helpful if the Author would elaborate that point, because it became so important in the analysis of complicated frameworks, and an obvious advantage of a model method was its application to such frameworks.

Mr. G. P. Manning said that the Author had stated that no more equipment was needed than splines, a packet of pins, and a hammer to knock them firmly into the board. But the Paper included a reference to microscopes, and Mr. Sparkes had made reference to Beggs's apparatus. Would the Author give a list of the apparatus used in her laboratory.

The accuracy of the calculated results from the measurements on the models was well within the limits required by the engineer. It might interest the Author to know that Mr. Manning had abandoned his last experiments with the models, not because of their inaccuracy, but because he was continually worried by what happened after the model stage. In his view, after the model stage two extrapolations had still to be made. Starting with a simple elastic model with definite loading and definite end conditions, one had to extrapolate to a large elastic structure with vague loads (only in the case of hydraulic structures did one really know the load) and vague end conditions. Then came the second extrapolation to a large reinforced-concrete structure with vague loads and vague end conditions, and, with variable material plus construction joints.

Some actual measurements had been made in America on large structures. Had the Author taken the actual measurements seen in the structure, made a similar model, and compared its measurements with those on the structure?

Mr. Manning believed that three people out of four were naturally clumsy with their fingers. Freedom from clumsiness was not connected with academic distinction or expert performance in sport, nor was it inherited. So far as he knew, no research worker was ever tested for manual dexterity before he was allowed to do research, and that fact seemed to throw very grave doubt on about three-quarters of the published research work on very small models. Could the Author suggest some actual dexterity test to which staff could be subjected?

Dr. K. Hajnal-Kónyi observed that Dr. Beaufoy had referred to the Author's statement that there was no inherent reason why the direct method should not be applied to a model in three dimensions. and had quoted some of the tests. Another example was the shell system of roofing which had been applied in Germany. Very large models had been made, and Dr. Hajnal-Kónyi had himself been concerned with tests made at Frankfort in 1927 on a one-third scale model of a market hall of 130 feet span, when the barrel vault was tested in every possible way and long-term observations were made.

Professor Pippard had mentioned a statistical method of comparing the accuracy of experimental results. Dr. Kann, who had been a lecturer at the technical college, Wismar, had issued a publication on that subject in 1924 [1] and, with his students, had done a great deal of experimental work. He set the class experiments on a simple system, varying the ratio of the moment of inertia, and particularly he investigated the effect of haunching up to one-third of the span of a two-pinned frame. Dr. Hajnal-Kónyi considered that the experimental method was extremely valuable at technical colleges for teaching purposes, because many students did not like the calculating method. They were much more interested in investigating the structure in an experimental way. The method had the disadvantage that it was difficult to reproduce. If somebody were to submit a calculation based on experiments to the authorities the latter would insist upon evidence of their correctness; but in cases where such reproduction was not required, and especially for extremely complicated systems, the method was particularly valuable. The classical example was the Grandfey viaduct, a steel bridge built in the 'sixties near Fribourg, Switzerland, and considered to be the most impressive bridge on the Swiss railway system. When the question of strengthening it in connexion with electrification of the railway was considered the bridge was rebuilt in such a way that the structural system was entirely changed and transformed into a series of arches with spandrels. It would not have been possible to investigate the combined effect of the spandrels and the arches by calculation. The experimental method gave the required results. Another example was the railway-bridge across the Rhine at Coblenz, which consisted of two steel arches and various approach arches in brickwork. It seemed that it would be necessary to strengthen the bridge, but by the experimental method it could be proved that the arches were strong enough, and that the approaches could support a greater load.

*** **Professor J. F. Baker** observed that the direct methods of experimental analysis mentioned in the Paper were not, in general, so pretty or so useful as the indirect methods. Beggs's work on the suspension-bridge [2] was, however, something of an exception. Ten years ago, experimental work on much the same lines was carried out in the Department of Civil Engineering of the University of Bristol. The intention, in the first place, was to make a structural model of the Clifton suspension bridge and to compare the results of tests with the behaviour of the prototype. That

[1] "*Beton und Eisen*," 1924, pp. 268–273.

[2] "Tests on Structural Models of the Proposed San Francisco–Oakland Suspension Bridge," University of California, Publications in Engineering, vol. 3, p. 59.

*** This and the following communication were received in writing.—SEC. INST. C.E.

scheme was dropped when examination of the bridge showed that the stiffening girders were not supported at their ends, as it was thought to be not sufficiently general a type for study by undergraduates. Instead, a three-hinged bridge was built with the object of investigating the validity of the assumptions made in the elastic and deflexion theories. The bending moments in the stiffening truss were determined by the Beggs method, but the strains in the suspenders were measured by means of mirror-type extensometers.

The apparatus and the results of the investigation were described by Mr. J. W. Roderick, Stud. Inst. C.E.,[1] in a Paper published in abstract form in 1940. The detailed information contained in the full Paper should be of particular interest to those in charge of structures laboratories.

Mr. R. J. Wilkins observed that, as stated by Professor Pippard, analysis by models would probably be found of the greatest use as a quick alternative method for comparison with the results of a lengthy calculation, and also in the early stages of the design of an indeterminate structure.

At the Northampton Engineering College, London, twenty-five students had each been engaged during the present session upon the design of a two-hinged welded steel portal frame bridge having dimensions of 80 feet 6 inches centre to centre of pins and 17 feet 4 inches from pins to roadway-level. The girders were of varying second moment of area, and a model of sheet xylonite to a geometrical scale of $\frac{3}{8}$ inch= 1 foot, but with the cube of the model depth proportional to the I of the girder, was prepared and used in the manner described in the Paper. The results obtained by operators without previous experience confirmed that no special technique was required. In most cases the agreement with the calculated values of the influence line for the horizontal thrust was within ± 5 per cent. The measurements were taken by means of a steel rule on outlines of the model in its initial and deformed conditions made by a very hard chisel-pointed pencil. The results obtained by the experimental method were plotted before commencing the calculations, in order to ensure that no prior knowledge of the results should influence the model readings.

It was found very satisfactory to have that rapid independent confirmation of a lengthy calculation, and the results obtained fully justified the use of models in other cases.

Mr. Guthlac Wilson observed that the Paper contained much that would be useful. The hints on the technique of model testing were particularly valuable.

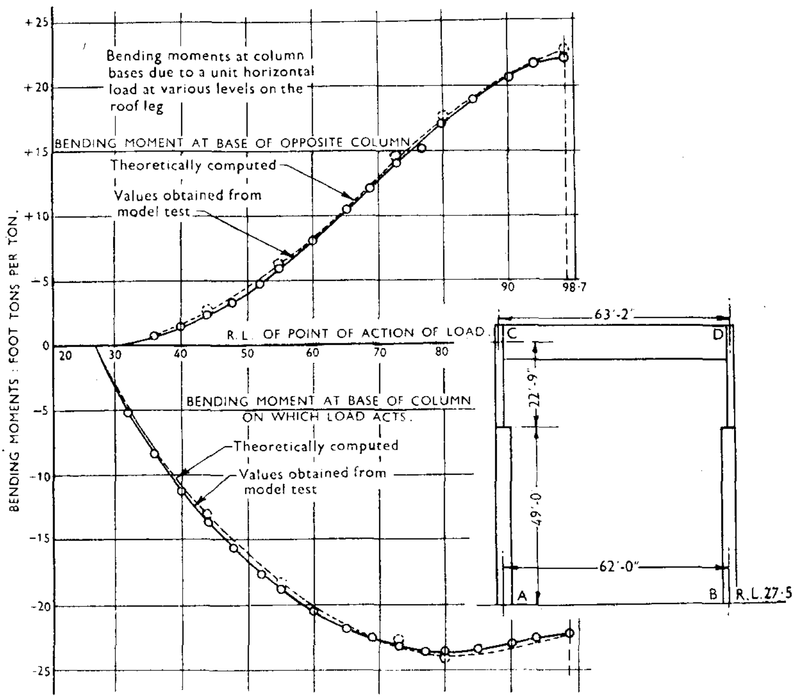
He had had occasion to use the indirect method when in India some years ago. The main portal frames of the Mulagore turbine-house of the Calcutta Electric Supply Corporation, for which Mr. W. C. Ash, M. Inst. C.E. was Consulting Engineer, were designed by the column analogy and

[1] "An Investigation of Stresses in a Three-hinged Stiffened Suspension-Bridge." Journal Inst. C.E., vol. 13 (1939–40), p. 343. (Feb., 1940).

checked by the indirect method described by Professor A. J. S. Pippard and Mr. S. R. Sparkes in a Paper which had just been published.[1] The agreement between the computed and experimental results was very gratifying, as would be seen from *Fig. 7.*

The Author, in reply, observed that she thought that Mr. Manning was confusing two distinct conceptions of the use of models when he said that

Fig. 7.

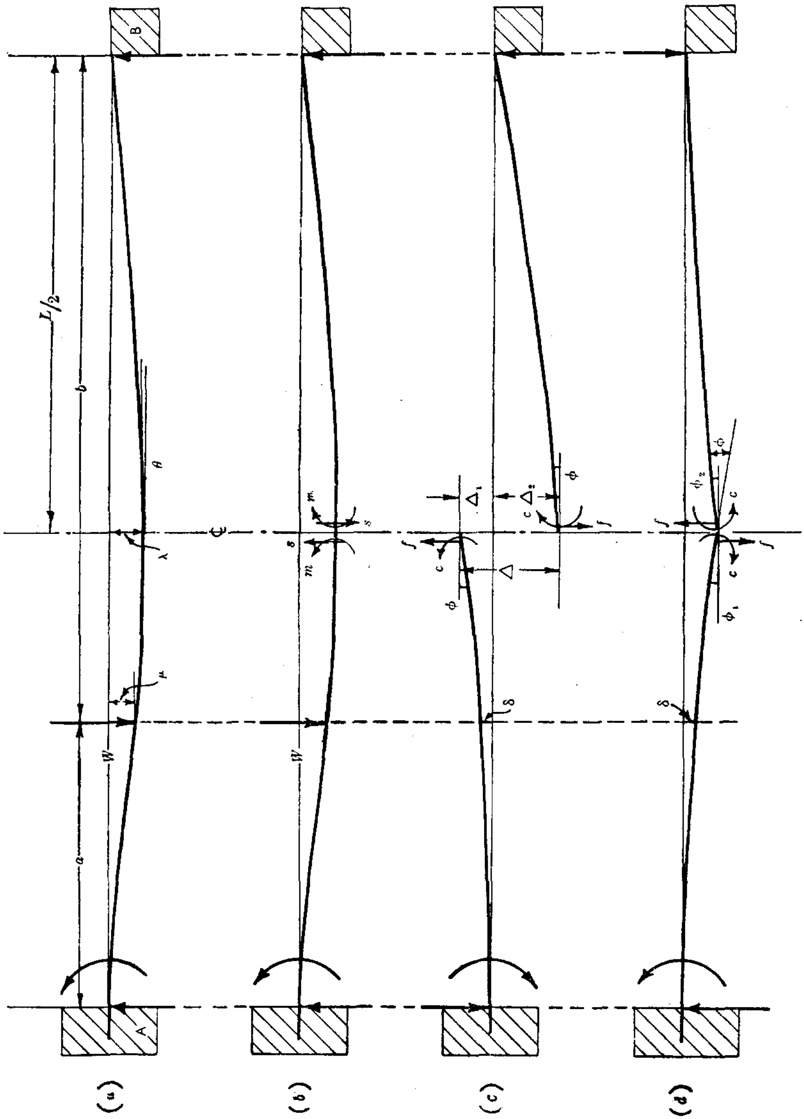


he had abandoned them because he could not make out how to proceed to the next stage. In any theoretical calculations made for purposes of design certain abstractions and sweeping assumptions were necessary before any progress could be made, and the difficulties were neither increased nor lessened if models took the place of arithmetical computation. The relation between the behaviour of the actual structure in the field and the abstraction was outside the range of the Paper, but a number of classical experiments on full-scale structures had been made which had either determined or confirmed the specific form of the assumptions usually made in those abstractions.

The Author wished to amplify the explanation of how a model was made

[1] References 3 and 4, p. 4, *ante*.

flexible by choosing different scales for the cross-sections and the configurations. If L, I, and A referred to the original and $k_L L$, $k_I I$, and $k_A A$ to the model, in order that the energies stored in bending and direct



Figs. 8.

compression might bear the same relation to one another in the model as in the original, k_A must be equal to k_I/k_L^2. That was satisfied in a true scale model, but not otherwise. Luckily, as explained, it was not often essential to satisfy that requirement, but one device for doing so was to make the member in two parts with slotted T-shaped ends: the heads of the T's were arranged to overlap and were bolted together. By altering the position of the two bolts the effective A could be changed without altering the effective I.[1]

Application of the indirect method to determine internal stresses had been discussed. The points of practical difficulty might not be obvious unless the underlying theory were clearly grasped. A simple example was afforded by a beam, encastered at one end and simply supported at the other. Suppose the shearing force and bending moment at the centre were required when a load W was applied at a distance a from the encastered end, as in *Fig. 8 (a)*. Imagine the beam to be cut at the centre and the internal reactions to be replaced by equal and opposite forces and couples so that the configuration of the beam was unchanged, as in *Fig. 8 (b)*, that constituted the first load system in the Clerk Maxwell reciprocal theorem. Next, as in *Fig. 8 (c)*, suppose the load W to be removed and the two cut ends forced apart a distance Δ, but kept parallel to one another; and suppose further that that was done in such a way that no resultant external force or couple was introduced at the section; that constituted the second load system. Taking first one half of the beam and then the other, by the reciprocal theorem,

$$(-W\delta+s\Delta_1+m\phi)+(s\Delta_2-m\phi)=(0\times\mu-f\lambda+c\theta)+(f\lambda-c\theta)=0,$$

whence $$s=\frac{\delta}{\Delta_1+\Delta_2}W=\frac{\delta}{\Delta}W.$$

If, by misadventure, a resultant external force F or couple C were introduced at the section, terms $F\lambda$ and $C\theta$ would be left on the right-hand side of the equation.

Similarly, as in *Fig. 8 (d)*, if the cut ends were kept together but forced to lie at an angle ϕ to one another (again without introducing any resultant external force or couple), by similar reasoning $m=\frac{\delta}{\phi}W$.

Extension of the method to the general case, say to any section of a framed building where there was an unknown thrust as well as shear and bending moment, was obvious. The practical difficulties were, firstly, to force the cut ends into the correct new positions without causing buckling of the structure and, secondly, to ensure that no resultant external reactions were introduced at the cut section through the medium of the apparatus used to deform it.

It was very gratifying to hear from both Mr. Wilkins and Mr. Guthlac Wilson of cases in which the indirect method had proved satisfactory as a rapid and independent check method in the drawing-office.

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[1] "Pre-Stressing Bridge Girders", H. J. Nichols. Journal Inst. C.E. vol. 5 (1936-7), p. 91 (Feb. 1937).

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