

The Author, in reply, observed that mechanical stabilization seemed to have the greatest potential future in the Colonies in view of the great distances which stabilizers such as cement or bitumen would generally have to be brought: the need for the type of plant mentioned in the Paper would, however, remain. It was true that graders provided a certain soil-mixing action, but that should not be taken to mean that they were an equally efficient substitute for soil-mixers.

At least two and probably three mobile water tankers with drivers and operators would be needed to feed and fetch water for the American machine referred to. It would seem more economical to have storage containers on site which could be provided in advance with sufficient water for a full day's requirements. The method of supplying water direct to a mixer did have advantages over other methods for intimacy of mixing. The Author agreed that a 12-foot lane was desirable, but it should be remembered that the larger the machine the greater was the loss of output if a mechanical fault developed as had been found with travel mixers. The water-sprayer shown in the film had since been modified to spray over a 6-foot width.

The value of additional passes using a tandem roller could only be proved by density tests. Such a roller would be valuable only if a thin surfacing over the base were contemplated. For a tarmacadam or concrete surfacing the finish left by a rubber-tired roller and a grader working in conjunction would probably be sufficient. A compressor might well prove more satisfactory than a broom drag prior to a priming. The Author agreed that 2,000 square yards per day per team might well be exceeded with an experienced team of operators working to a strict time-table, especially if the need for sheepfoot rollers could be avoided.

STRUCTURAL AND BUILDING ENGINEERING DIVISION.

Structural Paper No. 18.†

“Reduction of Ground Vibrations into Structures.”

By J. H. A. CROCKETT, B.Sc., and R. E. R. HAMMOND, A.M.M.I.C.E.

Correspondence.

Dr. Herbert Chatley observed that study of the vibrations in the soil might enable progress to be made in the determination of the bearing power of piles. In pile-driving a fraction of the energy of the moving weight was converted into vibration energy of the soil. That fraction might be very small, but at refusal it amounted to unity. The elasticity

† Discussed at Meeting on 11 March, 1947

of the soil was a determining feature and the difference between clay and sand was a major factor in producing ambiguity in pile-bearing formulae based on driving tests.

Secondly, it was possible that a study of "seismic" phenomena in the proximity of building on unknown foundations might enable the depth and character of those foundations to be deduced. What was still more important was that, just as such investigations had thrown light on the existence of rock in prospecting for oil and other minerals, so it was conceivable that the nature of a site for a proposed structure might be determined without the expensive and tedious process of boring.

Thirdly, the Authors had called attention to the lack of knowledge concerning gustiness of wind, with an implication that structures might be put into a state of resonant vibration by such gusts. Judging by the records of anemometers and personal observations of gusts, their periods resembled those of sea-waves and increased with the strength of the wind. The variations of wind-strength were of the order of 50 per cent. (plus or minus). Presumably eddies were continuously generated from the rugosities of the ground and were generally clockwise looking across the wind when the latter was blowing from left to right. Eddies with vertical axes could also develop from the angles of vertical obstructions. Resonance implied the persistence of gusts with fairly constant periods, and such constancy seemed to be rare; but there were several well-known cases in which bridges had been destroyed by excessive distortion which seemed to have arisen from prolonged synchronism of gusts and the oscillation-time of the structure. Delicate anemometers free of "hunting" could certainly give information on that matter, which was of interest also in aerodynamics, whilst meteorologists should be able to help. The soaring of birds and the behaviour of sail-planes were intimately related to gustiness.

Mr. R. S. Colquhoun observed that on p. 7 the Authors had introduced the term Soil Dynamics, whilst Figure 9 illustrated a series of wave-fronts travelling through the soil beneath a foundation which was subjected to a single vertical downward impulse. The form of the wave-fronts approximated to the Boussinesq "bulb of pressure" beneath a statically loaded foundation. How had that diagram been derived, and had any research been carried out on the distribution of dynamic stress waves in various soil types.

Such an investigation should be of more than theoretical interest, since it would have some bearing on the design of runways and aircraft standings (the failure of which was sometimes ascribed to "vibration"), and also on the use of vibration as a method of compacting earth fill.

Mr. J. Duff observed that he had been impressed by the wide range of subjects covered by the Paper. He had been associated with the first Author on some soil dynamics investigations carried out at Sevenhampton

and partially reported elsewhere. His observations would mainly be related to that work.

The relation between the velocities (V_p , V_s) of the P- and S-waves was given on p. 18 by:—

$$\frac{V_p}{V_s} = \sqrt{\frac{(1-\sigma)}{(1-2\sigma)}}$$

The corresponding relation between the two velocities (V_s , V_R) of the S- and-Rayleigh waves was:—

$$\left(\frac{V_R}{V_s}\right)^6 - 8\left(\frac{V_R}{V_s}\right)^4 + 8\left(\frac{V_R}{V_s}\right)^2 \left(\frac{2-\sigma}{1-\sigma}\right) - \frac{8}{1-\sigma} = 0$$

The extreme values of Poisson's ratio (σ) so far obtained would appear to be $\sigma = 0.15$ (reinforced concrete) and $\sigma = 0.45$ (lead), and therefore $\frac{V_p}{V_s}$ ranged between 1.56 and 3.32 and the corresponding range of $\frac{V_R}{V_s}$ was from 0.90 to 0.95.

It could be shown that for a single impulse such as that produced by the piling-winch and drop-hammer method mentioned on p. 13, the horizontal and vertical displacements due to the P- and S-waves set up could all be expressed in terms of a damped sine-wave function whose argument was the same but whose amplitude and damping were different. If m equalled $\frac{\text{maximum horizontal displacement}}{\text{maximum vertical displacement}}$ for any period and S denoted the difference in rate of damping of P- and S-waves, the expression connecting $\frac{V_R}{V_s}$, S , and m could be derived.

$\frac{V_R}{V_s}$ was a function of σ , whilst S was a function of σ and of a physical property of the strata. Assuming that $\frac{V_R}{V_s}$ and S varied slowly for small increases of depth, $\frac{V_R}{V_s}$ could be determined and hence σ .

For depths of up to about 21 feet below ground-level at Sevenhampton, a value of $\sigma = 0.167$ (say $\frac{1}{6}$) had been obtained by Mr. Duff, who hoped to describe his work later.

If $\sigma = 0.167$, $\frac{V_p}{V_s} = 1.58$, and $\frac{V_R}{V_s} = 0.905$

The idea of relating ultimate bearing pressures to frequencies was a useful line of advance in foundation practice, and enough material should be available in soil mechanics laboratories in Great Britain to enable an empirical relation to be worked out. Mr. Duff considered that it should be possible to link up Boussinesq's theory of stresses and displacements

in semi-infinite elastic bodies with the ultimate bearing pressures, through a method similar to the piling-winch frequency-determination referred to elsewhere. Had the Authors any information as to work in that direction?

Had the Authors considered the possibility of using a paving breaker in lieu of a piling winch for frequency measurements? Mr. Duff knew of one type with a lay-out similar to a cantilever piling frame, which was self-propelled and could easily be adapted for giving an impulse below ground-level. Both time and labour could be saved by the use of that machine.

The Authors had stated that cork did not expand under compression. Was that due merely to the fibres consolidating under pressure, or was it to be understood in the elastic sense? In the latter case, could the Authors state the Poisson ratio for the material?

The problem of wind-gust frequencies had been discussed in a recent Paper¹ by Mr. J. P. Harding, which included a bibliography.

A considerable amount of work on wind stresses was done in connexion with the Forth bridge, and Mr. Duff suggested that published accounts of that work and of similar investigations made on other large bridges built at about the same time might repay examination.

Mr. Archibald Eastmond inquired whether the Authors had carried out any research work with regard to the vibrations set up by stone-crushers and gravel-screeners, and whether further results could yet be given, particularly on quarrying plant?

Mr. J. L. King observed that the silencing of ship machinery formed an important part of the work of the Admiralty Research Laboratory. It was necessary to reduce the vibration (usually periodic) generated by a machine and to minimize the transmission of that vibration to the hull. The problems raised were similar to some of those discussed in the Paper.

As the Authors had pointed out, those problems were complex rather than inherently difficult. However, the familiar electrical analogy for mechanical systems was of assistance, as was illustrated by the following example.

To calculate the effect of resilient mountings, it was usually sufficient to assume that the vibrational power developed by the machine was dissipated only in the hull. Thus the machine itself could be represented simply by masses and stiffnesses—springs—and the only resistance term occurred in the hull. If the impedance of the hull were Z_0 , say, the mechanical circuit and its electrical equivalent might, in most cases, be represented as shown in *Fig. 53*.

A voltage was equivalent to a force and a current to a velocity, so that the impedance of a mass was $j\omega M$ and that of a stiffness was $-jS/\omega$, where $j = \sqrt{-1}$ and $\omega = 2\pi n$, n denoting the frequency.

¹ J. P. Harding. "The Reconstruction of Ten 305-foot Tubular Steel Masts in Reinforced Concrete," *J. Instn Civ. Engrs*, vol. 27 (1946-47), p. 113 (Dec. 1946).

If the impedance to the right of and including the r th section were Z_r , the two circuits shown in Fig. 54 were equivalent.

If V_r and i_r denoted respectively the voltage across and the current in Z_r , the following relations held :—

$$Z_r = j\omega M_r + \frac{1}{\frac{j\omega}{S_r} + \frac{1}{Z_{r-1}}},$$

$$\frac{i_{r-1}}{i_r} = \frac{1}{1 + \frac{j\omega Z_{r-1}}{S_r}},$$

$$\frac{V_{r-1}}{V_r} = 1 - \frac{j\omega M_r}{Z_r},$$

$$V_r = Z_r i_r, \text{ for } r = 0, 1, \dots, N.$$

Thus, starting from a given Z_0 , it was possible to calculate $Z_1, \dots, Z_N$ and also $V_0, V_1, \dots, V_N, i_0, \dots, i_N$ in terms of any one of those quantities (usually V_N was specified).

Fig. 53.

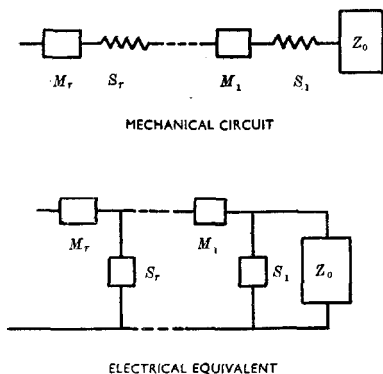
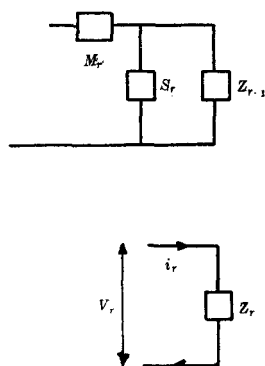


Fig. 54.



The vibrational power, P_r , dissipated in Z_r , was given by the real part of $V_r \times i_r$. In the present case, $P_N = \dots = P_1 = P_0$ but, if resistances were included other than in Z_0 , the power dissipated in the r th section was given by $(P_r - P_{r-1})$.

In that simple problem, the iteration method of calculation was adequate. For more complicated circuits, however, the calculations became too laborious and could be more easily carried out by the use of relaxation methods or even, possibly, by constructing the electrical network.

Calculation of the values of the various components of that circuit

was also laborious. The mass terms of *Fig. 53* were usually compounded of masses and moments of inertia, whilst stiffness terms, in most cases, were due to several beams or resilient mountings not vibrating in phase. The combination of those elements depended upon the mode of vibration and it was therefore necessary to consider separately not only each source of vibration but also each mode set up by that source. That gave rise to the well-known result that not all modes of vibration were equally affected by resilient mounting.

As an example, considering a rigid body of mass M with moments of inertia (I_1, I_2, I_3) about the centre of gravity supported on several springs. Using vector notation, and supposing that a spring subjected to a displacement $\mathbf{y} = (y_1, y_2, y_3)$ and a rotation $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ produced a restoring force $\mathbf{K}(\mathbf{y})$ and a couple $\mathbf{L}(\boldsymbol{\theta})$ where $\mathbf{K}(\mathbf{y}) = (K_1y_1, K_2y_2, K_3y_3)$, $\mathbf{L}(\boldsymbol{\theta}) = (L_1\theta_1, L_2\theta_2, L_3\theta_3)$; then if the springs were attached at points whose coordinates were $\mathbf{r}_1, \mathbf{r}_2, \dots \mathbf{r}_n$ relative to the centre of gravity, and a force $\mathbf{F}$ and couple $\mathbf{G}$ were applied at the centre of gravity, the equations of motion would give :—

$$M\mathbf{x}'' + \sum_{k=1}^n \mathbf{K}(\mathbf{x} + \boldsymbol{\theta} \times \mathbf{r}_k) = \mathbf{F},$$

$$\mathbf{I}(\boldsymbol{\theta}'') + \sum_{k=1}^n \mathbf{L}_k(\boldsymbol{\theta}) + \sum_{k=1}^n \mathbf{r}_k \times \mathbf{K}(\mathbf{x} + \boldsymbol{\theta} \times \mathbf{r}_k) = \mathbf{G},$$

where $\mathbf{I}(\boldsymbol{\theta}'') = (I_1\theta''_1, I_2\theta''_2, I_3\theta''_3)$, and $\mathbf{x}, \boldsymbol{\theta}$ defined the motion of the centre of gravity, dashes denoting differentiation with respect to time. It could be seen that the vanishing of any component of $\mathbf{F}$ or $\mathbf{G}$ did not imply the vanishing of the corresponding component of $\mathbf{x}$ or $\boldsymbol{\theta}$.

For resilient mountings, some simplifying assumptions could be made. For most mountings, $\mathbf{L}(\boldsymbol{\theta})$ could be neglected and, in mountings designed by the Admiralty Research Laboratory, $K_1 = K_2 = K_3 = K$, say. In order to keep all resonant frequencies as low as possible, it was desirable

that $\sum_{k=1}^n \mathbf{r}_k = \mathbf{0}$.

Then, finally, the equations simplified to :—

$$M\mathbf{x}'' + nK\mathbf{x} = \mathbf{F},$$

$$\mathbf{I}(\boldsymbol{\theta}'') + K\sum \mathbf{r}_k \times (\boldsymbol{\theta} \times \mathbf{r}_k) = \mathbf{G}.$$

The Laboratory intended to carry out experiments in order to determine how far the many simplifications implied in the theory sketched above would reduce its effectiveness. It was hoped that applications of the theory would prove of value in problems of ship vibrations. Ground vibrations raised further complications and might prove less amenable to solution by those methods, but at least qualitative information should be obtainable.¹

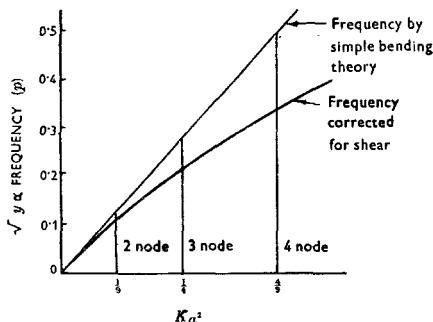
Dr. L. Knopp observed that the Authors had presented a valuable

¹ Crown Copyright reserved.

Paper in an attractive manner, with both interesting and informative illustrations. They had not, however, attempted any mathematical investigation into the oscillational behaviour of structures, and he hoped that that subject would form the basis of a further Paper.

In another field, some consideration had been given to the subject. It was obvious that the simple theory of stress was not accurate for beams which had transverse dimensions small in relation to their length, and that effect tended to become exaggerated in a hollow girder such as a building structure. Dr. Knopp suggested that the following extra term should be introduced with the differential equation of flexural vibration

Fig. 55.



of a uniform beam, in order to correct for shear, loaded along its top and bottom edges :—

$$h_a^2 = \frac{g \cdot E \cdot K}{2\omega(1 - \sigma^2)} \cdot (\tanh Kh \cdot \operatorname{sech} Kh)$$

where h_a denoted frequency in circular measure $\left(\frac{2\pi}{r}\right)$;

g	„	acceleration due to gravity ;
E	„	Young's modulus ;
ω	„	load per unit length ;
h	„	half-depth of beam ;
$2\pi/K$	„	wave-length of vibration ;
σ	„	Poisson's ratio.

When, as was usually the case in practice, Kh was small, the expansion gave $h_a^2 = \frac{g \cdot E \cdot K^4 h^3}{3\omega(1 - \sigma^2)} \cdot \left(1 - \frac{4}{5} K^2 h^2\right)$ approximately, the latter portion of which gave the reduction of frequency due to shear (lower figure).

Rayleigh had given the expression for the amplitude of vibration due to a load applied to any point of a uniform beam namely, $y = \frac{g \cdot P_1 T^2}{\pi^2 w}$

$\times \frac{u \cdot u_2}{u_c^2}$. The same form of expression applied with truth to a non-uniform girder, and the only modification necessary in the actual value was obtained by noting that $w \cdot v_c^2/T^2$ was proportional to the kinetic energy of vibration with end amplitude v_c . The amplitude of forced vibration due to a pulsating load of any frequency, or to an unbalanced force, might be obtained from Rayleigh's expression by substituting for P the successive force increments, by integration with respect to time. The resulting maximum amplitude, corrected for a non-uniform beam, was:—

$$y = \frac{g \cdot P_1 T^2}{\pi^2 c \cdot w} \times \frac{u \cdot u_c}{u_c^2} \times \frac{T_1^2}{T^2 - T_1^2}$$

T_1 denoted the period of the applied force and P_1 the maximum of that periodic force. If the linear damping were represented by the factor e^{-kt} for the damped force vibrations, the dynamic magnifier became

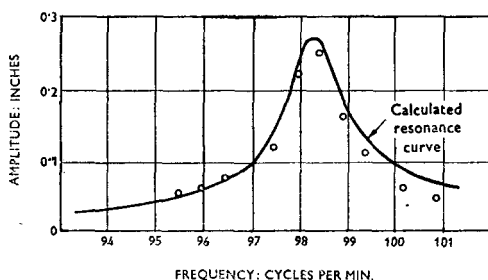
$$\sqrt{\frac{T_1^2}{(T^2 - T_1^2)^2 + \frac{K^2}{\pi^2} T_1^2 \cdot T^4}}$$

From experiments on complex structures it had been found that $K \propto N^2$ approximately. In *Fig. 56* the figures corresponded to

$$K = 10^{-7} \cdot N^{1.76}.$$

That curve related to a structure in which excessive vibration was ex-

Fig. 56.



perienced—caused by a large gas-engine revolving at 98 revolutions per minute. The curve showed the calculated resonance compared with observed readings and the peak resonance between 97 and 99 frequencies per minute was acute.

Dr. Knopp considered that further research and investigation into the natural resonant frequencies of fully stiffened or monolithic structures was a matter of importance, and that, until results were obtained, the

determination of the desired characteristics of resilient foundations would remain a matter of systematic guesswork.

Professor E. G. T. Liddell observed that it was an old observation that the fingertip could detect 1,500 frequencies per second from a rotating toothed wheel. It was also known that muscles were sensitive to a stretch of less than one-hundredth of a millimetre. The sensitivity to vibration in daily life would obviously depend upon whether the subject was protected by thick carpet and was standing on it or sitting on a padded chair on the carpet. If he were lying on his back in bed on a spring mattress, the bed itself being on a bare wooden floor, sensitivity might be very evident, and Professor Liddell's experience was that it was evident. He hoped that the Authors might be able to get the problem pursued further, possibly in collaboration with the Building Research Station.

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Dr. A. J. Ockleston observed that the growth of the forced vibration of a damped system (p. 5) might, perhaps, have been dealt with more fully. It was sometimes not appreciated that when the frequency of the disturbance was of the same order as, but not equal to, the natural frequency of the system there was a pulsating growth of amplitude due to beats between the free and forced component vibrations. As a result the amplitude, after one or two cycles, might exceed the amplitude of steady forced vibration and be little less than that due to the same number of cycles of a synchronous disturbance.

The velocities of stress waves in concrete given in Table I appeared to be based on the Code of Practice values for the elastic modulus of 1 : 2 : 4 ordinary grade and 1 : 1 : 2 special grade concrete. Those corresponded with the total (elastic and plastic) strains due to sustained loading. The higher initial tangent values of the elastic modulus would probably give a closer estimate of the stress-velocities.

Did the statement (p. 15) that "two harmonics below . . . those listed in Table III have been noted," imply that the natural oscillation usually excited was a higher mode, and that the fundamental mode of vibration was seldom induced? Seismograph records of horizontal earthquake motion had shown, in different localities, prevailing frequencies ranging from about 1 cycle to 10 cycles per second. Those had been considered to be the result of natural vibration of the ground.

With regard to the mode of vibration of buildings (p. 23), was the dynamical distortion similar in form and extent for all types of ground, or did the mode of vibration of a building depend upon whether it was founded on soft soil or, say, rock? Some seismograph records taken during earthquakes had indicated that there was no appreciable difference between the motion of the foundations of a building and that of the adjacent ground.

For buildings subject to ground oscillations of the frequencies indicated in *Fig. 41*, the Authors' suggestion of resilient foundations appeared to have practicable possibilities. For earthquake-resistant structures that

type of construction seemed to be less suitable, since in some earthquakes large accelerations had been given by waves having frequencies of only 0.2 cycle per second. It might, however, be possible to obtain a very low natural frequency of horizontal oscillation by using rollers between the foundation and the superstructure in conjunction with relatively soft springs to control the horizontal movement. Some form of weak link which could transmit the wind forces, but would fail when subject to the seismic loads, might be necessary.

Mr. F. W. Slatter observed that his firm had recently carried out investigations with a view to ascertaining the cause of settlement of a reinforced-concrete framed building which was constructed in 1936 on pads of cork, to alleviate vibration from road and rail traffic.

The pads consisted of pieces of cork in its natural condition, 1½ inch wide and 6 inches deep, laid to cover an area 6 inches wider than the column base: the grain of the cork was at right angles to the axial load.

When some of the cork was removed it was found to have been compressed permanently about 1½ inch adjacent to the edge of the column base, and it was tested in a laboratory under a load of 100 lb. per square inch—approximately the applied load from the column from which the samples were taken. The following results were obtained:—

Duration of applied Load :	Compression : inches.	
	Sample A.	Sample B.
5 minutes	1.84	2.77
10 "	1.91	2.82
30 "	2.02	2.90
2 hours	2.12	2.98
5 "	2.19	3.02
24 "	2.32	3.10
48 "	2.34	3.14

The density of the cork was approximately 9½ lb. per cubic foot, and according to Mr. W. A. W. Lankshear, of the Corkboard Information and Research Bureau, cork of that density should be loaded to only slightly over 40 lb. per square inch.

The samples which were tested, however, had practically all recovered from the effect of the compression tests, whilst the effect of the compression due to the weight of the building could still easily be seen.

When the cork was first removed it was found to be very damp along its lower edge, and appeared to be decomposing. With very slight rubbing it could easily be crumbled away and the decomposition, in part, seemed to be due to fungi. The samples had, however, been allowed to dry out under natural conditions, and the cork had once more returned to its natural state.

Professor Julio Valenzuela observed that the Paper formed a valuable contribution to the knowledge of vibratory phenomena in structures.

In dealing with earthquakes the Authors had been influenced by the

fact that they lived in a country free from such disturbances and had therefore not made a detailed study of that matter. Their mathematical analysis provided an excellent starting-point for the analysis of the vibration of structures during an earthquake. It was particularly important that they had not only considered the transmission of vibratory energy from the subsoil of the foundation to the structure itself, but also cases of resonance such as those in which the period of vibration of the soil was substantially different from that of the structure.

In dealing with the problem of vibrations produced by man, such as those due to traffic, forging-hammers, etc., the Authors had presented an erudite review of the procedure necessary to reduce the harmful effects which such vibrations were likely to have upon structures and on plant.

The Paper should help engineers towards a better knowledge of the problems concerning the vibration of foundations; the Tables giving practical results achieved by the Authors were very valuable.

The Authors, in reply, agreed with Dr. Chatley that pile-driving was a dynamic or vibrational and not a static problem; they believed that only if the driving methods could be properly related to the natural oscillation conditions of the ground would some satisfactory pile-bearing expression be obtained. Design methods so developed would necessarily include the effect of reflected stress waves, etc., within the pile and in the several adjacent strata. None of the existing pile bearing formulæ appeared to be linked with the dynamic properties of the ground, although Glanville, Grime, and Davies¹ had considered that problem in the piles.

Dr. Chatley's suggestion for the use of seismic methods in new and old foundation investigations had already been started by the Degebo in Germany and by Bergstrom and Linderholm in Sweden²; the results so far obtained were very promising and linked up with borehole work. The Authors had been told by the Swedish investigators that they hoped to instigate further research in that useful field.

Dr. Chatley's experience that "wind gusts with fairly constant periods . . . seemed to be rare" did not matter; a condition of near resonance could be almost as dangerous as true resonance, so that the "tuning-range" was fairly wide, particularly for the more complex structures. As an instance, one of the Authors had found that the frequency of a certain large machine was 12 cycles per second and that of the ground 16 cycles per second; serious near resonance occurred, even with that difference of 25 per cent. Professor H. A. Williams,³ working at Harvard University

¹ "The Behaviour of Reinforced-Concrete Piles during Driving." J. Instn Civ. Engrs., vol. 1 (1935-36), p. 150 (Dec. 1935).

² Translated copies of these reports may be seen in the Ministry of Works Library; the originals of the former are in the Science Library.

³ "Dynamic Distortions in Structures subjected to Sudden Earth Shock," Trans. Amer. Soc. Civ. Engrs (1937).

on models and also on water-towers, had found similar conditions in the zone of resonance and near resonance.

In reply to Mr. Colquhoun, the Authors believed that Mr. W. C. Andrews, M.I.C.E., Mr. Crockett, and Mr. James Duff, A.M.I.C.E., were the first to use the term "Soil Dynamics" to denote the oscillational behaviour of soil. Some German investigators and others had used it to denote the settlement of structures, but that use was clearly wrong, because "settlement" was exactly equivalent to creep and elastic failure of structures, their elements, and test-pieces under normal static loading.

Only limited research had been carried out so far on the distribution of dynamic stress-waves around the bulb of pressure, although much theoretical and practical knowledge existed of the more remote stress-waves in all three media, gas, liquid, and solid. The Authors were working on the results obtained by Andrews, Crockett, and Duff and hoped to publish them shortly. Ample scope existed for research into the relationship between the frequency of vibration of any material confined within boundaries, as, for example, freshly-poured concrete confined within its shuttering, or the compaction of earth embankments; a few aspects had been studied by the Degebo and others, and the Building Research Station wished to investigate the vibration of concrete. The vibration settlement of foundations would be discussed by the Authors in a forthcoming Paper; but at present it was only possible to make a good guess at the sub-foundation ground dynamic action.

The Authors were indebted to Mr. Duff for his information on the relative velocities of the three main types of waves, and they wished to add that the velocity of the Love, or horizontal shear, wave came between the "S" and the Rayleigh waves. Much of the work carried out by Crockett and Duff compared favourably with the Degebo investigations, and would be considered in detail shortly in other Papers. In the United States Professor Don Leet had measured ground vibrations produced by a portable paving breaker, and the Authors considered that the records he had obtained of the ground movement showed that the use of such a source of original impulse for purposes of testing would be very suitable. Perhaps a "pom-pom" soil rammer would also be good, but that had not yet been tried out.

Cork did not expand outward under compression, mainly owing to cellular compression, as shown under test conditions.

The Authors intended to convey in the Paper that, in their opinion, it was necessary to assess the wind-gust frequency individually for any new site where the information was not already available. They regarded that as important for the comfort of the occupants of the building, if not for the structural behaviour as such; even light-weight buildings of fairly low stiffness were often quite uncomfortable in a heavy gale, when there was certainly no resonance with the wind-gust period.

In reply to Mr. Eastmond, the Authors said there was a very loud

air-borne noise from stone-crushers and gravel-screens) which was not the subject of the Paper). Stone-crushers of the impulsive jaw type produced individual shocks into the ground, but gravel-screens induced a more steady train of oscillations, so that remedies would have to be considered for the appropriate type as described.

Mr. King had made some very interesting suggestions, particularly that the electrical analogy might be used to show what was wanted in any part of a foundation. The Authors agreed that the problems were complex and tedious rather than inherently difficult. Most engineers had neither the time nor the energy to employ laborious methods, so that any simplifications which did not reduce the effectiveness below a reasonable accuracy would be welcomed. Most building structures consisted of many different materials having non-linear coefficients, whilst the ground was also non-linear in its behaviour under and around a foundation; thus at present the complexities were great and unmanageable. The Authors would be glad to know of any further work which the Admiralty Laboratory might carry out in that important field, and would welcome the possibility of applying the results obtained to the problems of ground and structural vibrations.

In reply to Dr. Knopp, the Authors considered that any mathematical investigation into the oscillational behaviour of a modern building would be quite impossible at present. Work had been carried out at the National Physical Laboratory and at the Building Research Station for many years, but that had had a largely experimental basis. It was difficult to see how the position could be altered for structures consisting of a mixture of steel, concrete, brick, lime, cement, timber, plaster, asbestos and steel sheets, and many other materials, particularly when it was remembered that much of the constructional work and subsequent maintenance would of necessity have to be carried out by unskilled labour. The Authors agreed that "the desired characteristics of resilient foundations would remain a matter of systematic guesswork"; but they considered that that should be guided at every step by instrumental investigations together with the two chief principles: (*a*) where the wave-lengths were larger than the dimensions of the structure (the oscillational problem), and (*b*) where the wave-lengths were shorter (the reflection, refraction, and diffraction problem). They also wished to thank Dr. Knopp for pointing out that a structural beam had its natural frequency controlled by shear as well as by bending, and that the final result was not proportional for the higher harmonics. The springy-beam type of engine mounting had been successfully used many times, and that method appeared to have many practical advantages.

Particular thanks were due to Professor Liddell for his bibliography on the subject of Vibration Sense. One of the Authors had been kept awake at night by his bed being nearly in resonance with the ground vibrations set up by a large drop-hammer a mile away. Details of human perception

to industrial vibrations, if known, would be of great value to the engineer, and would doubtless become extremely important in legal actions, where it was often difficult to assess damage or annoyance inflicted on nervous people by vibration.

Mr. Slatter's information on the deterioration of some cork under a building was very interesting and clearly indicated that care should be taken against fungi. It also stressed that provision should be made for renewing such resilient materials at intervals.

The Authors agreed with Dr. Ockleston that the forced vibration of a damped system could have been dealt with at greater length; but suitable books had been listed in the bibliography. They were well aware that when the steady frequency of a disturbance was not in resonance, but was in near resonance, with the natural frequency of the system, beats occurred as a result of the addition of the free and forced vibrations. A record taken by Andrews and Crockett had been mentioned in the Authors' reply to Dr. Chatley.

Ground vibrations when originating from a single impulse, were somewhat aperiodic in each of the three groups, namely that of beginning ("P" and "S"), middle (mainly Rayleigh), and the final dying-out phase, which made the beat and other actions still more complicated. For those reasons, the Authors had recommended design engineers to keep well away from that troublesome "tuning-zone" by making the natural frequency of the structure, where possible, not greater than one-third of the inducing frequency.

They agreed that the higher initial tangent value of the elastic modulus for concrete would have been more suitable for computing stress-wave velocities.

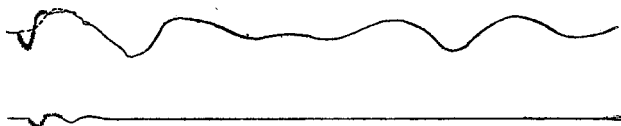
With regard to the natural oscillation of the ground, the Authors had no knowledge of the fundamental, lowest, or basic frequency of a particular ground, site, or stratum. It was clear that a range of possible frequencies, having a simple harmonic relation, had existed on those sites measured by Andrews, Crockett, and Duff and by the Authors. They had used the term "most stable self-frequency" to denote the frequency which was most often excited by impact. It seemed reasonable to suppose that the lower self-frequencies might be those occurring in earthquakes. The most stable self-frequencies induced by impact, measured by Andrews, Crockett, and Duff, using impact, were identical with the frequencies measured by the Degebo for similar types of ground when an oscillator was used; no other frequencies appeared to have been recorded when that method was used by the Degebo.

The Authors believed that dynamic distortion of ground took place adjacent to building foundations, because the modes of a building were mostly different in frequency from those of the underlying and adjacent ground, thus requiring some zone of distortion to provide the necessary connexion. Such a zone had been noticed, but not explored, by the first

Author around very large machines, comparable in size with small and medium buildings. Since it now appeared that there was a range of harmonically connected self-frequencies in the vertical mode of oscillation, that strongly suggested that there might be translational and rotational modes in the remaining five degrees. Observed behaviour of machine foundations appeared to confirm that. Presumably any bulging, or distortion, would be related to all of those modes and to the particular type of ground. With regard to the question of whether the mode of vibration of a building depended upon the type of ground upon which it was founded, the Authors had tried to indicate in the Paper that they believed that the ground/building system was a coupled mechanism, in which the ground bulb and the foundation formed one component and the superstructure of the building the other. That suggested that the combined mode would be different, or would at least have a different set of frequency responses, if the building were founded on soft soil or on rock.

In an earthquake where the wave-lengths were very great a particle on or just below the surface had extremely small rotational movement, which

Figs. 57.



would hardly be indicated by the ordinary translational measuring seismograph. Any translational movement of the ground which momentarily shifted a foundation might distort and bend the building structure immediately above it, particularly if the building was tall and narrow, thereby causing the foundation to be rotated appreciably, the amount of rotation being related to the bending couple thus set up and to the physical characteristics of the subsoil, such as soft soil or rock. If that rotation were centred about ground-level, then horizontally recording seismographs placed at that level might be expected to show much the same translational movements as other seismographs outside the zone of local disturbance connected with the building, and not to indicate rotational movements.

As an illustration of the rotational action, a machine weighing about 350 tons was set into oscillation by an eccentric blow so that rotational oscillations were induced as well as purely vertical ones. Another identical machine on a rather similar foundation was also given a perfectly central blow (actually slightly less intense); the vertical vibrographs for both were given in *Figs. 57*. (Both machines rested on similar thick and very stiff springs of timber on top of the concrete foundation.) In the second case the rotational natural frequencies of both machine and the foundation/ground systems were not near the resonance zones of each other, nor did the

central blow excite those oscillations appreciably. In the first case, however, the natural frequencies of both of those rotational modes were not far from each other and both came into action. The vertical vibrograph was taken at a point on the side of the machine nearly level with the mass centre and centre of rotation (which were not quite the same, owing to the method of support by the timber spring), so that both the purely vertical oscillation of the machine and the vertical displacement at that point due to rotation appeared superimposed. The former could be seen to be almost the same as that for the similar second machine, and quickly damped out, whilst the rotational oscillation continued for some time—about 2 seconds. That showed a distinct beat action of such amplitude (shown full size) that it could not have been produced with any other mechanism than that of the heavy machine itself, of 350 tons, and the foundation/ground of more than 2,000 tons. Similar actions had been noticed in other cases, which had made the Authors reasonably satisfied of the existence of a self-rotational frequency of the ground/foundation system.

The Authors wished to thank Professor Valenzuela for his kind remarks and for his expression of opinion that the Paper would prove a help to engineers towards a better knowledge of vibrations in foundations, and their effects.

MARITIME AND WATERWAYS ENGINEERING DIVISION.

Maritime and Waterways Paper No. 8.†

“The Problem of Coast Erosion.”

By JACK DUVIVIER, B.Sc.(Eng.), M.I.C.E.

Correspondence.

Mr. T. W. Marsh observed that the sketch picture of the English coast given by the Author was inevitably unduly brief. A very comprehensive study had recently been published by Mr. J. A. Steers.¹

The Author had referred several times to the travel of beach material as being due to the tidal currents and wind waves. Whether the tides or the waves formed the cause of the transportation of shingle was an old controversy which had enlivened many engineering and geological discussions in the past. The matter had been summarized by Mr. Marsh in 1924 in a Paper presented to The Institution² the substance of which had been published in 1925.³ The travel of shingle on the beach itself

† Discussed at Meeting on 25 March, 1947.

¹ J. A. Steers. “The Coastline of England and Wales” (Cambridge University Press, 1946).

² T. W. Marsh. “The Littoral Drift of Shingle,” Instn Civ. Engrs, Student’s Paper No. 785 (1924).

³ “Alongshore Drifting,” *Proc. Geol. Assoc.*, vol. xxxvi (1925), p. 434.