

Evaluation of virtual fixed points in the response spectrum analysis of a pile-supported wharf

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Response spectrum analysis is widely employed in the seismic design of pile-supported wharves, as it can easily determine the maximum response of a structure. The analysis typically uses virtual fixed points to model the structure, without modelling the ground, as the structure is assumed to be a frame. Virtual fixed points can adequately simulate pile responses under horizontal static loads; however, pile responses under seismic loads have not been well studied. Therefore, the dynamic centrifuge tests evaluate the applicability of response spectrum analysis, using virtual fixed points, to the seismic design of a pile-supported wharf. Comparison of results from scale models (3 × 3 groups of piles from sections of Pohang New Port, Korea) with those from the analysis showed that the response spectrum analysis using virtual fixed points does not adequately simulate the natural period or pile bending moment of a pile-supported wharf system.

KEYWORDS: centrifuge modelling; earthquakes; standards & codes of practice

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NOTATION

D	pile diameter
EI	bending stiffness of the pile
E_m	elastic modulus of the model pile
E_p	elastic modulus of the prototype pile
I_m	moment of inertia of the model pile
I_p	moment of inertia of the prototype pile
K_h	coefficient of horizontal subgrade reaction
N	average N -value of the ground, from its surface to $1/\beta$, as determined by standard penetration testing
n	scale factor applied to the experiment

INTRODUCTION

Elastic analysis using response spectra can calculate the maximum response of a structure based on the significant modes of its response spectrum curve (Kiureghian, 1981; Kiureghian & Neuenhofer, 1992; CEN, 2005). This approach is simple and therefore widely used in evaluating the performance of bridges and other structures. It also can combine multiple modes (CEN, 2005; Su *et al.*, 2006; Taghavi & Miranda, 2010), most typically by a complete quadratic combination (Wilson *et al.*, 1981; Jiahao, 1992; Su *et al.*, 2006).

The response spectrum analysis of a pile-supported wharf generally models each pile with a virtual fixed point or by using a spring method and does not model the ground (PIANC, 2001; PARI, 2009). Using virtual fixed points is similar to the equivalent cantilever model used to

approximate the static lateral pile-head response of an actual soil–pile system (Nair *et al.*, 1969; Chen, 1997; Chiou & Chen, 2007). This method uses a line corresponding to one half of the inclined angle of the ground surface as a virtual ground surface. It is assumed that the virtual fixed point is located at a distance $1/\beta$ below the virtual ground surface (PIANC, 2001; PARI, 2009), where β is calculated as follows

$$\beta = \sqrt[4]{\frac{K_h D}{4EI}} (\text{cm}^{-1}), \quad (1)$$

$$K_h = 0.15 N_{\text{ave}} (\text{N/cm}^3) \quad (2)$$

Virtual fixed points are widely used and can adequately simulate pile responses under horizontal static loads; however, little attention has been paid to pile responses under seismic loads (Nair *et al.*, 1969; Chen, 1997; Chiou & Chen, 2007). Applying a seismic load to a virtual fixed point model encounters the following problems. First, adequate simulation of the natural period of the actual soil–pile system is difficult, because this method models piles as a frame using virtual fixed points without considering the ground. Furthermore, if the natural period of the model structure is different from that of the actual structure, the actual responses might differ greatly. Second, because the ground is not modelled, an amplified seismic wave should be used as the input spectrum; however, different codes use different input ground acceleration positions, leading to confusion in determining the appropriate input ground acceleration considering the amplification (MOF, 1999; PIANC, 2001; PARI, 2009; MLTM, 2012).

To evaluate and analyse these issues, this study reports the results of the response spectrum analysis and dynamic centrifuge tests. Input ground accelerations for the analysis were obtained at different depths based on the results of the dynamic centrifuge tests, and the response spectrum analysis with virtual fixed points was then performed using these input ground accelerations. The natural period and pile moment results of the analysis and centrifuge tests are compared.

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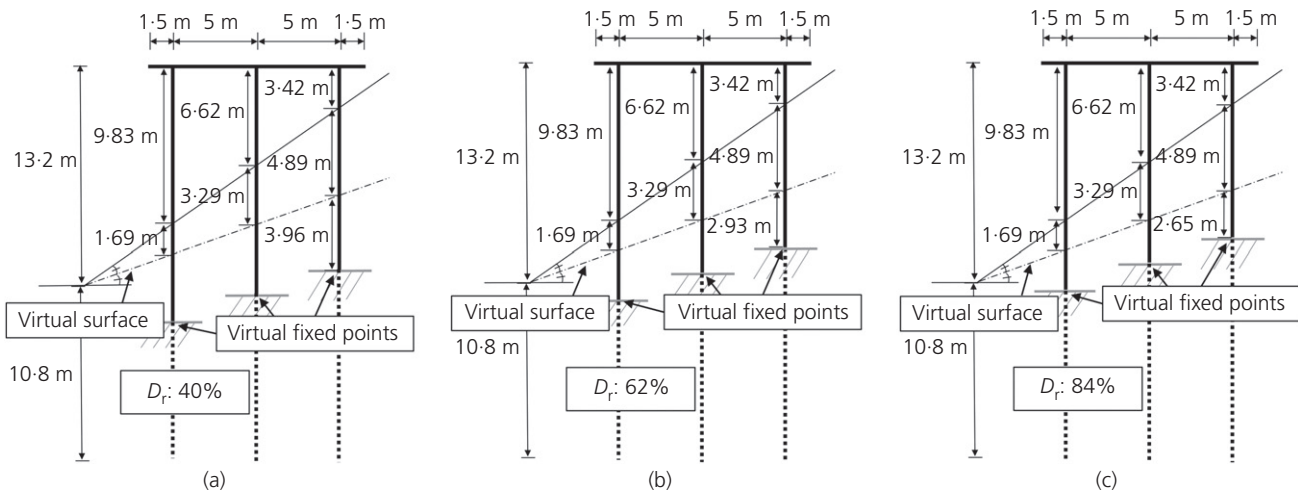


Fig. 1. Dynamic centrifuge test models and virtual fixed points for response spectrum analysis: (a) IA40 model, (b) IA62 model and (c) IA84 model

CONDITIONS FOR DYNAMIC CENTRIFUGE TESTS AND RESPONSE SPECTRUM ANALYSIS

Experiments were conducted at the Geo-Centrifuge Testing Center at KAIST, Daejeon, Korea. The geo-centrifuge testing machine has a radius of 5 m and can operate at a maximum of 240 g-ton (Kim *et al.*, 2013). The model box used for the experiment was an equivalent shear beam box 0.49 m long, 0.49 m wide and 0.63 m high (Lee *et al.*, 2013).

The models for the dynamic centrifuge tests were 3 × 3 groups of piles scaled from sections of Pohang New Port, Korea. The piles were fixed to the base plate to simulate the end bearing pile, and the ground composition was then added. The tests used a simplified ground condition comprising a sand surface sloping at 33° and adjusted to different relative densities (40, 62 and 84%; Fig. 1). Each model was at 1/48 scale, and the bending stiffness of the pile was determined using equation (3) (Schofield, 1981; McCullough *et al.*, 2007; Taylor, 2014). The model pile and deck were aluminium (A6063; $E_m = 68\,300$ MPa; Poisson's ratio, $\nu = 0.3$) (Table 1).

$$\frac{E_p I_p}{E_m I_m} = n^4 \quad (3)$$

Table 2 lists the properties of the silica sand used as the ground soil. The average particle diameter was about 0.3 mm, and the material was classified as poorly graded sand according to the Unified Soil Classification System (USCS). Its relative density was controlled through air pluviation, and the slope of the ground was formed using a vacuum machine. Four laser displacement meters, 17 accelerometers and 42 strain gauges were used as shown in Fig. 2.

Table 1. Properties of prototype and model (scaling factor, 48)

		Prototype	Model
Pile	Diameter: mm	914	19
	Thickness: mm	14	1
	Length: mm	2400	50
	Density: kN/m ³	78.5	26.9
	Flexural rigidity: kN m ²	8.42×10^5	0.157
Deck	Thickness: mm	1000	20
	Density: kN/m ³	24.5	26.9

Table 2. Properties of the silica sand

Soil type	Silica sand
USCS	SP
C_c	1.16
C_u	1.96
C_s	2.63
γ_{dmax} : kN/m ²	16.5
γ_{dmin} : kN/m ²	12.4

The input motion was an artificial wave with 5% damping ratio, produced as proposed by the Korean Ministry of Oceans and Fisheries (MOF, 1999) (shown in Fig. 3(a)). Figure 3(b) shows that the response spectrum curve of the artificial wave closely matched the Korean standard design response spectrum curve (MOF, 1999). The amplitude of the input acceleration was in the range of 0.04–0.23g (Table 3).

As described above, the response spectrum analysis used virtual fixed points, applied using equations (1) and (2), as shown in Fig. 1. The virtual fixed point locations were calculated using the relationship between the relative density and the N value suggested by Meyerhof (1956) (Table 4). The input properties for the response spectrum analysis are the same as those used for the prototype structure (Table 1). The finite-element analysis program Midas GEN 2016 ver. 1.4 was used for the response spectrum analysis (Midas FEA, 2016).

In general, the design standards propose to model the pile as a frame structure by using virtual fixed points without considering the pile–soil interaction and then propose to conduct the response spectrum analysis for the pile-supported wharf. However, in the actual ground, seismic waves are amplified during an earthquake, and for this to be taken into account, seismic response analysis should be performed using an amplified seismic wave as the input wave.

MOF (1999) and PIANC (2001) suggested that a one-dimensional seismic response analysis be performed during the response spectrum analysis to obtain the surface acceleration. In contrast, PARI (2009) and MLTM (2012) used the seismic response acceleration obtained at the depth of the virtual fixed point ($1/\beta$) from a one-dimensional seismic response (Fig. 4).

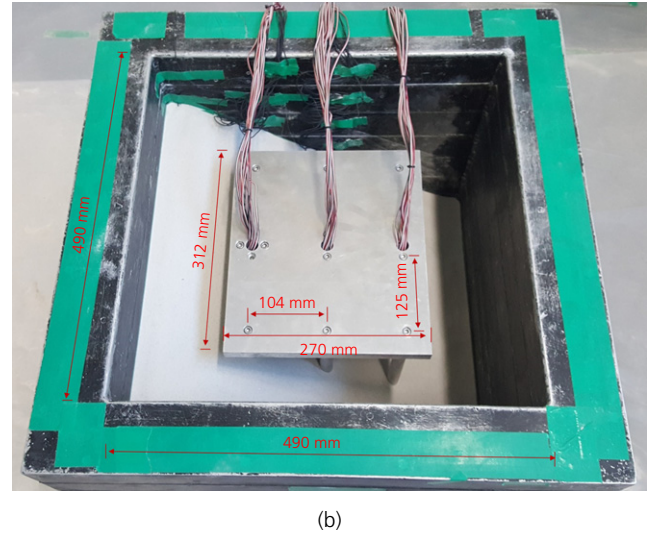
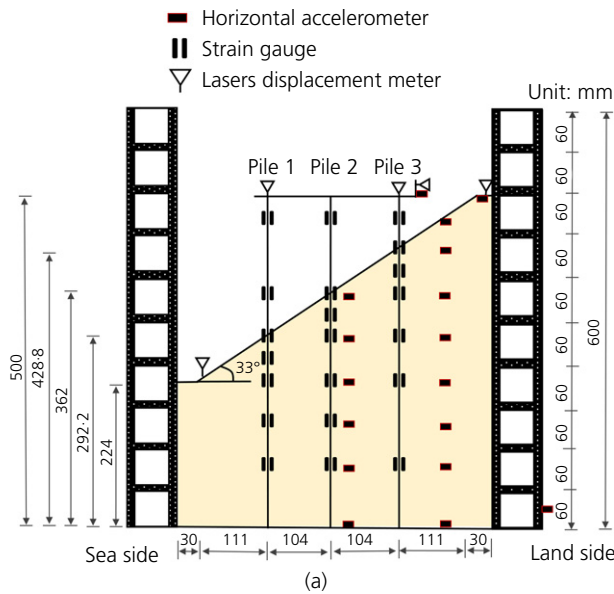


Fig. 2. Experimental model: (a) section view and (b) plan view

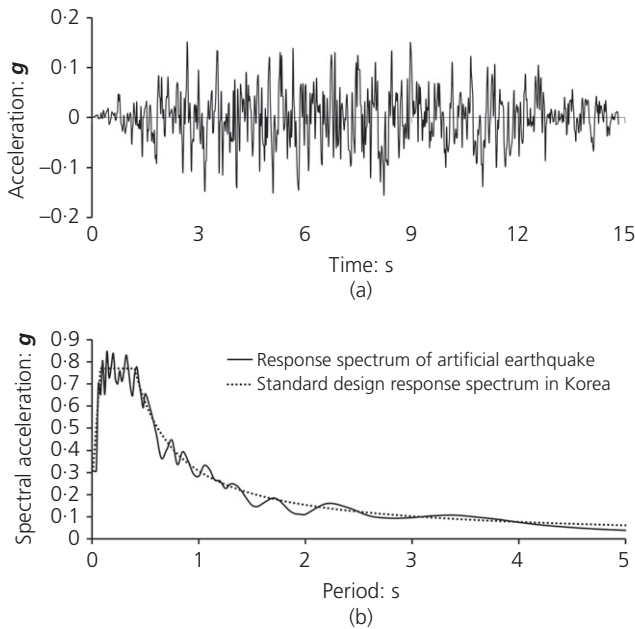


Fig. 3. Input seismic wave: (a) artificial earthquake wave and (b) comparison of response spectrum

Table 3. Experimental parameters for pile-supported wharf models

Model	IA40	IA62	IA84
Relative density: %	40	62	84
Input acceleration amplitude (g)	0.04 0.12 0.16 0.18 0.23	0.05 0.12 — 0.18 0.21	0.09 0.15 — 0.17 0.23

Figure 4 also represents the peak ground acceleration (PGA) at various depths derived from the dynamic centrifuge model tests. The PGA was about 0.12g in the bedrock, 0.151g near the virtual fixed point and 0.239g in the upper

ground. The analysis used the response spectrum curves derived from the accelerations obtained during the dynamic centrifuge model tests.

COMPARISON OF THE NATURAL PERIOD BETWEEN TESTS AND ANALYSIS

The natural periods of the prototype in the dynamic centrifuge tests and the virtual fixed point model for the response spectrum analysis were compared. The natural period of the centrifuge model was calculated using the acceleration response spectral ratio, which estimates the peak value as the peak acceleration response spectrum curve divided by the base acceleration response spectrum curve (Gazetas, 1987; Laurendeau *et al.*, 2013; Gazetas *et al.*, 2016; Ha *et al.*, 2017). In this study, the acceleration response spectrum of the deck (upper plate) and bedrock (base plate) were calculated to predict the natural period of the test model.

Eigenvalue analysis (Midas GEN 2016 ver. 1.4) was used to estimate the natural period when using virtual fixed points through a series of processes for solving the differential equations of a structure with multiple degrees of freedom. It can easily obtain the mode shape based on each vibration mode (Midas FEA, 2016).

Figure 5 compares the results of both methods with input accelerations of 0.21g and 0.23g as examples. For the dynamic centrifuge test, the soil–pile interaction was simulated, and for the response spectrum analysis, virtual fixed points were used to simulate the frame structure without modelling the ground. The natural periods of the centrifuge tests were 0.5–0.6 s, compared with 0.66–0.78 s for the response spectrum analysis. The shorter natural periods in the centrifuge tests were due to the consideration of the soil–pile interaction, the ground confining pressure and ground rigidity. The comparison of the two sets of results shows that the natural period consistently increased with decreasing relative density of ground. Furthermore, Fig. 5 shows that as the natural period increased, the spectral acceleration (i.e. the input acceleration) decreased, and thus that the response spectrum analysis underestimated the dynamic response. This trend is consistent for the response spectrum curves at the bedrock, the virtual fixed points and the ground surface. Therefore, the responses of the analysis

Table 4. Relationship between relative density and *N* value

Consistency	Very loose	Loose	Medium	Dense	Very dense
Relative density	<0.2	0.2–0.4	0.4–0.6	0.6–0.8	>0.8
<i>N</i> value	<4	4–10	10–30	30–50	>50

Source: Meyerhof (1956).

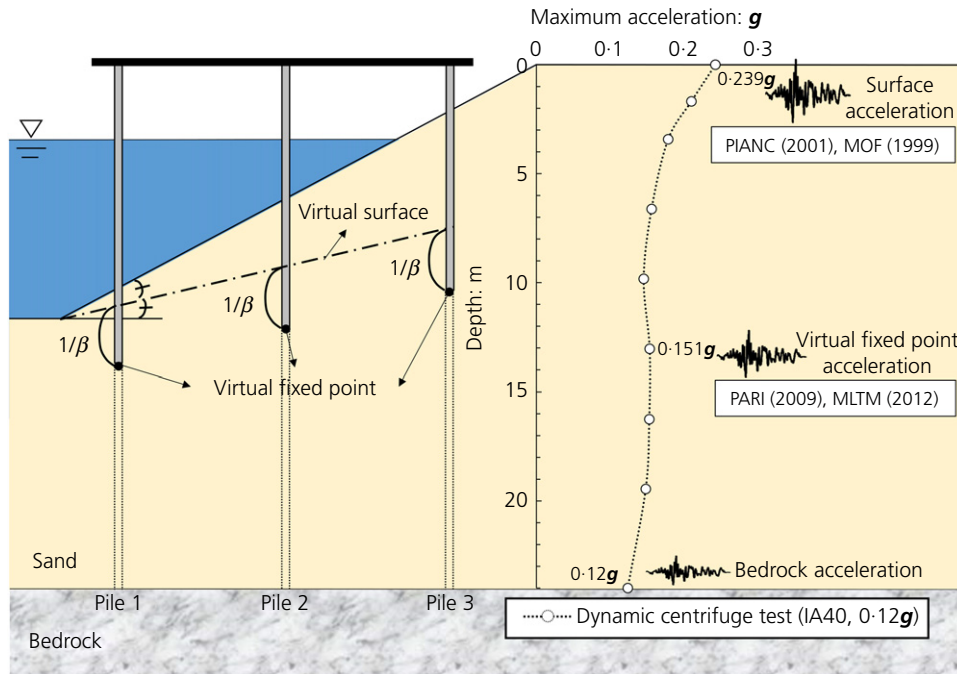


Fig. 4. Determination of input ground acceleration and amplified acceleration (IA40 model, input amplitude 0.12g)

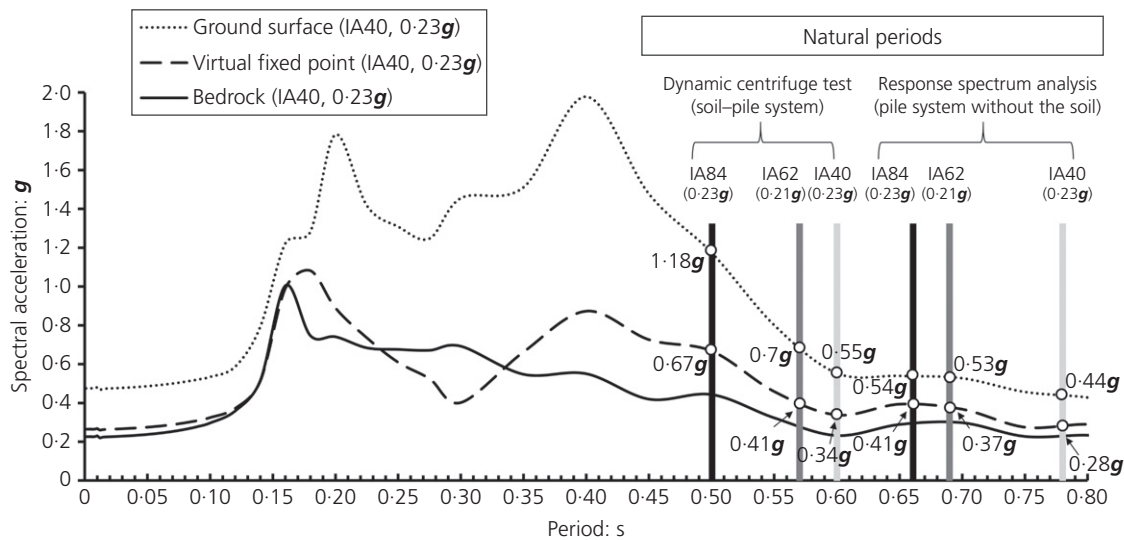


Fig. 5. Spectral accelerations with respect to depth (IA40, 0.23g) and the natural periods of the system in tests and analysis (0.21 and 0.23g)

using virtual fixed points were smaller than those of the centrifuge tests.

COMPARISON OF THE PILE MOMENT BETWEEN TESTS AND ANALYSIS

As piles can be severely damaged by the lateral response during ground shaking, evaluation of the pile bending

moment is generally important for a pile-supported wharf. Figure 6 shows the moment of pile 3 in Fig. 4 with respect to depth when the maximum moment occurred in the dynamic centrifuge tests and the response spectrum analysis at a prototype scale. The analysis applied the amplified acceleration at the virtual fixed points proposed by PARI (2009) and MLTM (2012), and at the upper ground surface proposed by MOF (1999) and PIANC (2001). In the

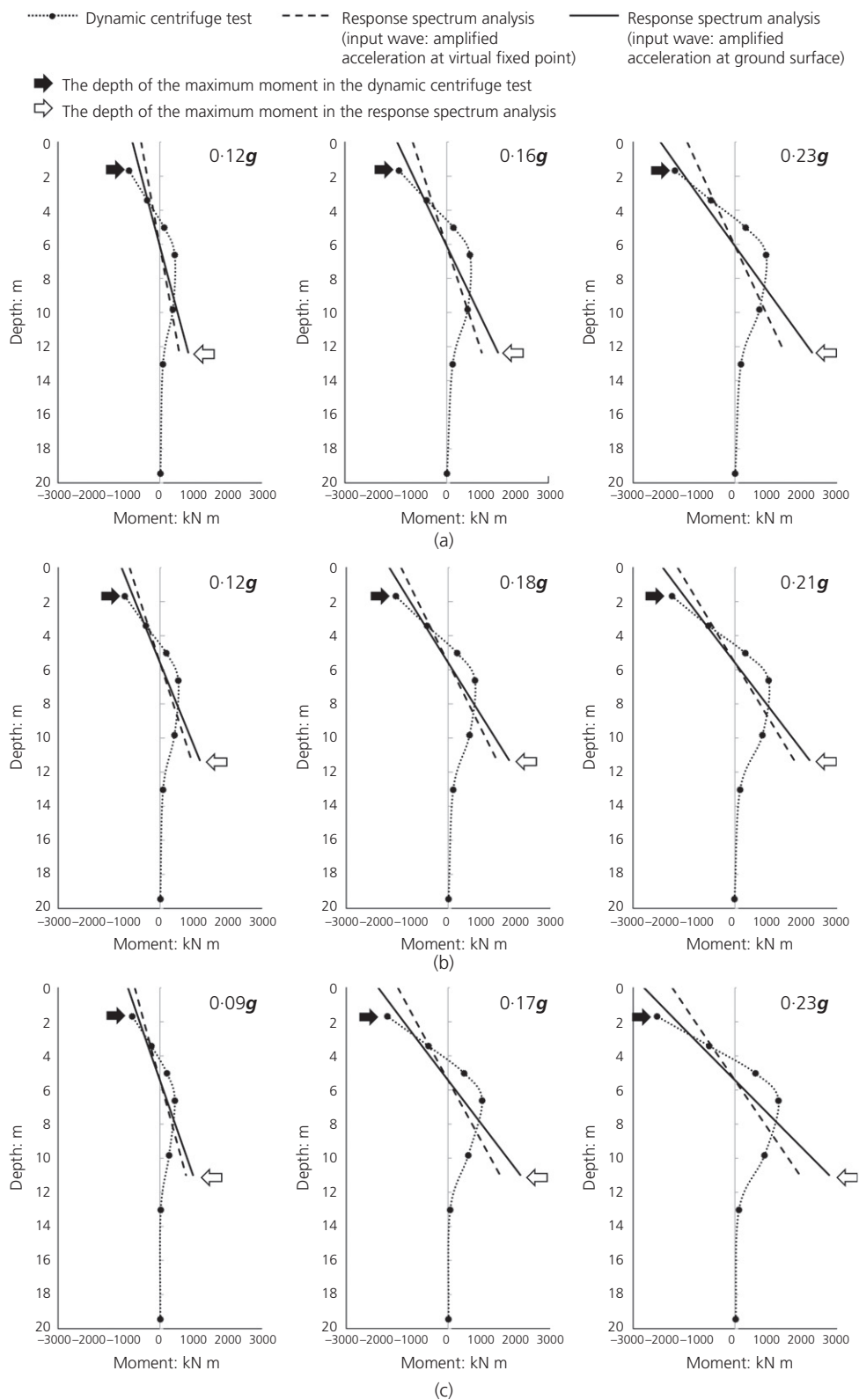


Fig. 6. Pile moments from dynamic centrifuge tests and response spectrum analysis (at prototype scale, pile 3): (a) IA40, (b) IA62 and (c) IA84

dynamic centrifuge tests, the maximum moment occurred at the top of the pile, and then decreased to zero at a depth of about 14 m (Fig. 6). As the intensity of the input acceleration increased, the moment result also increased. Conversely, the response spectrum analysis revealed that the maximum moment occurred at the bottom of the pile, and then changed linearly, because the analysis used virtual fixed points without considering the ground.

Figure 7 shows the pile moment with respect to depth when the maximum moment occurred in the test and analysis at a prototype scale for different piles of the IA40 model (0.23g). The test and analysis results show the position of the maximum moment. With increasing ground height, the position of the maximum moment also increased. In addition, the moment value increased from pile 1 to pile 3 due to the largest kinematic force of the ground being in pile 3.

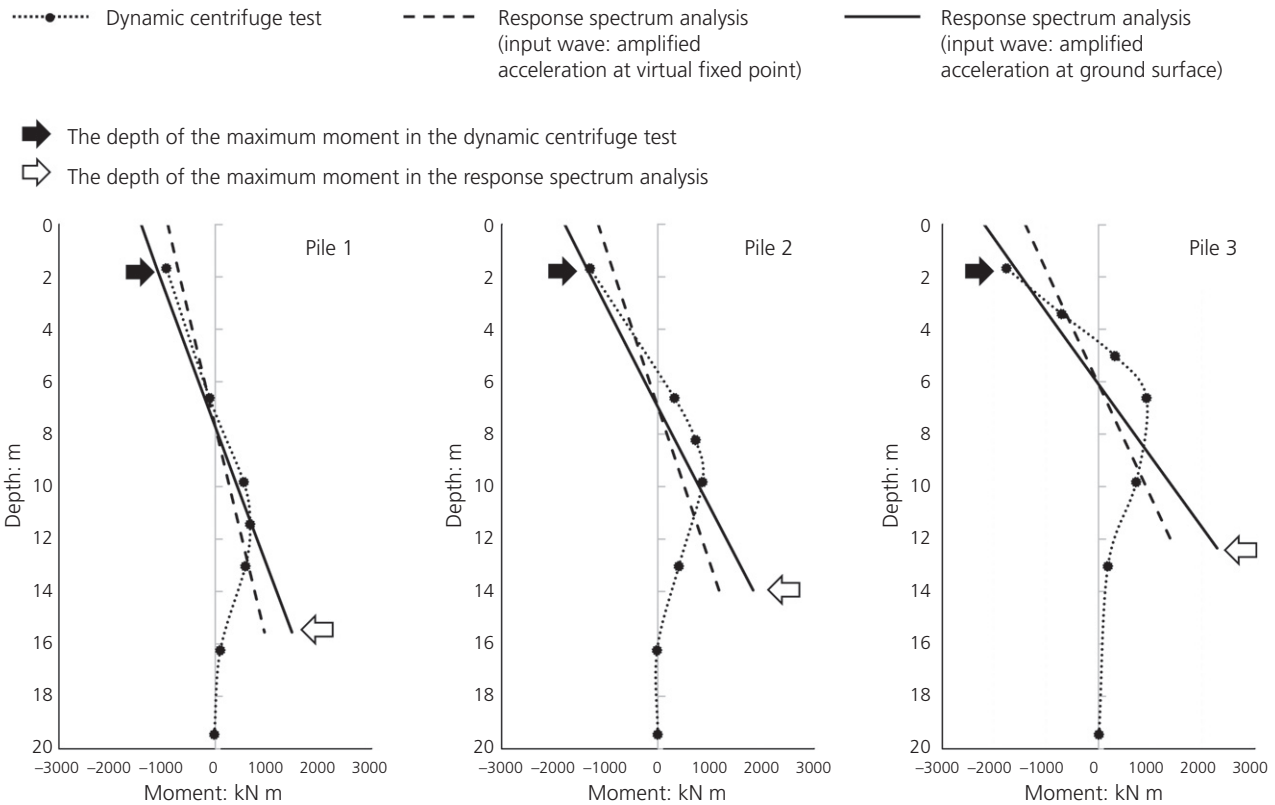


Fig. 7. Pile moments from dynamic centrifuge tests and response spectrum analysis (at prototype scale, IA40, 0.23g)

Table 5 compares the maximum pile moments obtained from the analysis and centrifuge tests. The analysis using amplified acceleration at the virtual fixed points yielded results 6–37% smaller than the centrifuge tests, because the natural period in the analysis was 0.15–0.2 s longer than that of the dynamic centrifuge tests. Consequently, spectral acceleration was reduced by 39% and the responses were underestimated (Fig. 5). In addition, the maximum pile moments of the analysis using the acceleration at the upper ground surface were 8–29% larger than those obtained by the dynamic centrifuge tests (except in three cases which were 6–8% lower), because the amplified acceleration at the ground surface was 32–57% larger than that at the virtual fixed point inside the ground where the pile was

embedded (Fig. 5). Analysis using amplified acceleration at either the virtual fixed point or the upper ground surface does not simulate the pile moment of the actual pile-supported wharf system. Overall, the response spectrum analysis using virtual fixed points appears unable to adequately simulate the pile response under the seismic load. Therefore, it is desirable to perform the analysis without using virtual fixed points and using a soil–spring model for an accurate seismic design.

CONCLUSIONS

Response spectrum analysis using virtual fixed points was compared with dynamic centrifuge model tests to evaluate

Table 5. Comparison of maximum pile moments (in prototype)

Model	Input acceleration amplitude (g)	Dynamic centrifuge test	Response spectrum analysis			
			Input seismic wave: amplified acceleration at virtual fixed point location		Input seismic wave: amplified acceleration at ground surface location	
			Moment: kN m	Moment: kN m	Difference: %	Moment: kN m
IA40	0.04	293.3	211.6	-28	277.0	-6
	0.12	907.4	572.3	-37	837.4	-8
	0.16	1404.6	1035.5	-26	1514.0	8
	0.18	1623.0	1190.1	-27	1759.1	8
	0.23	1767.8	1459.3	-17	2274.7	29
IA62	0.06	456.2	349.3	-23	428.3	-6
	0.12	1036.9	928.4	-10	1173.2	13
	0.18	1541.4	1434.1	-7	1795.1	16
	0.21	1852.3	1745.7	-6	2200.3	19
IA84	0.09	813.7	767.4	-6	974.6	20
	0.15	1441.5	1342.7	-7	1750.1	21
	0.17	1819.0	1531.9	-16	2132.4	17
	0.23	2296.5	1910.9	-17	2776.4	21

its applicability to the seismic design of pile-supported wharves. It could not adequately simulate the natural period of the wharf system including soil. In addition, analysis using amplified acceleration at the virtual fixed points proposed by PARI (2009) and MLTM (2012) yielded smaller pile moments than those measured in centrifuge tests. This result arose because the use of virtual fixed points increased the natural period and the pile moment values were underestimated due to the decrease of the spectral acceleration as the input acceleration. However, analysis using the acceleration at the ground surface proposed by PIANC (2001) and MOF (1999) generated larger moments than those observed in the centrifuge tests, because the amplified acceleration at the ground surface was larger than the amplified acceleration inside the ground. Overall, the response spectrum analysis using virtual fixed points cannot adequately simulate pile responses under seismic loads. Therefore, a soil–spring model should be employed to ensure accurate seismic design.

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