

the field of foundation engineering for a considerable time. I have tried various methods but none has been found absolutely satisfactory from every respect.

The easiest method of forming a loose sand bed seems to be by pouring dry sand through a funnel keeping the tip of the same at a constant height above the sand already poured. A dense bed can be produced if this sand is poured in layers and each layer is compacted using a tamper with a falling weight. The density can be varied by varying the thickness of each layer and the number of blows delivered per layer. The density was uniform as could be ascertained from penetration records obtained by tests using an arbitrary falling weight penetrometer. The sand used had effective size = 0.34 mm, uniformity coefficient = 1.53 and fineness modulus = 4.10. Loose density obtained by pouring was 97.2 lb/cu. ft ($n=41.5\%$) and a density of 105.2 lb/cu. ft ($n=36.6\%$) was obtained by delivering 36 blows/sq. ft on layers 3 in. thick using a tamper which had a base 6 in. \times 6 in., falling weight 2.5 lb and height of fall 18 in. This method even though satisfactory is time consuming.

To obtain better uniformity and speed of work an attempt was made to induce a draft on the soil in the funnel by letting in compressed air from an air compressor into the stem of the funnel in the direction of flow. The uniformity of the resulting bed was excellent but the density was found to be insensitive to air pressure and the maximum density obtained was 100 lb/cu. ft at a pressure of 60 lb/sq. in.

The best results in terms of maximum density and speed of work were obtained by using a concrete pin vibrator (2850 rpm) working on a three-phase supply. Densities of 104 lb/cu. ft could be easily obtained by centre vibrating a bed 3 ft square and 4 ft deep for 6 minutes. The needle was initially inserted to the bottom of the bed and it was found to be travelling up on being rejected by the bottom layers as they were getting compacted. A disadvantage of this method was that there was observed concentration of density around the point of vibration. To overcome this defect a concrete shutter vibrator (2950 rpm) working on a three-phase supply was tried by moving the same over sand poured in layers. The uniformity in the horizontal plane was excellent but for getting better uniformity in the vertical direction surface vibration had to be carried out on a number of layers. The results, however, were found to be quite satisfactory.

Attempts are now being made to employ the principle of fluidization using an aeropump for this purpose.

The problem of forming a large clay bed with uniform consistency at a reasonable speed is much more difficult and unless a suitable method is devised foundation research will continue to be confined to sand only.

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THE ANALYSIS OF THE STABILITY OF GENERAL SLIP SURFACES

(MORGENSTERN, N. R. & PRICE, V. E., *Géotechnique*, **15**, No. 1, 79-93)

In the correspondence by Spencer (1968) he disputed the claim made by the Authors that in this method all equilibrium and boundary conditions are satisfied. In particular he stated that the equations of vertical equilibrium are not satisfied. We believe that the arguments presented by Spencer are false for the following reasons.

The equation of equilibrium in two directions, N and S , given in equations (3) and (4) was combined with the failure criterion to give equation (7). Provided the force distributions E' and X satisfy equation (7) then normal and tangential forces dN' and dS on the base

of each slice can be found which satisfy equations (3) and (4) and which therefore satisfy the equilibrium condition in the vertical direction for every slice and hence for the complete body.

E_n and X_n are not the sums of the horizontal and vertical components of the resultants of the interslice forces. A claim to make them so would only be valid if the forces across the base and the body forces are ignored.

Spencer states that it is unreasonable to be able to satisfy the three conditions of equilibrium by adjusting the values of only two parameters, λ and F . However, by counting the total number of equations for each slice and the unknowns it can be seen, as follows, that two parameters are required.

Consider a body with n slices having vertical boundaries with co-ordinates x_0, x_1, \dots, x_n . Assume the normal conditions for a slip surface in a slope stability problem that the height of the body is zero at x_0 and x_n and that no external force is exerted at x_0 and x_n on the outer slices. Thus $E_0 = E_n = X_0 = X_n = 0$. The total number of equations to be satisfied is $4n$, namely the three equations of equilibrium for each slice and the equation of limiting equilibrium at the base of each slice. The unknown forces number $4n - 3$, namely the normal and tangential forces on the base of each slice and the inter-slice forces $E_1 \dots E_{n-1}$ and $X_1 \dots X_{n-1}$. The positions of the E forces introduce a further $(n-1)$ unknowns, y_i . Thus, at this stage there are $4n$ equations and $5n - 3$ unknowns. In order to make the problem statically determinate further assumptions about the internal forces have to be made. By assuming the relationship between E and X at $x_1 \dots x_{n-1}$ a further $(n-1)$ equations are introduced bringing the total number of equations to $5n - 1$. Hence a further two unknowns, λ and F , have to be introduced to make the total number of unknowns the same as the total number of equations.

As may be seen from the preceding argument it is not necessary to assume that the equation connecting E and X holds at x_n , but it holds at all points infinitely close to x_n and at such points neither $f(x)$ nor E tend to infinity. E tends to zero and $f(x)$ is finite thus giving a value of X which tends to zero as required.

REFERENCE

- SPENCER, E. (1968). Correspondence on the analysis of the stability of general slip surfaces. *Géotechnique* **18**, No. 1, 92-93.

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