

- PARKIN, A. K. (1965a). The application of discrete unit models to studies of the shear strength of granular materials. Ph.D. thesis, University of Melbourne.
- PARKIN, A. K. (1965b). On the strength of packed spheres. *J. Aust. math. Soc.* **5**, Part 4, 443-452.
- PARKIN, A. K. (1967). Discussion on Incremental strain rate ratios and the strength of sand in the triaxial test. *Géotechnique* **17**, No. 2, 171-172.
- PRAGER, W. (1956). *Finite plastic deformation*. Chap. 3, Rheology, theory and applications (Eirich, F. R., ed.), pp. 1-761. New York: Academic.
- RENNIE, B. C. (1959). On the strength of sand. *J. Aust. math. Soc.* **1**, Part 1, 71-79.
- ROSCOE, K. H. & SCHOFIELD, A. N. (1964). Discussion on Stress-dilatancy, earth pressures and slopes. *Proc. Am. Soc. civ. Engrs* **90**, SM1, 136-150.
- ROWE, P. W. (1962). The stress-dilatancy relation for static equilibrium of an assembly of particles in contact. *Proc. R. Soc. A* **269**, 500-527.
- ROWE, P. W. (1964a). Discussion on some experiments on the influence of strain conditions on the strength of sand. *Géotechnique* **14**, No. 4, 361-364.
- ROWE, P. W. (1964b). Closure to discussion on Stress-dilatancy, earth pressures and slopes. *Proc. Am. Soc. civ. Engrs* **90**, SM3, 145-180.
- SCOTT, R. F. (1964). Discussion on Stress-dilatancy, earth pressures and slopes. *Proc. Am. Soc. civ. Engrs* **90**, SM1, 133-136.
- TROLLOPE, D. H. & PARKIN, A. K. (1963). Discussion on Stress-dilatancy, earth pressures and slopes. *Proc. Am. Soc. civ. Engrs* **89**, SM6, 129-133.

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A SEMI-EMPIRICAL METHOD FOR DETERMINING STRESSES BENEATH EMBANKMENTS

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As will be admitted by the Author, Clough and Woodward (1967) have shown that an incremental analysis of an embankment yields values of elastic displacements which are very different from values obtained using a single-lift analysis. It follows that calculated displacements in the foundation will also be different and therefore cannot be estimated using the method proposed. The only application that the Author can claim for his semi-empirical approach is the approximate computation of consolidation settlements in the foundation, which he bases on changes in vertical stress only. This is a reasonable approach for wide embankments on shallow foundations; otherwise a better estimate of consolidation settlement can be facilitated by the calculation of principal stresses and hence an estimation of initial excess pore pressures. Only beneath the centre of a symmetrical embankment will those stresses be vertical and horizontal. In general they can only be calculated after an evaluation of shear stresses and those the Author says cannot be calculated by his method.

Further, we have reservations regarding the Author's method of estimating vertical stress changes. Bishop (1952) concluded, from his relaxation solution, that the vertical stress at any point within the embankment was approximately equal to the 'vertical head of soil' above that point. For the two points considered in the Author's Fig. 1(a), if h_1 and h_2 represent the vertical head of soil at the interior and the base respectively, it is easy to show that $(H-d)/H = h_1/h_2$. The Author's equation (1) therefore appears to represent a similar but more restricted conclusion in that it refers only to a ratio of stresses of unspecified magnitude. However, the distribution of vertical stress across the base of the embankment (Author's Fig. 3) lies close to the distribution given by a loading diagram of similar shape to the embankment. The Author's equation (1) may therefore be a re-statement of Bishop's conclusion. If this is so, then the Author's solution for vertical stress in the foundation only differs significantly from that obtained using Jürgenson's (1934) influence coefficients because of the poor approximation the Author makes when he represents the embankment by three

layers. Certainly it is not shown, even for a common shape of embankment, that approximating this shape to a triangle and using the Author's empirical equation (3) will give a more reliable solution than using the actual embankment shape as a loading diagram. It is interesting that a good approximation to Bendel's (1962) solution for vertical stress (Author's Fig. 6) can be obtained using the embankment as a loading diagram and considering compression in the foundation to be one dimensional. This gives vertical lines of equal stress which lie closer to Bendel's values than do the Author's.

We do not agree with the Author's statement that his method is simple enough to require no specialized training for its application. We also feel that he is not realistic when he suggests that numerical solutions, such as that of Poulos (1967), may become available for stresses beneath strip loads on non-homogeneous and non-linear foundations thus enabling his method to be used. Having noted the limitations of his method and having used the finite element method, we would recommend any engineer to make use of a finite element computer program. Such programs should become widely available in the near future and the engineer will require little training in their use.

REFERENCES

- BENDEL, H. (1962). *Die Berechnung von Spannungen und Verschiebungen in Erddämmen* (Calculation of stresses and displacements in earth dams), pp. 1-106. Zürich: E. T. H., No. 55.
 BISHOP, A. W. (1952). Stability of earth dams. Ph.D. thesis, University of London.
 CLOUGH, R. W. & WOODWARD, R. J. (1967). Analysis of embankment stresses and deformations. *J. Soil Mech. Fdns Div. Am. Soc. civ. Engrs* **93**, No. SM4, 529-549.
 JÜRGENSON, L. (1934). The application of theories of elasticity and plasticity to foundation problems. *Boston Soc. civ. Engrs, Contributions to Soil Mech.* (1925-1940), pp. 148-183.

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* * *

In their discussion on this Paper, Chowdhury and King have raised a number of points.

They point out that analyses by Clough and Woodward (1967) have shown that an incremental construction analysis of an embankment yields values of elastic displacements which are very different from the values obtained using a single lift analysis, and argue that the calculated displacements in the foundation will also be different and therefore cannot be estimated using the method proposed in the Paper. This argument, however, overlooks the fact that the Author's method is one for calculating stresses and not displacements directly. Although the two quantities appear to be interrelated, that a conclusion regarding the stresses cannot be reached by a comparison of displacements has been demonstrated by Clough and Woodward (1967) by the fact that, although the displacements as given by the two methods of analysis are strongly different, the stresses themselves are not so different. Hence, within the limits of accuracy of the method proposed by the Author, not only can the foundation displacements be calculated correctly using the stresses obtained by the Author's method, but also if desired the displacements in the embankment itself can be estimated taking into account the incremental construction procedure, as the proposed method enables the horizontal and vertical stresses beneath any embankment layer to be determined.

The rest of the argument in the first paragraph of the discussion by Chowdhury and King is a re-statement of the last paragraph on p. 203 of the Paper.

Their suggestion that using the shape of the embankment as a loading diagram, for the calculation of vertical stresses in the foundation, leads to better estimates of stress than does