A simple method based on the energy used to drive a given body into the soil offers a temptation to correlate the results of the input of energy with useful parameters such as ultimate pile load, bearing capacity and even soil properties. As is well known the correlation is ineffectual but the temptation never dies. During the formulation of the specification, suggestions for changes were supported by strong opinions, detailed analyses and highly theoretical approaches but all had to be overcome in order to preserve the simple equipment and procedures of the method. Many attempts were made to include, as part of the specification, the interpretation of results not only as a measure of soil parameters but as correspondence to results of other sampling instruments, different weights, height of drop, dimensions and boring procedures. Admittedly the standard penetration test is a crude, pragmatic and empirical method whose origin, growth and development are the result of the need to explore economically the soils at the bottom of a boring and do it consistently.

It should be admitted that the split-tube is the cheapest and perhaps the most effective method of sampling sandy soils. Over the years accumulated experimental data have been used to relate the number of blows to the relative density of sands and sandy soils without pretence to theory or rigid analysis. Many attempts have been made to rationalize the method but to no avail. Without this test soil engineers would be seriously handicapped; modifications, substitutions and changes would create chaos. Hence the need for a standard specification which, in the U.S., gives it respectability.

A controversial feature introduced in 1956 was adopted: the number of blows required to effect the first 6 in. of penetration is not considered in recording the total number for a penetration of 12 in., i.e. the N value. This procedure eliminates from the count the effects of soil disturbance at the bottom of the hole and compensates for the presence of soil accidentally accumulated after cleaning out the boring before sampling. As De Mello *et al.* (1960) point out there may be a correlation between seating or not seating the sampler over the first six inches. The standard makes that correlation unnecessary. Brazilian engineers using samplers of different dimensions, other hammers and heights of drop have gained local experience. They can communicate among themselves but cannot translate their results to standard penetration test results. It is to facilitate a universal language that method D-1586 was standardized by ASTM in 1966.

Although the standard penetration test was originally devised to explore all types of soils and the results were used in a variety of ways to predict the type and behaviour of foundations, its present use is more modest. It is restricted primarily, and perhaps exclusively, to sample sands and sandy soils and to appraise their state of relative density. Other methods and devices are used to sample effectively cohesive soils.

As long as soil mechanics remains the art that it is, this simple and crude test will be an essential adjunct to the exercise of the few scientific facts that judgement and experience employ to decipher the properties of sand below ground surface.

REFERENCE

De Mello, V. F. B., Souto Silveira, E. B. & Quaresma, A. R. (1960). Some field correlations on dynamic penetration resistances in exploratory borings in Geotécnica, Brazil. *Proc. 1st Pan Am. Conf. Soil Mech., Mexico City.*

STRESSES AND DISPLACEMENTS IN A CROSS-ANISOTROPIC SOIL

(Barden, L. (1963). *Géotechnique* **13**, No. 3, 198–210)

G. P. Raymond, Queen's University, Ontario

This correspondence is concerned mainly with the undrained settlement of a cross-anisotropic soil. Unlike the Paper, however, no restriction is placed on the value of the independent shear modulus. Specific solutions are not given since a large number of general solutions are already available (e.g. Gerrard, 1968, 1969).

The five independent constants are taken as

E_v elastic modulus in the vertical direction
E_h elastic modulus in the horizontal direction
ν_v vertical Poisson's ratio relating imposed vertical strain to the resulting horizontal strain, ν_{vh}
ν_h horizontal Poisson's ratio relating an imposed horizontal strain to the resulting horizontal strain at right angles to the imposed strain, ν_{hh}
G_v vertical shear modulus relating an imposed vertical/horizontal shear stress on a vertical/horizontal plane to the resulting shear strain (complementary shear stress)

The relationships between stress and strain using only these five constants and taking the x and y axes as horizontal and isotropic are

$$\epsilon_x = \frac{\sigma_x}{E_h} - \frac{\nu_h \sigma_y}{E_h} - \frac{\nu_v \sigma_z}{E_v} \qquad (1)$$

$$\epsilon_y = -\frac{\nu_h \sigma_x}{E_h} + \frac{\sigma_y}{E_h} - \frac{\nu_v \sigma_z}{E_v} \qquad (2)$$

$$\epsilon_z = -\frac{\nu_v \sigma_x}{E_v} - \frac{\nu_v \sigma_y}{E_v} + \frac{\sigma_z}{E_v} \qquad (3)$$

$$\gamma_{xy} = 2\tau_{xz}(1+\nu_h)/E_h \qquad (4)$$

$$\gamma_{xz} = \tau_{xz}/G_v \qquad (5)$$

$$\gamma_{yz} = \tau_{yz}/G_v \qquad (6)$$

As pointed out by the Author the theory of elasticity places certain restrictions on the elastic constants by the fact that the strain energy function must be positive (or zero). The first of these requires that none of the elastic constants is negative, and the second is

$$1 \geqslant \nu_h + 2\frac{E_h}{E_v}\nu_v^2 \qquad (7)$$

the equality resulting from a zero strain energy function.

In addition, by requiring dilation to be the same sign as the applied stress

$$\tfrac{1}{2} \geqslant \nu_v \qquad (8)$$

$$1 \geqslant \nu_h + \frac{E_h}{E_v}\nu_v \qquad (9)$$

The equality in both cases results from zero dilation, which restricts the value of E_h/E_v for zero dilation to a maximum of 2.

The Author gives the equation for the vertical stress below a point load on the surface of a semi-infinite mass in terms of α and β. In their general form α and β are expressed by

$$\alpha\beta = \frac{E_v}{E_h}\frac{1-\nu_h^2}{1-E_h\nu_v^2/E_v} \qquad (10)$$

$$\alpha+\beta = \frac{E_v - 2G_v\nu_v(1+\nu_h)}{G_v(1-E_h\nu_v^2/E_v)} \qquad (11)$$

The product of α and β is positive due to the restriction of the strain energy function being positive. Imposing a restriction that all stresses must be real, then the upper limit of G_v results from

$$(\alpha+\beta)_{\min} = 2\sqrt{(\alpha\beta)} = 2\alpha = 2\beta \qquad (12)$$

Therefore

$$G_v \leqslant \frac{E_v}{2\nu_v(1+\nu_h)+2\sqrt{\left[\frac{E_v}{E_h}(1-\nu_h^2)(1-E_h\nu_v^2/E_v)\right]}} \qquad (13)$$

and for the cross-anisotropic case of $E_v=E_h$, $\nu_v=\nu_h$, isotropy occurs when G_v is at its upper limit as restricted by equation (13). (G_v is an independent constant so that $E_v=E_h$ and $\nu_v=\nu_h$ do not automatically mean isotropy.)

The settlement of the surface of a semi-infinite mass subjected to a vertical point load on the surface is

$$\rho_r = \frac{P}{2\pi r E_h}\left\{\left(1-\frac{E_h}{E_v}\nu_v^2\right)\frac{E_h}{E_v}\left[\frac{E_h}{G_v}+2\sqrt{\left[\frac{E_h}{E_v}\left(1-\frac{E_h}{E_v}\nu_v^2\right)\left(1-\nu_h^2\right)\right]}-2\frac{E_h}{E_v}\nu_v(1+\nu_h)\right]\right\}^{1/2} \qquad (14)$$

which is always positive provided the previously stated restrictions are satisfied. Since G_v is an independent constant cross-anisotropy will permit an infinite surface settlement in the limit of $G_v \to 0$ irrespective of the value of E_h/E_v. It may be seen that minimum settlements will occur when G_v has its maximum value; thus isotropy will be associated with minimum settlement ranges. For the special case of zero dilation

$$\rho_r = \frac{P}{2\pi r}\sqrt{\left[\frac{1-\frac{E_h}{4E_v}}{E_v G_v}\right]} \qquad (15)$$

Thus for any given fixed values of E_v and G_v the ratio of maximum to minimum undrained settlements will be $\sqrt{2}$, and the variation in the ratio E_h/E_v has only a minor influence on undrained settlements. It may be concluded that major errors in immediate settlement analysis resulting from cross-anisotropy will occur from not measuring and accounting for the difference in G_v.

Since the settlement of a vertically loaded surface area is the integration of the point load solution these arguments and ratios of resulting settlements apply for any fixed area and shaped surface vertical loading (on a semi-infinite mass).

In conclusion it may be stated that

(*a*) theoretically insignificant improvement in undrained settlement analysis will result by assuming a soil to be cross-anisotropic as opposed to isotropic and using the vertical elastic modulus unless the independent shear modulus is determined

(*b*) theoretically the determination of the independent shear modulus is also of immense importance in predicting drained settlements if anisotropic elasticity is assumed for the soil skeleton

(*c*) standard isotropic undrained settlement predictions correspond closely to minimum undrained cross-anisotropic settlement predictions; thus cross-anisotropy cannot explain overestimation of immediate settlements in saturated clays.

REFERENCES

GERRARD, C. M. (1968). Supplement XS-14 to The axisymmetric deformation of a homogeneous cross-anisotropic elastic half space. Highway Research Board, Record 223, 36–44.

GERRARD, C. M. (1969). Supplement XS-24 to The analysis of a homogeneous, cross-anisotropic elastic half space undergoing deformations that possess a vertical plane of symmetry. Highway Research Board, Record 282, 41–47.