Driven piles in clay—the effects of installation and subsequent consolidation

RANDOLF, M. F., CARTER, J. P. & WROTH, C. P. (1979). *Géotechnique* **29**, No. 4, 361–393

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The Authors have made an important contribution to the analysis of the installation of single piles driven into clay and the effects on the distribution of stresses and displacements in the soil near the piles immediately after driving and during subsequent consolidation of the clay. This analysis shows that the initial pore pressures in the clay at the pile shaft are about 3–4 times the initial cohesion c_u of the clay, which is somewhat smaller than the initial range of 5–7 c_u measured close to driven piles in the field (Meyerhof, 1976). The corresponding initial effective radial stress on the pile shaft is estimated by the Authors to be about 2·5 c_u and would thus correspond to an estimated coefficient of earth pressure on the shaft K_s of about 1·5 times the earth pressure coefficient at rest K_0 for normally consolidated clay.

It is further estimated that this effective radial stress increases to about twice the initial value at the end of consolidation, which would correspond to an estimated final earth pressure coefficient K_s of about $3 K_0$ for normally consolidated clay and more than twice this value for heavily overconsolidated clay. However, these estimated final values of K_s are roughly twice those deduced from the observed skin friction of full-scale pile load tests in such clays, which gave a final effective radial stress of about 1·5–2 c_u on driven piles (Clark & Meyerhof, 1972) corresponding to about 1–1·5K_0 (Burland, 1973; Meyerhof, 1976). This difference between theoretical and observed radial stresses on the pile shaft may be due to a much greater compressibility than estimated for remoulded and reconsolidated clay near driven piles and the simplifications made in the present analysis of the complex and variable mechanical properties of the soil near driven piles, including the effects of the skin friction on the pile shaft influencing the stresses and displacements in the soil near the piles.

REFERENCES

Burland, J. B. (1973). Shaft friction of piles in clay—a simple fundamental approach. *Ground Engineering* **6**, No. 3, 30–42.

Clark, J. I. & Meyerhof, G. G. (1972). The behaviour of piles driven in clay. An investigation of soil stress and pore water pressures as related to soil properties. *Can. Geotech. Jl* **9**, No. 3, 351–373.

Meyerhof, G. G. (1976). Bearing capacity and settlement of pile foundations (11th Terzaghi Lecture). *J. Geotech. Engng Div., Am. Soc. Civ. Engrs* **102**, No. GT3, 195–228.

Dr D. M. Wood, Cambridge University

The approach to the modelling of the installation of driven piles by analogy with the expansion of a cylindrical cavity is an interesting one which may in due course have beneficial effect on the design of such piles. My comments relate to the stress path which has been deduced for the elements of soil around the installed pile.

The Authors have illustrated the relevance of the cylindrical cavity analogy and have correctly noted that the path applied to elements of soil in this process—a strain controlled path—consists, for a typical clay deposit of large lateral extent, of one-dimensional consolidation (deposition of the soil) followed by plane strain constant volume shearing, with the plane of shearing being perpendicular to the previous direction of consolidation (rapid installation of the pile).

The Authors have used an isotropic model of soil behaviour, the Cam-clay model, to predict the effective stress path resulting from this strain path. True triaxial apparatus, such as that at Cambridge (Pearce, 1970; Wood, 1974), can be used to apply the strain path to soil samples and observe the resulting effective stress path. A group of tests on samples of kaolin at three different overconsolidation ratios was described by Wood & Wroth (1977). Two further tests, on reconstituted Boston blue clay, were performed by Wood (1978) for Woodward-Clyde Consultants as part of a co-operative study of predictions of axial capacity for offshore piles sponsored by Amoco Production Company.

The observed effective stress paths in these two tests on Boston blue clay, subjected to precisely the same history and strain path, are shown in Figs 1(a), (b) and (c) as three different representations of three-dimensional principal stress space. Figs 1(a) and (b) show two orthogonal views of stress space: Fig. 1(b) shows a view on the π-plane, that is a view down the stress space diagonal $\sigma_1' = \sigma_2' = \sigma_3'$; and Fig. 1(a) shows an orthogonal view which contains

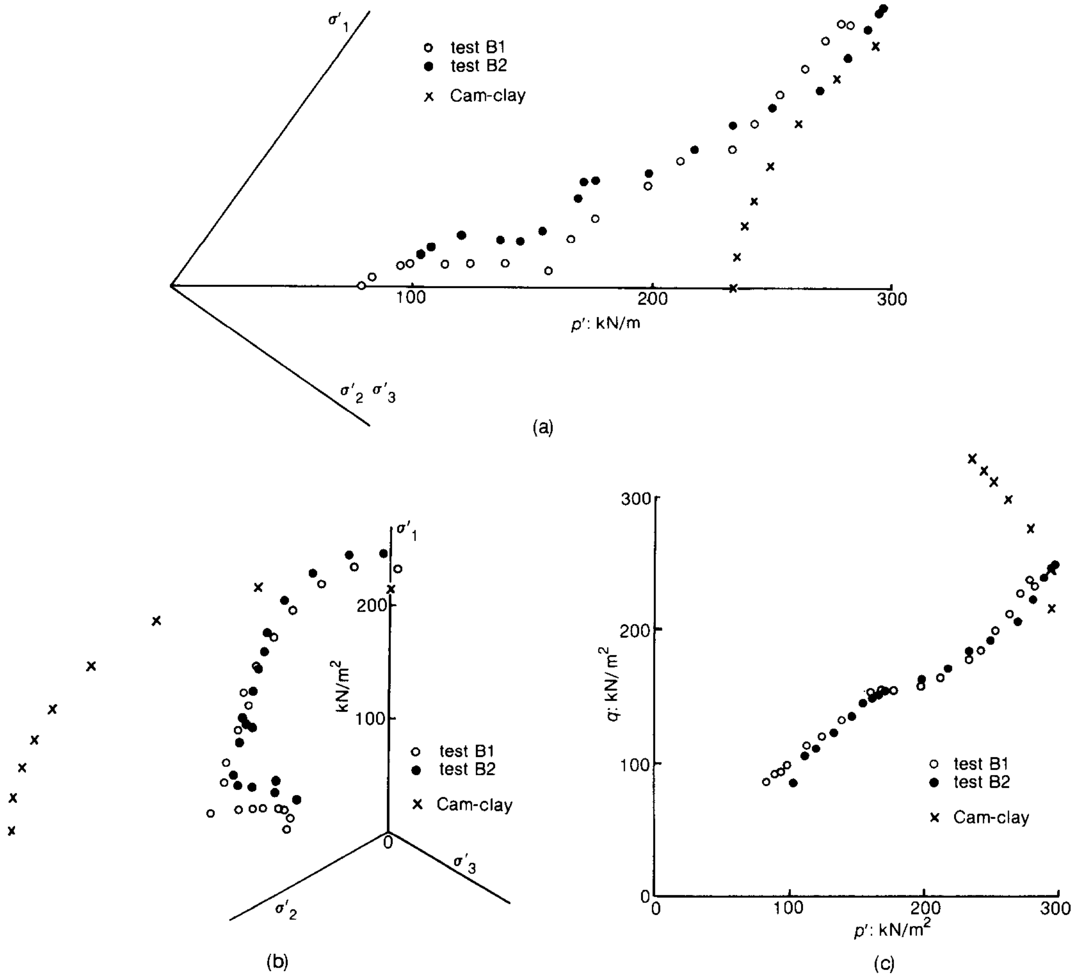

Fig. 1. Comparison of results of true triaxial tests on Boston blue clay with predictions on Cam-clay model: (a) orthogonal view containing true view projections of the σ_1' axis and the stress space diagonal; (b) projection on the π-plane; (c) results plotted in $q:p'$ space.

true view projections of the σ_1' axis and of the stress space diagonal. Fig. 1(c) shows the stress paths plotted in $q:p'$ space, where

$$q = [\tfrac{1}{2}(\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1') + (\sigma_1' - \sigma_2')^2]^{\frac{1}{2}}$$

and

$$p' = \tfrac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3')$$

where the three suffixes 1, 2, 3 denote reference axes of principal stresses, and correspond to the z, r, θ directions round the expanding cavity, or pile being installed. Because of the definition of q, Fig. 1(c) is not a view perpendicular to the π-plane (*pace* the Authors' Fig. 13). The stress path predicted by the Authors' Cam-clay model, with parameters for Boston blue clay, is also shown in Fig. 1.

The divergence between the experimental observations and model predictions arises primarily because the predictions have assumed isotropic hardening—a feature of material response of which the initially one-dimensionally consolidated soil is quite unaware. As a consequence there is little resemblance between predictions and observation. Cam-clay shows a steady increase of q and of the ratio q/p' with only a small decrease in p' during the undrained shearing. This occurs because the initial stress state is rather close to failure for the assumed shape of yield locus, and the combination of assumed isotropic elastic shear behaviour with assumed isotropic hardening of the yield locus guarantees that further hardening and increase of stress ratio must occur in order to match the applied strains.

The lack of success of the model on this particular strain path can be contrasted with its apparent success for the case of embankment loading on soft clay (Wroth, 1977). In that situation, however, while the effect of the large rotation of principal axes that occurs (for which an extrapolatory assumption in the model that rotation of axes has no effect is conventionally made) is uncertain, the stress paths in principal stress space for the elements of soil which contribute most to the deformation of the embankment foundation are largely loading paths, activating a rather restricted portion of the supposed isotropic yield surface both in $q:p'$ space and in the π-plane projection (Wood, 1980). Consequently, the error introduced in assuming isotropic hardening is likely for this embankment loading to be slight. However, where, as here, major explorations of stress and strain space are involved for the deforming elements the errors introduced will be severe.

The Authors present analyses for the dependence of the effective stress state of soil elements adjacent to the pile after installation on overconsolidation ratio of the soil before pile installation. They conclude that, provided the influence of possibly different water contents is taken into account, by some appropriate normalization, then the memory of earlier history of the soil, as indicated by its overconsolidation ratio, is eliminated by the pile installation—because this installation subjects the soil elements to such major remoulding.

Wood & Wroth (1977) reporting tests on kaolin use a different normalization from that used by the Authors to accommodate differences in water content. It appears that there is a decrease in the non-dimensionalized value of q reached (and possibly also in the non-dimensionalized value of p' reached) with increasing overconsolidation ratio.

The test results presented by Wood & Wroth (1977) and also those of Wood (1978) suggest also that even after considerable shearing the soil still retains some memory of its original stress-induced anisotropy, induced by the original one-dimensional consolidation of the soil.

Models of the Cam-clay type have proved extremely valuable in matching observed stress–strain response of soils. However, most of the data against which they have been matched have been obtained from triaxial compression tests which offer rather restricted scope for stress or strain space exploration. Their success under these conditions should not lead one to expect that similar success and relevant results will be obtained, without additional modification, in predicting response of soils to more general stress or strain paths.

REFERENCES

Pearce, J. A. (1970). *The behaviour of soft clay in a new true triaxial apparatus.* PhD thesis, Cambridge University.

Wood, D. M. (1974). *Some aspects of the mechanical behaviour of kaolin under truly triaxial conditions of stress and strain.* PhD thesis, Cambridge University.

Wood, D. M. (1978). *True triaxial tests on Boston blue clay.* Report submitted to Woodward-Clyde Consultants, March 1978.

Wood, D. M. (1980). Yielding in soft clay at Bäckebol, Sweden. *Géotechnique* **30**, No. 1, 49–65.

Wood, D. M. & Wroth, C. P. (1977). Some laboratory experiments related to the results of pressuremeter tests. *Géotechnique* **27**, No. 2, 181–201.

Wroth, C. P. (1977). The predicted performance of soft clay under a trial embankment loading based on the Cam-clay model. In *Finite elements in geomechanics*, Gudehus, G. (ed.), 191–208. New York: J. Wiley.

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The Paper presents useful data of practical measurements and of time-dependent phenomena. I have not seen the Authors' unpublished reports (Carter, Randolph & Wroth, 1979; Randolph & Wroth, 1978), but I think that the plane stress condition for the radial movement of the clay around a pile is more reasonable because the vertical normal stress at depth z can be assumed to be γz, where γ is the bulk unit weight of the clay. I have presented (Nishida, 1966) an expression for the excess pore pressure for a group of piles, symmetrically arranged with a space of $2b$ between two pile centres. By using the same symbols as the Authors my expressions become as follows. In the plastic failure zone where $R \geqslant r \geqslant r_0$

$$u = c_u\left\{\frac{4}{3}\left(\frac{1}{2}\frac{m+1}{m-1}\frac{R^2}{b^2}+\ln\frac{R}{r}\right) + (A-\tfrac{1}{3})\sqrt{\left[3+4\left(\frac{1}{2}\frac{m+1}{m-1}\frac{R^2}{b^2}+\ln\frac{R}{r}\right)^2\right]}\right\} \quad (1)$$

In the outer elastic zone when $b \geqslant r \geqslant R$

$$u = c_u\left\{\frac{2}{3}\frac{m+1}{m-1}\frac{R^2}{b^2}+(A-\tfrac{1}{3}) \times\sqrt{\left[3\left(\frac{R^2}{r^2}\right)^2+4\left(\frac{1}{2}\frac{m+1}{m-1}\frac{R^2}{b^2}\right)^2\right]}\right\} \quad (2)$$

where $1/m$ is Poisson's ratio and A is Skempton's pore pressure coefficient of the clay. R is the radius of the plastic failure zone around a pile and it is given by

$$r_0{}^2+\frac{c_u}{G}\left(\frac{R^2}{b^2}-1\right)R^2+\int_{r_0}^{R}\frac{c_u}{G_p}\left[\left(\frac{R^2}{b^2}+1\right) +\left(\frac{m-1}{m+1}\right)2\ln\frac{R}{r}\right]r\,\mathrm{d}r = 0 \quad (3)$$

in which G_p is the shear modulus of the clay in the plastic failure zone.

For the plane strain condition (the same as the Authors') it follows that in the plastic failure zone when $R \geqslant r \geqslant r_0$

$$u = c_u\left(\frac{m+1}{3m}\left[\frac{2(m+1)}{m-1}\frac{R^2}{b^2}+4\ln\frac{R}{r}\right] +(A-\tfrac{1}{3})\sqrt{\left\{3+\left[\frac{(m+1)(m-2)}{m(m-1)}\frac{R^2}{b^2} +\frac{2(m-2)}{m}\ln\frac{R}{r}\right]^2\right\}}\right) \quad (4)$$

In the outer elastic zone when $b \geqslant r \geqslant R$

$$u = c_u\left(\frac{2}{3}\frac{(m+1)^2}{m(m-1)}\frac{R^2}{b^2}+(A-\tfrac{1}{3}) \times\sqrt{\left\{3\left(\frac{R^2}{r^2}\right)^2+\left[\frac{R^2}{b^2}\frac{(m+1)(m-2)}{(m-1)m}\right]^2\right\}}\right) \quad (5)$$

In equations (4) and (5) R is given by

$$r_0{}^2+\frac{c_u}{G}\left[\frac{(m+1)(m-2)}{m(m-1)}\frac{R^2}{b^2}-1\right]R^2 +\int_{r_0}^{R}\frac{c_u}{G_p}\left[\frac{m-2}{m}\left(\frac{m+1}{m-1}\frac{R^2}{b^2} +2\ln\frac{R}{r}\right)+1\right]r\,dr = 0 \quad (6)$$

The equations corresponding to the case for a single pile can be obtained by putting $b \to \infty$ in the above equations (Nishida, 1963). The Authors' equation (7) can be obtained by putting $b \to \infty$, $m = 2$ and $A = \frac{1}{3}$ in equations (4) and (6) above.

REFERENCES

Nishida, Y. (1963). A basic study for pore pressure on a pile driven in clay. *Trans. Japan Soc. Civ. Engrs* No. 98, 10–14 (in Japanese).

Nishida, Y. (1966). Excess pore pressure by group piles. *Trans. Japan Soc. Civ. Engrs* No. 129, 6–14 (in Japanese).

Authors' reply

Professor Meyerhof raises questions concerning the magnitude of the excess pore pressures predicted at the time of pile installation, and the magnitude of the final radial effective stress acting on the pile after reconsolidation. In relation to the first question, the expansion of a cylindrical cavity (e.g. in pressuremeter testing) gives rise to a limiting total radial stress typically of 6–7 times the undrained shear strength c_u. If it is assumed that this figure applies also in the pile case, then the excess pore pressure at time of installation will depend on the current effective stress state in the remoulded soil close to the pile. As has been pointed out by Dr Wood, the Cam-clay model is not particularly successful at predicting effective stress paths in cases where the major principal stress changes orientation. However, the precise values of the effective stresses in the soil immediately after pile installation are not of fundamental importance because the final effective stress state (after reconsolidation) is largely governed by the consolidation phase, for which the Cam-clay model—linking as it does consolidation stress, density and strength of the soil—provides a powerful and reasonably accurate soil model.

The figure of 3–4 c_u for the excess pore pressure at time of pile installation is likely to be a lower limit, since it is based on an ideal, insensitive soil. For soil with sensitivity greater than unity, the effective stresses in the remoulded soil (as multiples of the peak strength) will be lower, giving rise to higher excess pore pressures. In the example calculation given in the Paper for pile tests in soil of sensitivity ~7 (Eide, Hutchinson & Landva, 1961), an excess pore pressure of ~$6c_u$ is postulated, which is in keeping with the values quoted by Professor Meyerhof.

The assumption of plane strain conditions during the reconsolidation phase is questionable, particularly for heavily overconsolidated clays, and this may lead to overestimates of the radial effective stress acting on the pile at the end of reconsolidation. However, the mistake should not be made of equating this value of the radial effective stress with that acting on the pile shaft at failure during a subsequent load test. By analogy with simple shear tests on clays, there are indications that the radial effective stress acting on the pile shaft may decrease by 50% or more during a load test (Randolph & Wroth, 1981). This would give radial effective stresses at the point of failure of 2–3c_u, more in keeping with the findings of Clarke & Meyerhof (1972).

Without having access to Dr Nishida's original work and the basis for his expressions, we are unable to comment in detail on his interesting remarks. The two reports to which he refers have been published as Carter, Randolph & Wroth (1979) and Randolph & Wroth (1979).

Dr Nishida's suggestion that conditions of plane stress may be more relevant than plane strain is not borne out by observations that total pore pressures measured close to driven piles are sometimes greater than the total overburden stress γz even within pile groups (e.g. Koizumi & Ito, 1967). In reality, neither condition is entirely appropriate, and there will be variation with depth alongside a driven pile. In his final statement, Dr Nishida suggests a value of 1/3 for Skempton's pore pressure parameter. For an isotropic elastic material, this value is only correct for conditions of axial symmetry; for plane strain it should be 1/2. In the plastic zone the relevant value of A will depend on the overconsolidation ratio of the clay in question.

The points made by Dr Wood are well taken, and form an important reminder that the Cam-clay model has limitations. At present there are inadequate good-quality data to provide reliable information about the behaviour of clay under general three-dimensional conditions of stress with rotation of the principal axes. Even if this information were forthcoming, any mathematical model that incorporated such behaviour would be unusually complex. The virtue of the Cam-clay model is that it is a complete, self-consistent model which represents the salient features of the behaviour of clay and, in particular, up-dates the current undrained strength as consolidation occurs. It is believed that the work discussed in the Paper is the first in which pile capacity can be estimated both in terms of effective stresses by taking account of the effects of driving and subsequent consolidation, and as to how it changes with time after installation. This has been achieved with the use of one consistent model for all the stages of installation. It is not claimed that the results are other than qualitatively relevant to the problems of the design of piles, much further research is needed before accurate quantitative predictions can be made.

REFERENCES

Carter, J. P., Randolph, M. F. & Wroth, C. P. (1979). Stress and pore pressure changes in clay during and after expansion of a cylindrical cavity. *Int. Jl Num. Anal. Meth. Geomech.* **3**, 305–322.

Eide, O., Hutchinson, J. N. & Landva, A. (1961). Short and long term test loading of a friction pile in clay. *Proc. 5th Int. Conf. Soil Mech., Paris*, 45–54.

Koizumi, Y. & Ito, K. (1967). Field tests with regard to pile driving and bearing capacity of piled foundations. *Soil Fdn* **7**, No. 3.

Randolph, M. F. & Wroth, C. P. (1979). An analytical solution for the consolidation around a driven pile. *Int. Jl Num. Anal. Meth. Geomech.* **3**, 217–229.

Randolph, M. F. & Wroth, C. P. (1981). Application of the failure state in undrained simple shear to the shaft capacity of driven piles. *Géotechnique* **31**, No. 1, 143–157.

Application of the calculus of variations to the vertical cut off in cohesive frictionless soil

DE JOSSELIN DE JONG, G. (1980). *Géotechnique* **30**, No. 1, 1–16

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The Author is to be congratulated on the introduction of the second variation in the analysis. In effect, the Legendre, Jacobi and Weierstrass conditions cannot be omitted in the search for an extremum. Luceño (1979) and Luceño & Castillo (1980a, b) introduced these conditions in the analysis of several existing variational methods in soil mechanics problems and demonstrated that Baker & Garber's and Chen's methods do not lead to extrema.

With respect to the Paper, the Author demonstrates that functional (21a) subject to (21b) and (21c) does not attain a minimum because, in an analysis of the second variation, the Jacobi condition is violated. This conclusion would have been better obtained by an analysis of the first variation which includes not only the Euler equation but also the transversality condition. This last condition is not satisfied by the solution given in the Paper, as it will be shown.

A straightforward check of the transversality condition leads to some problems because the auxiliary function H depends on α_F, but due to the fact that the Kötter conditions (32) and (33) must be satisfied by every potential slip line, the class of these lines can be initially reduced to those satisfying Kötter's conditions. With this assumption α_F and β_F become constants and the Euler equation remains unchanged. So the same class of extremals (25) is obtained.

The transverality condition now becomes (Bolza)

$$\bar{x}'H_{x'}+\bar{y}H_{y'}|_F = 0 \tag{D1}$$

where

$$\begin{aligned}\bar{x}(s) &= s\\ \bar{y}(s) &= 2h/3\end{aligned} \tag{D2}$$

are the equations of the free surface CD in parametric form.

Substitution of expressions (23) and (D2) in (D1) leads to

$$1-g_2\,y-\frac{2}{1+(y'/x')^2}\frac{y'}{x'}\left[(g_1+g_2\,x)-(1-g_2\,y)\frac{y'}{x'}\right] + 2\left[\beta_F+\arctan g\left(\frac{y'}{x'}\right)\right](g_1+g_2\,x)|_F = 0 \tag{D3}$$

Taking into account (25a) and introducing (34) in the left-hand side of (D3) one gets a value of 1·865 which is obviously not zero. In consequence, the first variation does not vanish because the transversality condition is violated.

This fact demonstrates that Kötter's conditions are not compatible with the transversality condition in the point F.