dynamic coupling between the tube and the surrounding soil (J' being a function of velocity). A zero value of J' indicates a decoupling, i.e. a pure shear or friction surface between pile and soil.

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Authors' reply

Ever since D. W. Taylor undertook his classical research on the secondary consolidation of clay there has been a growing awareness amongst geotechnical engineers of the significance of viscosity in soil behaviour. In the many different branches of geotechnical engineering where viscosity is encountered it is not uncommon to find it referred to by a variety of specialist names, e.g. in consolidation theory it is referred to as 'secondary consolidation', in pile driving it is often referred to as 'damping factors'. These names are not always suggestive of the phenomenon being considered and are even less suggestive of any physical connection with the other applications. None the less, there is still a substantial concensus that in real soils, and in clays in particular, viscosity is present. The Authors are therefore surprised to learn that Mr Mizikos has found clay soils which, under some circumstances, appear to be free of viscous effects. It may well be that the viscous forces are small and almost undetectable as in sands for instance. However, for clays this is not the case, and in particular under pile-driving conditions the viscous forces may even become dominant.

Mr Mizikos's basis for concluding J' is zero appears to be based on the back-analysis of pile-driving data. It may be that Smith's model has led to the anomaly since as the Paper shows the viscosity law is, in practice, significantly different from that which Smith assumed.

A constant velocity of penetration was used in all the tests since as the Paper shows this readily allows curves of resistance against velocity to be plotted from which the viscous parameters may be deduced. It was for this reason that no tests on impact driving were undertaken.

The purpose of the tube tests was to provide an independent check on the general conclusion that $J < J'$. This conclusion was not entirely unexpected since several other investigators have observed the same when back-analysing wave equation data. While it might be as Mr Mizikos suggests that the results will have been influenced by the lateral displacement of the soil particles as the tube penetrates it was hoped that disturbances of this origin would have been minimized by using thin-walled tubes. However, such effects would not affect the sign of the slope of the line which is the only consideration required to indicate the relative magnitudes of J and J'.

Elastic solutions for a deep circular tunnel

M. J. PENDER (1980). *Géotechnique* **30**, No. 2, 216–222

J. E. Gumbel, Mott, Hay & Anderson

In discussing the stresses and displacements around a circular opening in elastic ground, Pender has set out clearly the analytical basis for the different effects of loading applied at a distant boundary (load case 1) and loading applied at the periphery of the opening (or unloading, as in the case of tunnel excavation) (load case 2). The solutions presented for an unlined opening may further be interpreted to indicate the relative response to these two load cases of a lined opening.

The elastic analysis of ground–liner interaction was first developed for load case 1 (Burns & Richard, 1964; Höeg, 1968); the solution for load case 2 was obtained by somewhat different logic (Curtis, 1976) and a complete alternative derivation has recently been put forward (Einstein & Schwartz, 1979). However, liner response to load case 2 may be deduced directly from the solutions for load case 1 without the need for any separate interaction analysis.

RELATIVE LINER RESPONSE TO LOAD CASES 1 AND 2

In either load case the ground–liner system is subject to uniform (symmetric) and distortional (asymmetric) components of applied stress.

Uniform component $\sigma_z = \frac{1}{2}(\sigma_v + \sigma_h)$
Distortional component $\sigma_y = \frac{1}{2}(\sigma_v - \sigma_h)$

These components may be identified in the Author's equations (4)–(6). The aspects of liner response relevant to design may be expressed as

Radial deflexion $w = w_z + w_y \cos 2\theta$
Circumferential (hoop) thrust $N = N_z + N_y \cos 2\theta$
Ring bending moment $M = M_y \cos 2\theta$

where the subscript z indicates an effect of uniform loading, and the subscript y indicates an effect of distortional loading. Interaction forces may be expressed in a similar form if required (see, for example, Einstein & Schwartz, 1979). In order to determine the relative values of w_z, w_y, etc. for load cases 1 and 2 it is sufficient to consider the stresses reaching a buried, rigid cylinder when uniform and distortional loads are applied in turn to the ground at a distant boundary.

From the Author's equation (21), the unrestrained radial displacement of a cylindrical opening when subject to a uniform stress σ_z applied at a distant boundary would be

$$u_{az} = 2\frac{(1-\nu^2)a}{E}\sigma_z$$

Thus, to maintain the original shape of the opening, a rigid cylindrical inclusion must exert a uniform stress σ_{zi} at its periphery as indicated by equation (28)

$$\sigma_{zi} = \frac{Eu_{az}}{(1+\nu)a} = 2(1-\nu)\sigma_z$$

If the rigidity of the cylinder is then relaxed, and it is replaced by a liner of finite stiffness, the radial stress σ_{zi} will be shared in some proportion between ground and liner so as to produce compatible deformations. Following the logic of Curtis (1976), the resulting interaction stresses, and hence also the response of the liner, will be identical to those generated by a uniform stress σ_{zi} applied directly to the unstressed system at the ground–liner interface.

By similar reasoning, a distortional stress σ_y applied at a distant boundary is found to produce the same interaction effects as a distortional stress σ_{yi} applied directly at the ground–liner interface, where

$$\sigma_{yi} = \frac{4(1-\nu)}{3-4\nu}\sigma_y$$

Thus for any given ground–liner system for which the interface shear transfer conditions are defined (either no slippage or full slippage), liner response to distant boundary loading (load case 1) and interface loading (load case 2) differs only in the relative magnitudes of the uniform and distortional effects.

To convert any load case 1 solution for ground–liner interaction, whether for a thin liner (Burns & Richard, 1964) or for a thick liner (Dar & Bates, 1974), to the corresponding solution for excavation unloading (load case 2), it is simply necessary to apply a factor

$$\frac{\sigma_z}{\sigma_{zi}} = \frac{1}{2(1-\nu)}$$

to all uniform effects (w_z, N_z, etc.), and a factor

$$\frac{\sigma_y}{\sigma_{yi}} = \frac{3-4\nu}{4(1-\nu)}$$

to all distortional effects (w_y, N_y, M_y, etc.).

These simple proportional relationships are evident in the plots comparing the load case 1 and 2 solutions presented by Einstein & Schwartz (1979). They are not immediately apparent from the rather complex algebraic expressions because no two authors have used exactly the same notation; but by making the necessary substitutions they may readily be confirmed, for example by comparing the respective expressions to be found in Höeg (1968) and Einstein & Schwartz (1979).

PRACTICAL RELEVANCE OF LOAD CASES 1 AND 2

The Author states that load case 1 has only limited application in geotechnical engineering. One obvious application he has apparently overlooked it that of culverts under high embankments. For buried pipes in general, it is necessary to apply a combination of load case 1 and load case 2 solutions to account for the separate effects of backfill weight, groundwater pressure and surface loading (Gumbel & Wilson, 1980). In the field of tunnelling it would seem no more prudent now to dismiss the relevance of load case 1 than it was for the original analysts to neglect load case 2.

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Author's reply

The scope of the Technical Note was limited to a statement of the effect of differing boundary conditions on the displacements about a circular opening in an elastic medium. Gumbel has extended this by outlining one way in which the interaction analysis between an elastic soil and an elastic tunnel liner can be made. By making use of the solutions given in the Note, Gumbel has arrived at the factors required to convert solutions for the soil–liner interaction for load case 1 to those for load case 2.

With regard to the behaviour of buried pipes and culverts, Fig. 3 of the Note was drawn with that situation in mind. However, recourse to the literature shows that an analysis which is based on the idealization of the pipe or culvert being surrounded by a uniform elastic material may not be relevant. Höeg (1968) reviews a number of cases in which the measured pressure distributions do not have the symmetry about the horizontal diameter predicted by the simple elastic analysis. He explains that the problem is related to the method of placing the backfill around the pipe. The densities beneath the springline tend to be lower than those above, so that the stiffness of the material surrounding the bottom half of the pipe is less than that over the top half. Höeg (1968) and Valsangkar & Britto (1979) found that when special placement techniques which gave a uniform backfill were employed, the simple elastic analysis gives a good description of the observed behaviour. However, these idealized placement techniques are not relevant to the field situation. Also in many field cases there are additional complications due to the cutting of a trench for the pipe or the presence of a stiffer material beneath the bottom of the pipe. Thus, the assumption of a uniform elastic surrounding material is even less likely to be true for the pipe or culvert situation that it is for the tunnel case.

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Reinforced earth structures situated on soft foundations

C. J. F. P. JONES and L. W. EDWARDS (1980). *Géotechnique* **30**, No. 2, 207–213

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The Authors discuss the role of the surrounding soil in the design of a reinforced earth retaining wall. They draw attention to a paradox, namely that reinforced earth walls have proved to be successful when founded on soft soils whereas it is possible to demonstrate that a conventional reinforced concrete retaining wall theoretically mobilizes less damaging base stresses than its more novel counterpart. This is no paradox. The forward projection of a reinforced concrete base beyond the face of a conventional wall obviously has the beneficial effect of displacing the centroid of that base in the direction required to resist the applied overturning moment. Almost any textbook will advise such an offset to counter moments due to dead loads on foundations. If a designer of reinforced earth structures wished to use this technique he could do so by constructing a projecting base platform in reinforced concrete, or even in reinforced earth. This issue has little to do with the selection of a wall type to stand on poor soils. The particular merit of most reinforced earth systems is that the bold joints between the facing panels invest the structure with an almost unrivalled capacity to deform without this being noticed. This leads to the satisfactory use of a reinforced wall with little or no expenditure on foundations, in conditions which would demand the use of piles if a conventional smooth-faced monolith were required.

The Authors go on to claim that 'the global failure criterion is not one of overturning about the toe, as the application of some design methods suggest, but that of a rotational slip through the retained embankment'. In this argument they seemingly confuse the concept of physical rotation with that of the moment of a force. Their contribution evidently seeks to cast doubt by innuendo on the method of enhancing the vertical base stress supposed to act under the toe of a reinforced earth wall by a factor $1+k_a H^2/L^2$ due to over-turning stresses $k_a \gamma Z$ which are taken to act horizontally on the reinforced zone. This calculation is, of course, a manifestation of engineers' beam theory, and invites the designer of a retaining wall to consider the foundation soils on which it stands as equivalent to the cross-section of a wide column. If the wall has to carry a moment of $k_a \gamma H^3/6$ from its backfill, then it must create a counterbalancing reaction at its base. On the further assumption that the induced-stress distribution is linear, the result is an increment of $k_a \gamma H^3/L^2$ to add to the overburden pressure γH at