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Bedding error in triaxial tests on granular media

R. W. SARSBY, N. KALTEZIOTIS and E. H. HADDAD (1980). *Géotechnique* **30**, No. 3, 302–309

Mr R. J. W. McDermott, *Gwynedd County Council, North Wales*

The Authors have shown that lubricated ends can be calibrated for a range of particle sizes and this therefore further substantiates their benefit to triaxial testing.

As indicated in the Authors' Fig. 6, a number of repeat calibrations should be made to take account of the effects of any random variations. I have found that a useful expedient in standardizing the preparation of lubricated ends is to preload them overnight by placing a 20 kgf weight (giving 25 kN/m^2 for a 100 mm dia. platen) on top of the lubricated ends interspersed with a flat 100 mm dia. Perspex or metal block with tissue paper at either end to prevent sticking. The result the next morning is a uniform flat rubber disc with a uniform layer of grease beneath the rubber disc perceptibly free of air bubbles (McDermott, 1972).

I have studied the effects of axial bedding error and boundary membrane penetration for fine sand in both isotropic (CR = 1) and anisotropic ($1 < CR = 4$) consolidation tests, as part of a study to formulate a self-consistent model for the recoverable component of deformation for granular material. I was concerned with the recurrent bedding error due to particle embedment both at the lubricated ends (axial) and at the membrane boundary (radial) which affects first load and in particular cyclic strain data.

Consider a sand sample undergoing small strains in an axisymmetric (triaxial) stress system ($\sigma_2' = \sigma_3'$). Then

$$v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$
$$= \varepsilon_1 + 2\varepsilon_3 \quad (1)$$

where v is measured natural volumetric strain, ε_1 is measured natural axial strain and $\varepsilon_{2,3}$ are deduced natural radial strains. For an isotropic material in a CR = 1 test

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3$$

and from equation (1)

$$\varepsilon_1 = \varepsilon_3 = v/3 \quad (2)$$

Figure 1 shows that the as measured strains for dense fine sand after a few cycles in a CR = 1 test follow equation (2) closely and this implies that the bedding errors in the axial and radial directions are either insignificant or equal in magnitude. Similar results indicating the isotropic nature of a range of sands in CR = 1 tests after cyclic loading are given by El-Sohby (1964) and Andrawes (1964) (reported in El-Sohby & Andrawes, 1973).

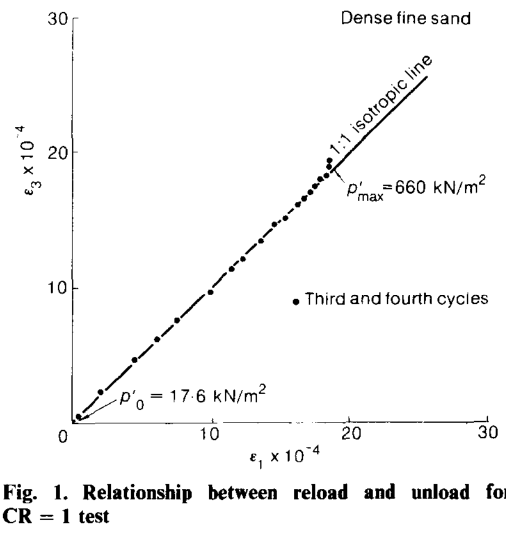

Fig. 1. Relationship between reload and unload for CR = 1 test

Axial bedding results from a series of tests on dense fine sand using single, double and triple lubricated ends are given in Fig. 2(a) and, in agreement with the Authors' results, indicate that when considering small cyclic strains the axial bedding error is not insignificant. Furthermore, the axial bedding error, which as mentioned by the Authors is independent of sample height (but for a known sample height can be expressed as a strain), is directly related to the logarithm of the major principal effective stress as seen in Fig. 2(b) (McDermott, 1972). Equation (1) can be usefully invoked if the bedding relationship is expressed in terms of strain. The use of the parameter S suggested by the Authors makes it easier to apply corrections for axial bedding error because the parameter can be incorporated in a computer program to give corrected strain data over an applied stress range.

I assessed the extent of errors arising from membrane penetration by adopting an extended method of Roscoe, Schofield & Thurairajah (1963) and El-Sohby & Andrawes (1972) in that several dummy, aluminium, solid right circular cylinders of different diameters (but equal heights) were incorporated in the sample (McDermott, 1972). All these independent studies found that a semi-logarithmic relationship held between the membrane penetration (volume) and the mean effective pressure p' ($p' = \sigma'_3$ in CR = 1 tests).

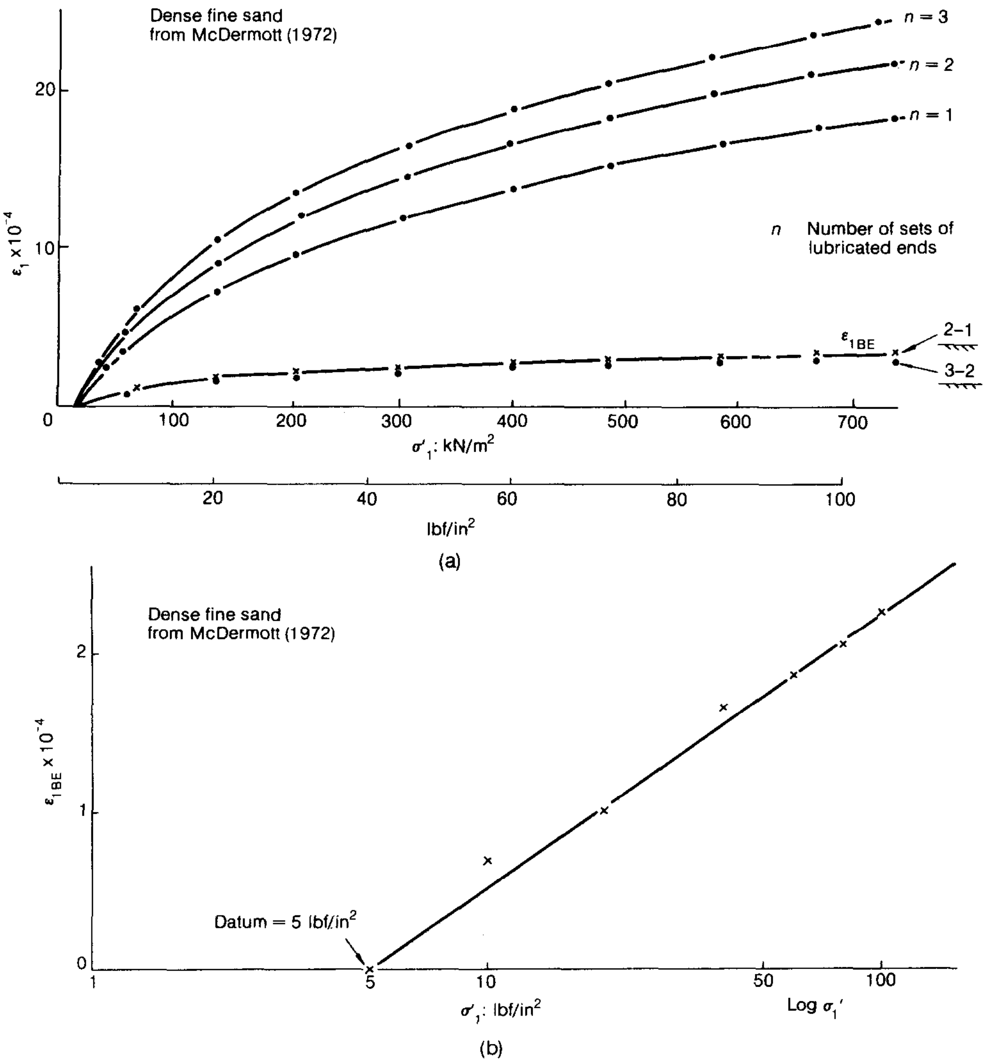

Fig. 2. (a) Bedding error at lubricated ends from CR = 1 tests; (b) relationship between bedding error (strain) and log σ_1'

Applying equation (1) to the bedding strains and rearranging gives

$$\varepsilon_{3BE} = \tfrac{1}{2}(v_{BE} - \varepsilon_{1BE}) \quad (3)$$

where v_{BE} is the volumetric strain due to both boundary membrane penetration and axial bedding. As v_{BE} and ε_{1BE} are measured, ε_{3BE} can be determined. Fig. 3 shows that the ratio of v_{BE} (membrane penetration tests) to ε_{1BE} (axial bedding tests) is 3 to 1. Therefore, from equation (3), the radial bedding strain–minor principal effective stress relationship will be (within the limits of the experimental data) of identical shape to the axial bedding strain–major principal effective stress relationship and therefore of semi-logarithmic form. This result is inherent in the equality of the as measured sample axial and radial strains given in Fig. 1.

As very small axial total strains were encountered with CR = 1 tests (less than 0·3% to $\sigma_1' = \sigma_3' = 800\,\mathrm{kN/m^2}$), natural strains ε_n are for all intents and purposes equal to engineering strains ε_e ($\varepsilon_n = \ln(1+\varepsilon_e)$). It is simpler to work in terms of engineering strains.

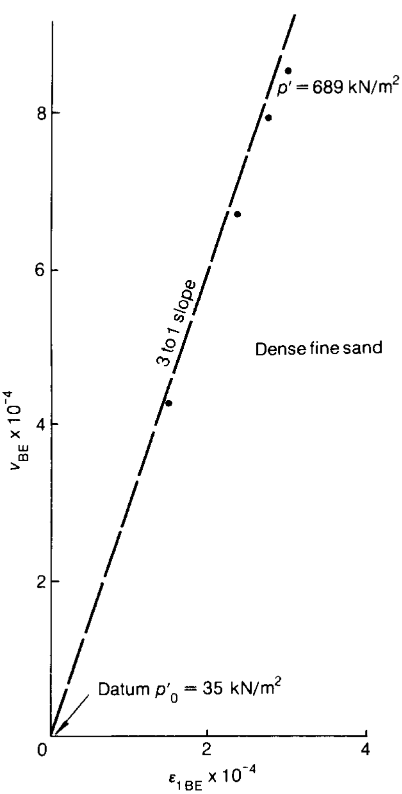

Fig. 3. Relationship between volumetric strain v_{BE} and axial bedding strain ε_{1BE}

Let D and H be the diameter and height of a sand sample respectively under an applied isotropic stress in a CR = 1 test. In the case of fine sand with a very large population of sand particles in contact with each lubricated end, a mean axial embedment Δh (equal to the axial bedding error) can be designated for a given applied stress. Likewise, let Δx be the mean depth of penetration at the boundary membrane. Using $H = D$ for samples with a 1 to 1 height to diameter ratio, the total volume error due to circumferential membrane penetration and axial bedding is given by

$$\pi D H \Delta x + 2\left(\frac{\pi D^2}{4}\right)\Delta h = \pi D^2\left(\Delta x + \frac{\Delta h}{2}\right) \quad (4)$$

The volumetric error strain is

$$v_{BE} = \frac{\pi D^2(\Delta x + \Delta h/2)}{(\pi D^2/4)H} = \frac{4}{D}\left(\Delta x + \frac{\Delta h}{2}\right) \quad (5)$$

Axial bedding strain due to lubricated ends is

$$\varepsilon_{1BE} = 2\Delta h/H = 2\Delta h/D \quad (6)$$

Applying, from Fig. 3, $v_{BE} = 3\varepsilon_{1BE}$, equations (5) and (6) give

$$\frac{4}{D}\left(\Delta x + \frac{\Delta h}{2}\right) = 3\left(\frac{2\Delta h}{D}\right) \quad (7)$$

and hence $\Delta x = \Delta h$, i.e. for fine sand particles the depth of membrane penetration at the sample boundary is equal to the embedment of the particles at each lubricated end (disc cut from same material) and is independent (to within the resolution of the test apparatus) of the adjacent boundary condition, i.e. whether it is flexible or rigid. This is probably explained by the fact that the relative thickness of the boundary rubber membrane (usually about 0·33 mm) to individual particle size (the fine sand tested was in the range 0·10–0·15 mm) was sufficient to prevent the rubber membrane penetrating into the interstices between the particles beyond a certain distance linked to the size and shape of the particles.

In anisotropic consolidation tests, because the axial and radial effective stresses are not equal, the magnitude of the bedding errors in the corresponding directions will not be equal. However, because the bedding error in the axial and radial directions is directly related to the magnitude of the corresponding principal effective stress, the unique axial bedding error–axial effective stress relationship (derived from CR = 1 tests) can be applied both in the axial and radial directions to anisotropic consolidation tests (on fine sand) (McDermott, 1972, 1980). When testing significantly larger grain size materials, because the boundary membrane is able to penetrate further into the interstices of the boundary particles, it is expected that the radial bedding Δx (flexible boundary) will be greater than the axial bedding Δh (rigid boundary). At what size of particle is this effect detected?

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Dr O. Daramola, *University of Ife, and* **Dr P. R. Vaughan,** *Imperial College of Science and Technology*

The contribution to bedding error is not restricted to the compressibility of the free ends as indicated by the Authors. In fact there are two major categories of bedding error

(*a*) the compressibilities in the measuring equipment, including the lubricated platens, load cell, porous stones and top cap

(*b*) the bedding between elements of the apparatus in contact, and between the soil and apparatus (Fig. 1(a)).

By careful calibration, some of group (*a*) can be ascertained and deducted from gauge measurements (Wesley, 1975; Daramola, 1978; Buri, 1978), but group (*b*) is of a rather subjective nature (Daramola, 1978). The degree can vary from one material to another, from one test sample to another and from one experimenter to another. Even though this can be reduced by careful sample preparation and set-up, it cannot, except in a few circumstances (Daramola, 1978; El-Ghamrawy, 1979), be eliminated.

In the soil mechanics laboratories of Imperial College a technique has been developed for determining axial deformations that are relatively free of bedding error (Daramola, 1978). Essentially, two linear variable differential transformers (LVDTs), each with an average resolution of $\frac{1}{4}$ µm were mounted on the samples at a quarter and three quarters of the height respectively (Fig. 1(b). This enabled the axial deformation of the middle half of the sample to be deduced (Fig. 1(c)), thus bypassing the bedding at the top and bottom ends, which normally form part of the overall axial deformations. Samples of Ham River sand at various densities were subjected to isotropic or K_0 consolidation history before subsequent shear.

Figure 2 shows the typical pattern of strains (Fig. 1(c)) in the samples. In the figures σ_{vi}' is the effective vertical stress before shear, σ_{vo}' is the previous effective vertical stress from which samples were rebounded and e_c is the void ratio after consolidation. The difference between the overall strain ε_o and the middle portion strain ε_a

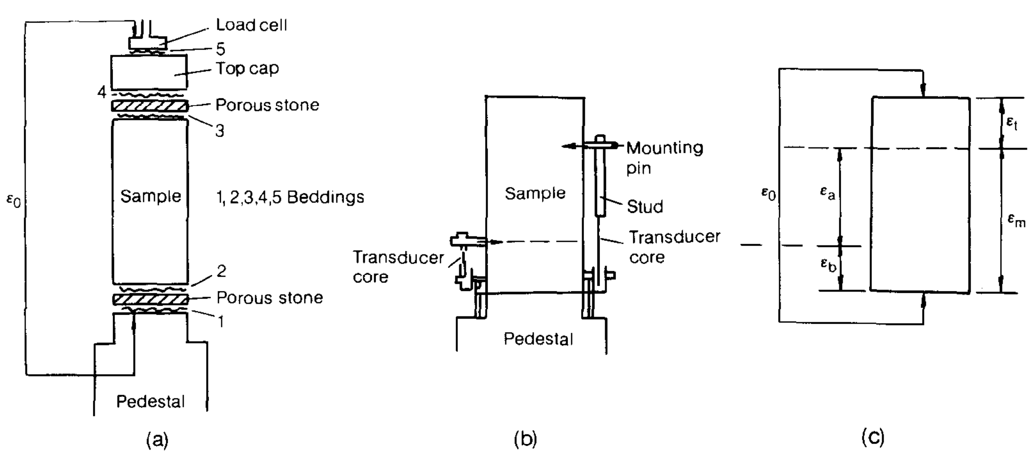

Fig. 1. (a) Bedding between elements; (b) installed internal displacement transducers; (c) measured and deduced strains

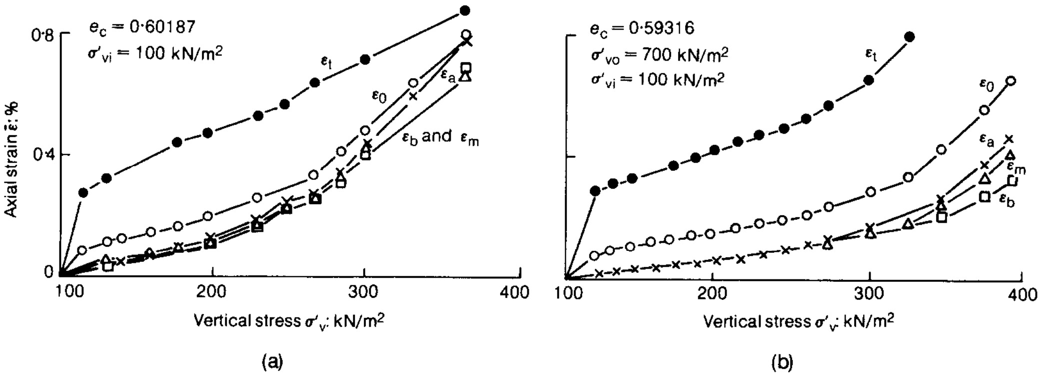

Fig. 2. Vertical stress plotted against axial strains: (a) isotropically normally consolidated sample; (b) isotropically overconsolidated sample

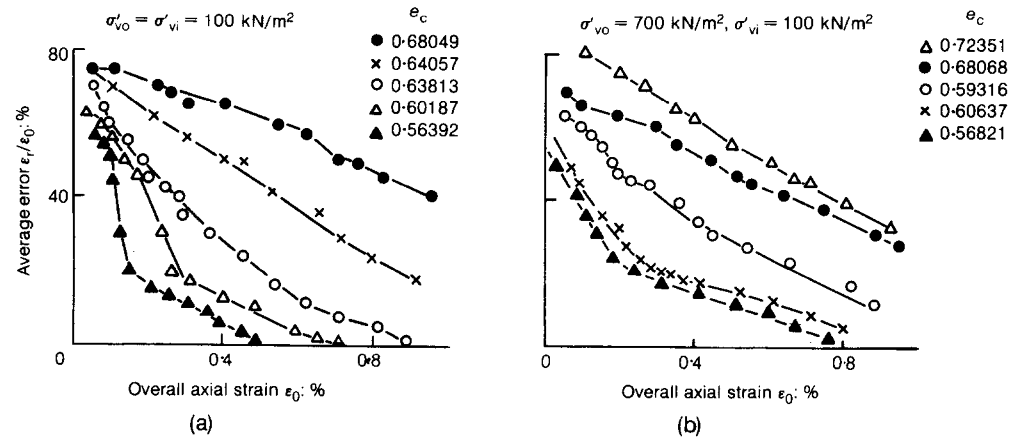

Fig. 3. Average errors in typical direct compression tests preceded by isotropic consolidation: (a) normally consolidated samples; (b) overconsolidated samples

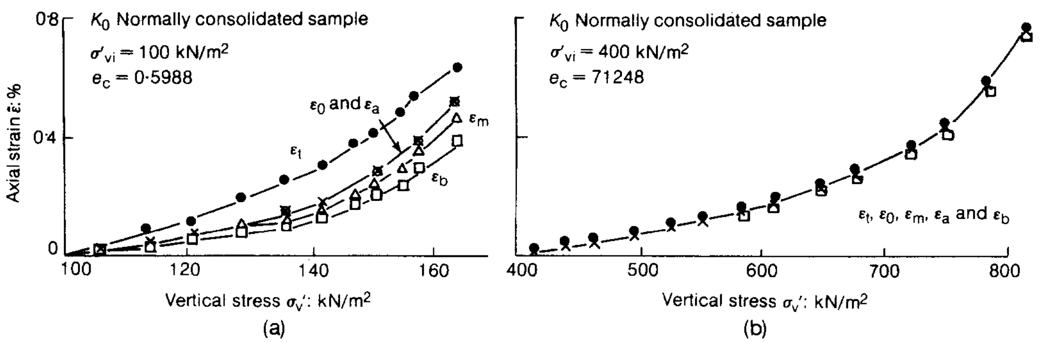

Fig. 4. Vertical stress plotted against axial strains

was considered to present an estimation of the bedding error ε_r in the test results. Such errors in a typical series of isotropically normally consolidated and overconsolidated samples are shown in Fig. 3. The pertinent aspects of these and other similar tests can be summarized as follows.

(*a*) Density of the samples is an important factor affecting the average bedding error; loose samples have a greater average error than dense samples. This may have been due to the extra difficulty involved in the preparation of loose samples.
(*b*) The contribution to bedding error was greater at the top than at the bottom.
(*c*) Higher isotropic consolidation stress or isotropic overconsolidation did not produce a dramatic reduction in the average bedding error during subsequent shear, nor did it change the pattern of greater errors being associated with looser samples.
(*d*) Irrespective of isotropic consolidation stress level, initial errors at about 0·2% average strain can be as high as 80%, and in most cases may not be less than 20%, the density and history of the sample notwithstanding.

There is, however, a marked reduction in the average bedding error in the case of samples with K_0 (anisotropic) consolidation history (Fig. 4). At low stress levels when the bedding errors were highest, the error at 0·2% overall strain was usually less than 40%, and in some cases the error at this strain was practically negligible. At higher stress levels, bedding error at all strains becomes insignificant (Fig. 4(b)).

The case of some samples which were K_0 unloaded past the isotropic stress state was instructive. The average bedding errors during the subsequent shear of these samples were higher than those in samples normally consolidated to the same effective vertical stress before shear. Thus, K_0 unloading past the isotropic stress state resulted in recovery of bedding which had been dissipated during the preceding anisotropic consolidation. As bedding errors changed with consolidation history, it seems probable that they were due to soil behaviour rather than surface to surface contacts within the apparatus.

Although there are similar trends in the work of the Authors and our work, it is felt that some of the observations by the Authors become clearer when explained in terms of, and in conjunction with, errors in category (*b*), and particularly in terms of bedding effects within the soil itself.

The degree of imperfect contact can be expected to be greater with coarser materials, hence causing an increase in the bedding error. This is confirmed by tests on clay, which show a similar pattern of error distribution, but with smaller values than in granular soils (Costa-Filho, 1979).

Unloading, especially to isotropic stress state, causes relaxation of surfaces already in contact; thus errors are still involved in reloading curves.

It is, however, surprising that the bedding error parameter S for loose samples is just, on average, 1·11 times that of dense samples. Even then this porosity factor, although small, is not reflected in equation (1) or equation (5). In addition, the scatter in the proposed relationship for S is said to be due to probable differences in methods of preparing the lubricated ends. All these give credence to the subjective characteristics of bedding error. Various factors govern the load–deformation response of a granular medium (Daramola, 1978, 1980), and that this is why granular soils, more often than not, defy empirical rules (Daramola, 1974, 1978). Thus, we are of the opinion that care should be exercised in the use of general numbers deduced from the behaviour of granular materials.

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Mr C. Sagaseta and Mr J. M. Sánchez, *Universidad de Santander*

The Authors point out two main causes for the bedding error initial lack of fit between the sample

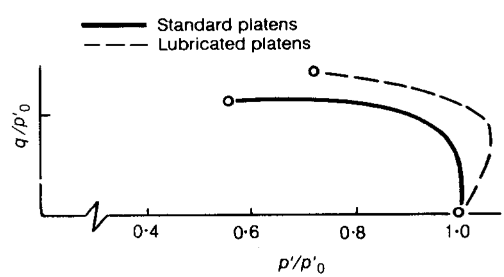

Fig. 1. Effective stress path in CIU tests on a soft clay (Sánchez & Sagaseta, 1981)

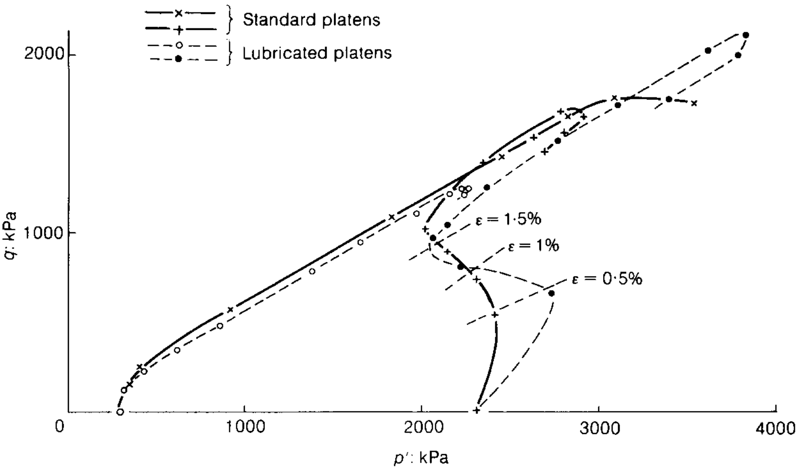

Fig. 2. Effective stress path in CIU tests on Sacramento River sand (Lee, 1978)

surface and the lubricated platen, and the penetration of the grease and disc layer by the constituent particles of the specimen. They relate the resulting bedding error to stress level and stress ratio, particle size and porosity.

In our opinion the thickness and deformability of the rubber membrane used at the ends must also be taken into account. The second of the causes indicated by the Authors affects the recorded strains in two ways: directly, because membrane penetration and compression produces an apparent vertical strain, and indirectly, because the penetration causes an increase of the sliding resistance at the specimen ends, thus implying a further restraint to the deformation of the specimen. Both effects are strongly influenced by membrane thickness. The first way is evident; the second has been analysed by Lee (1978), whose results show the influence of both particle size and membrane thickness in the sliding resistance. As the Authors give no data about the membranes used, it must be inferred that all the tests have been run with the same membrane thickness, regardless of the particle size of the soil tested.

The use of lubricated ends has been emphasized for accurate determination of shear strength of granular soils. For clays, the advantages of this system have often been questioned, because no significant difference has been found from the observed behaviour in conventional tests, regarding both strength and deformability (see, e.g., Andersen, Pool, Brown & Rosenbrand, 1980). Nevertheless, in triaxial tests carried out on a soft clay, we have found that the use of lubricated platens affects to a great extent the shape of the effective stress path during undrained loading. In Fig. 1, two effective stress paths are shown corresponding to two identical normally consolidated isotropic samples tested with smooth and rough bases. In this figure, $q=(\sigma_1-\sigma_3)/2$, $p'=(\sigma_1'+\sigma_3')/2$ and p_0 is the effective isotropic consolidation pressure. The effective stress path obtained with lubricated ends shows a bilinear shape which is partially masked in the conventional test (Sánchez & Sagaseta, 1981). This result has also been obtained in sands. Fig. 2 shows a similar plot from the tests reported by Lee (1978) on Sacramento River sand. It can be seen that, for samples which exhibit soft behaviour (in this case due to the high confining pressure), there is the same difference between the effective stress path in conventional and special tests, mainly at small strains.

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Dr F. Molenkamp, *Laboratorium voor grondmechanica, Delft*

The Authors distinguish between three components contributing to the bedding error of lubricated loading platens

(*a*) initial lack of fit between the sample (top) surface and the lubricated (top) platen
(*b*) penetration of the latex rubber disc by the constituent particles

(*c*) penetration of the thin layer of high vacuum silicon grease by the latex rubber disc due to point (*b*).

The Authors found that the bedding error during virgin loading is on average 30% larger than the bedding error during unloading and reloading.

I assume that this difference will probably be caused by components (*a*) and (*c*) and that the recoverable bedding error will probably be connected to the elastic deformation of the latex disc. To be able to support these assumptions with a theoretical mechanical model can the Authors state the roughness of the top surface of the sand on which the (top) platen is placed, the thickness of the latex disc, the thickness of the layer of grease, the viscosity of the grease and the period of a cycle of loading?

Authors' reply

We agree with Mr McDermott that for fine materials the bedding error is the same magnitude as the membrane penetration (expressed as an equivalent average deformation) at the sample periphery. As particle size increases, however, the radial bedding becomes greater than the axial bedding. For the materials tested by the Authors the limiting particle size, above which radial and axial bedding components cease to be equal, was 0·3 mm approximately, i.e. fine sand size.

In reply to Dr Daramola and Dr Vaughan, we would like to emphasize that all deformations were measured within triaxial cells using underwater dial gauges. Consequently the material present between the two points, the relative displacements of which were being measured, consisted of upper and lower platens, grease and membrane layers of the lubricated ends, the granular material under test, and any discs on which were mounted additional free ends. The bedding error was determined by comparing the axial deformation of samples containing different numbers of lubricated ends. Consequently the compressibility of the platens was eliminated from the estimation of bedding error. The discs included were made from duralumin so that even at maximum principal effective stress (3000 kN/m^2 approximately) their compression would only give rise to vertical movement of approximately $0{\cdot}06 \times 10^{-3}$ mm—even for the finest material tested this value represents only 0·8% of the deduced bedding error. The bedding error correction we determined does not eliminate any vertical movement due to compression of the loading platens. Thus, this compression will be included in the values of deformation for the particulate medium. However, because the upper platen was made from duralumin and the lower platen was made from stainless steel, under the maximum applied stresses this erroneous deformation represents (at most) only 1% of the axial deformation of the granular material.

Dr Daramola and Dr Vaughan state that the difference between their overall strain ε_0 and the middle portion strain ε_a was considered to present an estimation of the bedding error. According to their Fig. 1 they used porous stones at the upper and lower boundaries of their samples and these would give rise to lateral restraint at the ends of the sample and hence non-uniform deformation of the end zones. Consequently the strain conditions at the ends of samples would not be compatible with those of the middle portion which may, or may not, deform uniformly. Consequently the difference between the strains they measured is not due just to true bedding but is also a function of their apparatus. The lubricated platens developed at the University of Manchester eliminate restraint at the sample extremities and permit uniform deformation over the full height of the test specimen.

We agree that bedding errors occur during unloading and reloading. However, multi-cycle loading tests have indicated that after several cycles of load there is negligible change in the magnitude of either the bedding error or the true material deformations.

We also agree with Dr Daramola and Dr Vaughan that care must be exercised in the use of values deduced from the testing of granular materials. Furthermore, regardless of the corrections which may be applied to experimental data, it is essential to use apparatus and testing techniques that ensure a uniform distribution of stresses and strains within a test specimen.

In the tests we reported we did use the same membrane thickness ($0{\cdot}38 \pm 0{\cdot}05$ mm) as inferred by Mr Sagaseta and Mr Sánchez. Some tests were conducted so that the effects of membrane thickness on membrane penetration could be observed and these indicated that an increase in thickness of 40% produced a reduction in penetration of at most approximately 6%.

In reply to Dr Molenkamp, the latex disc was $0{\cdot}35 \pm 0{\cdot}05$ mm thick. The layer of grease, before the application of test loading, was approximately 0·05–0·10 mm thick. The viscosity of the grease was not applicable and the penetrometer test value for the grease was 180–210. The loading cycle was 2 h. The roughness of the top surface of the granular material cannot be specified without details of the definition of roughness.