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Centrifuge scaling considerations for fluid–particle systems

T.-S. TAN and R. F. SCOTT (1985). *Géotechnique* **35**, No. 4, 461–470

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The Authors have presented a detailed analysis of soil movement within a fluid subjected to the high acceleration field of a centrifuge. Their treatment of the problem, particularly in attemping to relate model test data to prototype scale, indicates that the centrifuge is an excellent tool for physical modelling and that in liquefaction studies it will be perfectly satisfactory for investigating the onset of liquefaction. It is also apparent that post-liquefaction behaviour will be modelled correctly when using fine sands and a model pore fluid with a viscosity that is n times greater than that of the prototype.

The Authors also present two subtly opposing views concerning steady seepage through a centrifuged soil model. Following the common practice of using the same prototype soil for the small-scale model experiments, the Authors show that a soil property, the coefficient of permeability k, is modified under the high acceleration field of the centrifuge such that it is increased by a factor n (equation (23)). This statement is misleading. For instance, it implies that in a zero gravity environment soil becomes impermeable.

Figure 1 shows a simple seepage experiment with valves B–B′ closed and A–A′ open. Under Earth's gravity a hydraulic head Δh is maintained across the sample of permeability k causing water to seep through the soil. If the model experiment is taken to a zero gravity environment, the Authors would contend that the permeability falls to zero and that seepage flow would cease. On this latter point they are correct. Since the water no longer has any weight there is no reason why water should flow from the stand-pipe at side A to that at side B. The hydraulic gradient $\Delta h/\Delta x$ is unable to force water through the sample.

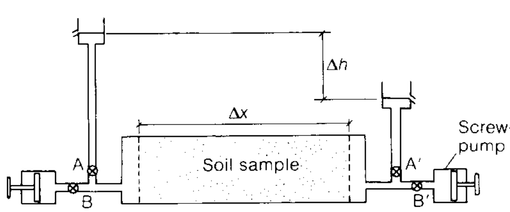

Fig. 1. Simple seepage experiment

However, suppose that valves A–A′ are now closed and B–B′ opened. Operation of the screw-pumps will force water through the sample, clearly showing that the soil is not impermeable.

The problem arises from relating Darcy's permeability to gravity-forced seepage with the traditional view of hydraulic gradient related to a drop in head of water between stand-pipes at different locations. While this is satisfactory for the majority of applications, it is a definition of limited value when seepage in different gravity environments is considered. A definition of hydraulic gradient in terms of pressure (Roscoe, 1968) instead of head eliminates the problem and is therefore to be preferred.

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Roscoe, K. H. (1968). Soils and model tests. *J. Strain Anal.* **3**, 57–64.

Authors' reply

It is apparent that some clarification and explanation are in order. The Authors' intention was simply to point out that the definition of coefficient of permeability k (length per unit time) as used in the geotechnical field bears with it the implication of its variation with the gravitational acceleration field, as well as its more frequently cited dependence on the permeating fluid density and viscosity. In the Paper, a few authorities for equation (22) and indirectly for Darcy's law in the form of equation (21) where h (dimensions of length) is the total head at a point in the fluid. (For a reference which presents the permeability expressions in essentially the same form, see Bear (1979) (note the error in equation 4-13).) Some researchers (Verruijt, 1970; Polubarinova-Kochina, 1962) combine the fluid density and viscosity in the usual way to form the kinematic viscosity ν so that g appears quite explicitly in the expression

$$k = Kg/\nu \tag{1}$$

The fundamental granular material property K is called variously the 'intrinsic', 'physical' or 'specific permeability' with dimensions of length squared. Bear (1979) refers to it as the 'permeability'.

In the production of oil, the variability in the fluid viscosity is important, and since pressures are commonly the reference variable Darcy's law is written in terms of pressure. Muskat (1949) gives the velocity components in the form

$$v_x = -\frac{K_x}{\mu}\frac{\partial p}{\partial x} \quad (2)$$

$$v_y = -\frac{K_y}{\mu}\frac{\partial p}{\partial y} \quad (3)$$

$$v_z = -\frac{K_z}{\mu}\left(\frac{\partial p}{\partial z} - \rho_1 g\right) \quad (4)$$

where x and y are the two co-ordinate directions in the horizontal plane (z is downwards), p is the pressure in the fluid and the material property K is represented for an orthotropic material with x, y and z being the principal directions. The second term on the right-hand side of equation (4) accounts for the change in elevation head. Since Dr Taylor also refers to Roscoe (1968), it is appropriate to quote the relevant portion of Roscoe's discussion. For Darcy's law, Roscoe wrote '$Q = kiA$, where Q is the volume of fluid flowing per unit time through an area A perpendicular to a hydraulic gradient i, and k is the permeability of the medium'. This conforms to conventional geotechnical usage; it is the following statement in the same paragraph which introduces a different point of view '... the hydraulic gradient is given by $i = (\mathrm{d}/\mathrm{d}L)\,(u - \gamma_f z)$'. On the previous page, Roscoe remarked '... the seepage force per unit volume is $i = \mathrm{d}/\mathrm{d}L(u - \gamma_f z)$, where u is the pore water pressure and z is the depth below the water table ...'. The terms L and γ_f in Roscoe's discussion are the length and the unit weight of the fluid respectively. It is apparent that the property 'permeability', as used by Roscoe, is neither that used conventionally in soil mechanics nor the specific (etc.) permeability used by other researchers cited earlier.

Since this topic emerged in the context of geotechnical centrifuge testing, it is worth noting what the Russian investigators Pokrovsky & Fyodorov (1975) say. They give Darcy's law in the form (p. 37)

$$v = k\,\Delta H/L \quad (5)$$

where 'v is speed of seepage', 'k is coefficient of seepage' and 'ΔH is the difference between the pressure heads bearing upon a section of seepage channel of length L'. These are the words of the translator, since we do not have the original Russian version. However, the meaning seems to be clear, as they later say that ΔH is the same in the model and the prototype, so that $v_m/v_p = n$ because $L_p = nL_m$. (In centrifuge scaling, pressures and stresses are the same in the model and the prototype.) In this form, the representation of Darcy's law seems incorrect. The change in elevation head must be accounted for. As other researchers (Harr, 1977; Bear, 1979) have pointed out, if the seepage experiment shown in Dr Taylor's Fig. 1 is arranged so that the soil sample inclines downwards from left to right (or is vertical), and a constant head test is run under gravity without the screw-pumps, and with the left-hand reservoir higher than the right-hand reservoir, the water will flow from left to right (top to bottom), under conditions which may be such that the pressure head at the right-hand end (bottom) of the soil sample is greater than that at the left-hand end (top). Schofield's (1980) remarks, cited in the Paper, have elements in common with that of Pokrovsky & Fyodorov (1975).

In summary, the conventionally employed coefficient of permeability k is related as currently defined to the acceleration of gravity g as well as to the fluid density and viscosity. If the connection to g causes conceptual difficulties, in particular at zero gravity, then a solution may be to define g in equations (22) in the Paper or equation (1) here as a constant, equal to the accepted value at the Earth's surface and independent of the level of real gravity or gravity-simulating acceleration in a particular test. This makes the permeability to water, say k_E, constant for one soil in all tests (at constant temperature) at different g levels, but Darcy's law, equation (21) in the Paper, would require to be rewritten

$$v = -k_E n \frac{\partial h}{\partial x} \quad (6)$$

to account for the real velocity change in tests where n is other than unity and to preserve the lack of dimensions of the (total) head gradient. If an experiment is to be carried out at zero or very low gravity on suitably confined soil, the flow, as Dr Taylor has pointed out, would have to be pressure driven, and an appropriate modification to Darcy's law would be required, but in this case k_E could still be employed.

Alternatively, Darcy's law could be reformulated in a pressure gradient form, as in equations (2)–(4) with the fluid viscosity alone incorporated in the permeability as implied by Roscoe (1968). This would require a considerable amount of reprogramming of textbooks, tables of permeability coefficients and geotechnical engineers' memories. If a general change of this nature were to be made, it would be preferable to make it fundamental by utilizing the specific (etc.) permeability K of equations (22) in the Paper and equations (1)–(4) here, which is solely a property of the granular material, although it should be noted that it varies with the permeating fluid for a variety of reasons. With this usage, any form of Darcy's law that is appropriate to a particular problem could be employed.

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