Davies, T. G. & Budhu, M. (1987). *Géotechnique* **37**, No. 4, 529–532

Non-linear analysis of laterally loaded piles in heavily overconsolidated clays

T. G. DAVIES and M. BUDHU (1986). *Géotechnique* **36,** No. 4, 527–538

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Davies and Budhu have presented a wealth of data on the non-linear response of laterally loaded piles, derived from a boundary element analysis where the soil is modelled as an elastic continuum and the pile as an elastic flexural member. By assigning limiting values of tensile and compressive normal traction and shear traction that act on the pile, an incremental form of the equation of flexure of the pile is used to define the nature of the redistribution of interaction tractions to interface elements that are still linear elastic. The first Writer has also studied this type of phenomenon using a modified boundary element model, rather more similar to that of Poulos (1971) than the model used by Davies and Budhu.

This discussion of the Paper by Davies and Budhu considers four points

(*a*) the need for more details of their incremental formulation
(*b*) whether negative plastic work has been considered by the Authors
(*c*) the concept of the effective length of a pile
(*d*) the predictions of the head load–deflexion response.

After studying the section entitled 'Incremental algorithm', it is not clear how the inverted soil compliance matrix, with appropriate rows and columns deleted, may be used in equation (6) to deal with limiting stress conditions at the pile–soil interface.

It may be that further details about the assumption of pile–soil compatibility at yielded soil nodes would aid understanding. In particular, in the non-linear analysis there will be a mismatch in the deflexions of the pile and soil nodes at interfaces which have reached the limiting stress condition. In the technique used by Hull (1987) the mismatch in deflexion which would have occurred in the elastic soil (under the total interaction traction distribution with soil yield) and the pile (under an equal but opposite set of tractions) is evaluated directly. This mismatch is an integral part of the solution and is a measure of the plastic deflexion. It is not clear from the Paper how the Authors' incremental algorithm treats this problem. This clarification of the assumption used would also be helpful in explaining when the soil might be allowed to return to an elastic state, i.e. 'elastic unloading'.

It is not possible to determine from the Paper whether negative plastic work, the definition of which varies according to the non-linear model used, has occurred in the Authors' analysis. In the method proposed by Hull (1987), this quantity is evaluated by multiplying the incremental mismatch displacement by the total interface traction. It has been suggested that negative plastic work has been incorrectly allowed in many finite element elastic–plastic analyses (Davis, Ring & Booker, 1974) and that the elimination of negative plastic work can produce significant improvements in the validity of the resulting solutions. Since negative plastic work is an indication that soil stresses are wrongly held on the yield surface, it will happen when the soil should be returning to an elastic state. Thus, for yielded elements, the sign of incremental plastic work and elastic unloading are inextricably linked.

In their results Davies and Budhu propose an equation to calculate the pile length beyond which all (similar cross-section) piles (in the same soil) will behave identically. A comparison of the effective lengths from use of their equation (12) with the critical lengths predicted by both Randolph (1981) (for $v = 0{\cdot}5$) and the first Writer is made in Table 1, for a range of appropriate ratios of pile–soil modulus ($K = E_p/E$).

The values of effective length calculated by equation (12) are inconsistent with the values quoted by Davies and Budhu. Except for larger values of K, their calculated values are too small and perhaps a typographical error has occurred.

The Writers have undertaken a brief comparison of results from Hull (1987), Poulos & Davis (1980) and Davies and Budhu for the example of a pile in a soil with the properties given in Fig. 1. The results, also in Fig. 1, show the non-linear load–deflexion response of the head according to Davies and Budhu (equations (10), (13) and (39)), Poulos & Davis (1980) and Hull (1987), for a pile with an installed length of 10 diameters. The results from Poulos & Davis (1980) and Hull (1987) are derived from dimensionless curves

Table 1

Researcher	Equation	Values for the following values of K		
		$K = 100$	$K = 1000$	$K = 10000$
Hull	$L_c/d = 2{\cdot}09K^{1/4}$	6·61	11.8	20.9
Randolph	$l_c/d = 1{\cdot}25K^{2/7}$	4·66	9·00	17·4
Davies and Budhu	$L_e/d = 0{\cdot}50K^{4/11}$	2·67	6·16	14·2

designed to modify elastic predictions of response. Hull's correction curves are presented in Fig. 2 and show the variation in the ratio of head deflexion allowing for soil yield to linear–elastic head deflexion F_u for various values of the ratio of dimensionless head moment to shear M/HL_c as a function of a normalized measure of head load R/R_u.

In Fig. 1, the results of Poulos & Davies (1980) and Hull (1987) agree well, as might be expected,

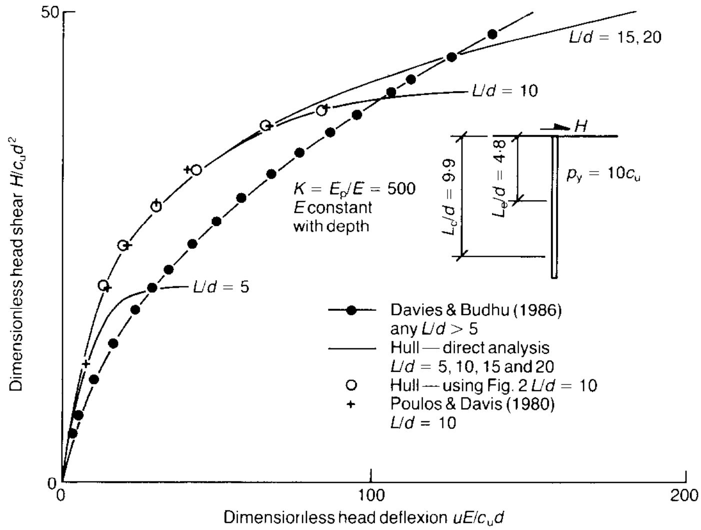

Fig. 1. Dimensionless head response to head loading

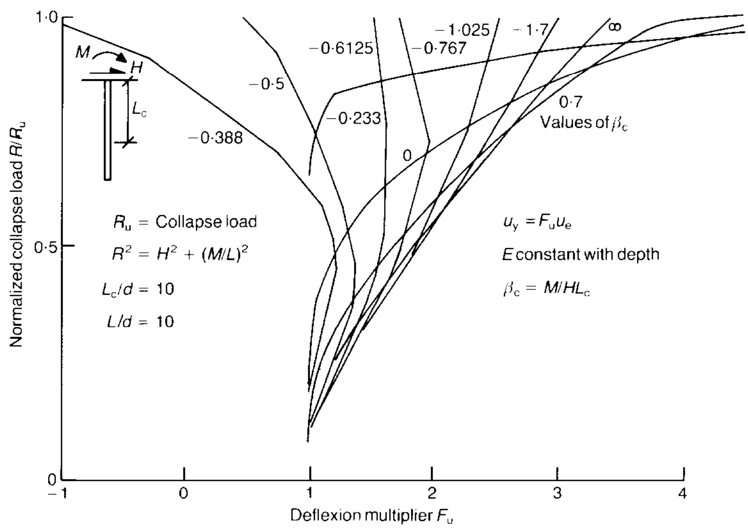

Fig. 2. Correction multiplying factors for head deflexion due to shear and moment loading for non-linear soil

but Davies and Budhu's results exhibit an earlier departure from elastic behaviour and a larger deflexion until near the ultimate collapse load (considering failure of the soil only).

Davies and Budhu's sets of curves and equations are intended to be applied to piles that are longer than their predicted effective length ($L_e/d = 4{\cdot}8$). This is also true of the results from the first Writer's analysis, provided that the critical length proposed by Hull (1987) is used (see Table 1). As an illustration, Fig. 1 shows results from direct use of Hull's analysis for four piles with the same properties as the example pile, but now with values of length-to-diameter ratio of 5, 10, 15 and 20. For these cases 20 elements were used to model either the critical length or the actual installed length of the pile (whichever was smaller, i.e. 'effective').

From Fig. 1 the three curves for $L/d = 10$, $L/d = 15$, and $L/d = 20$ are essentially the same for loads up to about 80% of the collapse load for piles with an actual length equal to the critical length, i.e. $H/C_u d^2 = 41{\cdot}4$. Piles longer than the critical length will have collapse loads that are larger than $H/C_u d^2 = 41{\cdot}4$, and for such cases (i.e. $L/d = 15$ and $L/d = 20$) the responses are almost the same (see Fig. 1) until their individual collapse loads are approached. Davies and Budhu's predictions, which essentially provide a 'backbone' curve for all 'flexible' piles, are more conservative than those of Hull for low loads, but a comparison of the responses in Fig. 1 demonstrates a reversal of this when loading exceeds Hull's collapse load based on critical length.

This again brings into question the effective length proposed by Davies and Budhu. Hull's analysis predicts a sensibly different linear response for piles of length five diameters and 10 diameters, but according to Davies and Budhu's effective length they should behave identically. Even allowing for the differences between the models used to analyse the non-linear response of piles in a yielding soil the results of the two techniques are inconsistent. More details concerning the formulation of Davies and Budhu's analysis and the characteristics of the resulting solution, e.g. a description of the growth and nature of the non-linear regions, would help to clarify the suitability of the method used. Further, a comparison of the non-linear correction curves of Davies and Budhu with those of Hull, in Fig. 2, suggests that a more complex non-linear form may be required than the essentially parabolic load–deformation relationship (linear correction) that they propose. Although some of the curves of Fig. 2 may be closely approximated by straight lines, e.g. for head loads with a positive moment ratio, by no means can the curves with $M/HL_c < 0$ be simplified to linear corrections.

The results presented by Davies and Budhu represent a new method in a field that has very little established work with which to make comparisons. The widely used *p–y* approach does not represent a solution with which the formulation of the method may be assessed: it only represents a different method of analysis of the same problem. Hopefully, the type of comparison made in this discussion represents the best way in which to establish the elastic–plastic method as a reliable and convenient alternative to the more established and empirical *p–y* methods of analysis.

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Authors' reply

The first Discusser states that he has undertaken an analysis of the problem using a model that is more similar to Poulos's (1971) than to the Authors'. It is therefore not surprising that his results are also at variance with the Authors'—this discrepancy was noted in the Paper. The Authors have not seen the paper by Davis, Ring & Booker (1974) but are familiar with a paper presented the previous year at Cambridge (Davis & Booker, 1973) which, in part, addresses the issue of negative plastic work (for more recent work on the same subject, see Schreyer (1987)). Negative plastic work may be of concern when dealing with strain softening materials and in algorithms where stresses are allowed to drift into inadmissible stress space, but in this case the soil is, conceptually, an elastic–perfectly plastic material and the Authors ensured that 'only one more segment reaches the yield conditions at the end of each increment', with the intention of preventing such excursions.

The Authors' technique for dealing with elastic unloading, which occurs primarily at high load levels as the centre of rotation of the pile displaces downwards, is as follows. If unloading tends to occur, as a result of an incrementally negative displacement with respect to the current displacement, then, in a second iteration, the associated soil segment is assumed to be elastic and the calculation is repeated. On occasion, recalculation renders the segment plastic and in such cases plasticity is enforced. Greater sophistication is not warranted in view of the model's simplicity. Further, it is unclear whether a consideration of negative plastic work as advocated by the Discussers is relevant to this problem—particularly since the Discussers' results do not differ much from Poulos's.

The Discussers rightly point out that equation (12) of the Paper is inconsistent with the text. It is also inconsistent with fig. 1 of the Paper from which it was derived. The equation should read

$$L_e/D = 1{\cdot}5K^{3/11}$$

Accordingly, equation (20) becomes

$$L_M/D = 0{\cdot}6K^{3/11}$$

Thus the corrected *elastic* effective length is eight diameters, approximately. In consequence, much of the Discussers' subsequent discussion which dwells on effective lengths becomes redundant. The concept of an effective length must be employed judiciously with regard to its definition, i.e. with respect to load level, and if load–displacement results are presented up to collapse (see Fig. 1) the concept of (a fixed) effective length becomes meaningless.

In Fig. 1 the Discussers have extrapolated the Authors' data for free-head piles from $H/C_u D^2 = 20$ up to $H/C_u D^2 = 50$. This extrapolation is invalid and leads to the incorrect conclusion, among others, that there is no failure load (defined as failure of the surrounding soil). The Authors have obtained complete load–displacement curves of the type shown by the Discussers but chose to omit the data at the higher load levels for several reasons, not least that pile yielding becomes increasingly important at these load levels and renders such results useless for practical purposes.

In practice, engineers will be concerned with load levels that are well within the ranges depicted in the Paper (i.e. $H/C_u D^2 < 20$ for free-head piles) and should therefore not be tempted into error. Equally, although the Discussers' Fig. 2 is interesting from a purely academic point of view (again pile yielding beyond working load levels is neglected), it does not negate the Authors' findings, namely that a linear correction factor is valid at working load levels (typically $R/R_u \approx 0{\cdot}25$). Further research work in this area is indicated.

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