Wu, T. W. & Lee, I.-M. (1989). *Géotechnique* **39**, No. 3, 561–562

DISCUSSION

Stability of gravity platforms on clay: reliability analysis

T. W. WU and I.-M. LEE (1988). *Géotechnique* **38**, No. 1, 101–116

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The Authors have presented a practical Paper on the application of probabilistic method in analysing the stability of offshore gravity platforms. The Authors also elucidated the step-by-step procedure for crystallizing all the uncertainties into a single useful index of risk measure, the reliability index β. However, discussion is needed on the Authors' claim that statistical correlation between spatial average strengths over different segments of the slip surface was negligible.

In performing a geotechnical reliability analysis, it is important to consider the statistical correlation of input soil parameters, which arises from three causes.

The first is that a soil property usually exhibits a positive autocorrelation in the field. In consequence, spatial averages of the property will also be correlated spatially.

The second cause is that mean values of the spatial averages over different spatial domains within a homogeneous soil profile are often estimated by the same sample mean value calculated from a common set of data. If the sample mean value underestimates (or overestimates) the true mean value of a spatial average over a specific spatial domain, the mean values of the spatial averages over other domains will all be underestimated (or overestimated) by the same amount. Therefore, the uncertainty arising from the estimation of the mean value of the random field, hereafter called the sampling uncertainty, will give rise to a positive correlation between the so-called sample spatial averages.

The third cause is that, in general, corrective factors, such as those arising from sample disturbance, the effect of cyclic loading in the stress states and the like as discussed by the Authors, have to be applied to the measured soil properties before they can be used as input soil parameters for analysis. A common set of corrective factors is often applied to spatial averages of different spatial domains, for instance, to average strengths of different segments of the slip surface. Like sampling uncertainty, if the estimated corrective factors underestimate the magnitude of the true corrective factors, all corrected soil properties will be affected to the same extent and similarly when the corrective factors overestimate the true values. This will give rise to a positive correlation between the input soil parameters.

To illustrate in more detail the correlation arising from different causes, consider a homogeneous random field $x(t)$, where t is a location parameter; $x(t)$ can be decomposed into a constant mean trend μ and a random component $\delta(t)$ with zero mean value and constant variance σ^2, i.e.

$$x(t) = \mu + \delta(t) \tag{1}$$

Suppose $\bar{m}$ is a sample estimate of μ, the 'true' spatial average $\tilde{x}_v$ and the sample spatial average $\bar{x}_v$ over a spatial domain are defined respectively as

$$\tilde{x}_v = \frac{1}{V}\int_V [\mu + \delta(t)]\ \mathrm{d}t \tag{2}$$

$$\bar{x}_v = \frac{1}{V}\int_V [\bar{m} + \delta(t)]\ \mathrm{d}t \tag{3}$$

If $\bar{m}$ is an unbiased estimate of μ, both $\tilde{x}_v$ and $\bar{x}_{v'}$ will have the same mean value of μ. Following the procedure by Li (1987) and Li & Lumb (1987), the covariance between the spatial averages over two different spatial domains V and V' is given as

$$\begin{aligned}\operatorname{cov}\{\bar{x}_v, \bar{x}_{v'}\} &= E\left\{\left[\frac{1}{V}\int_V [\bar{m} + \delta(t)]\ \mathrm{d}t - \mu\right] \times \left[\frac{1}{V'}\int_V [\bar{m} + \delta(t')]\ \mathrm{d}t' - \mu\right]\right\} \\ &= E\left\{\left[(\bar{m} - \mu) + \frac{1}{V}\int_V \delta(t)\ \mathrm{d}t\right] \times \left[(\bar{m} - \mu) + \frac{1}{V'}\int_{V'} \delta(t')\ \mathrm{d}t'\right]\right\} \\ &= \operatorname{cov}\{\tilde{x}_v, \tilde{x}_{v'}\} + \operatorname{var}\{\bar{m}\}\end{aligned} \tag{4}$$

in which the correlation between the soil property at the sample points and that within the domains V and V' is neglected. In particular, if the separation distance between the sample points in the field are large in comparison with the scale of fluctuation, the property at the sample locations can be regarded as statistically independent. In this case, var $\{\bar{m}\}$ reduces to the well known relation of

$$\operatorname{var}\{\bar{m}\} = \frac{\sigma^2}{n} \tag{5}$$

which is used in equation 11(b) of the Paper. The first term in equation (4) is a result of autocorrelation of the soil property, which cannot be reduced, and the second term is attributed to sampling uncertainty, which can be minimized by taking more samples within the field.

Turning to correlation arising from the third cause, if $N = \Pi_{j=1}^{N} N_j$ represents the overall corrective factor, the input soil parameter will become

$$\hat{x}_V = N\bar{x}_V \tag{6}$$

Using first-order-second-moment theory, it is easily shown that

$$\begin{aligned} \operatorname{cov}\{N\bar{x}_v, N\bar{x}_{v'}\} &= \bar{N}^2 \operatorname{cov}\{\bar{x}_v, \bar{x}_{v'}\} + \bar{m}^2 \operatorname{var}\{N\} \\ &= \bar{N}^2\{\operatorname{cov}\{\tilde{x}_v, \tilde{x}_{v'}\} + \operatorname{var}\{\bar{m}\}\} \\ &\quad + \bar{m}^2 \operatorname{var}\{N\} \end{aligned} \tag{7}$$

Equation (7) is the general equation for evaluating the correlation between input spatial averages under the assumptions stated above. Equation (7) becomes equation 11(b) in the Paper when the domains V and V' coincide. The Authors have stated that correlation between the average strength s_i and s_j, computed by the relations given in Vanmarcke (1977), could be ignored due to the fact that the scale of fluctuations δ_z was small compared with the separation distances of the segments. Based on this assumption, the Authors concluded that the correlation between the input spatial average strength at different segments of the slip surface can be ignored. However, it should be remarked that the relations given in Vanmarcke (1977) are developed on the assumption that the true mean value of the property is known. They are solely for evaluating the correlation arising from spatial variability (i.e. cov $\{\tilde{x}_v, \tilde{x}_{v'}\}$) rather than the correlation between input spatial averages, which is given by equation (7). The fact that cov $\{\tilde{x}_v, \tilde{x}_{v'}\}$ is small does not necessarily imply that other terms of equation (7) can be neglected in the analysis. Otherwise, the Authors would not have spent a major part of the Paper discussing the estimation of var $\{N\}$ which also appears in equation (7).

The information given in the Paper does not allow me to recalculate the reliability index to take into account the correlation of average strengths. Therefore, it is not clear to what extent the reliability index will be overestimated by neglecting the correlation between average strengths, although my limited experience on probabilistic slope analysis suggests that the difference is likely to be significant. Information on this by the Authors would be welcomed.

Authors' reply

Mr Li has provided details on spatial variation and spatial correlation, which could not be presented in the Paper because of limitation on length. The correlation between the average strengths of two segments k and l can be calculated with the simplified relations of Vanmarcke (1977), which are similar to Li's equations. The correlation coefficient is

$$\rho_{A_k A_l} \frac{A_0{}^2\Gamma^2(A_0) - A_{0k}{}^2\Gamma^2(A_{0k}) - A_{0l}{}^2\Gamma^2(A_{0l})\Gamma^2(A_{0kl})}{2A_k A_l \Gamma(A_k)\Gamma(A_l)} \tag{8}$$

where the As are areas as shown in Fig. 1 and the Γs are variance functions as defined by equation (2) of the Paper. Using the values given in Table 2 in equation (8), gives $\rho \cong 0$ for segments 3 and 4. Thus, the covariance between $N\bar{x}_v$ and $N\bar{x}_{v'}$, Li's equation (7), is dominated by the last term in the equation. The correlation between the other segments is zero because the soil properties in the different layers are assumed to be statistically independent.

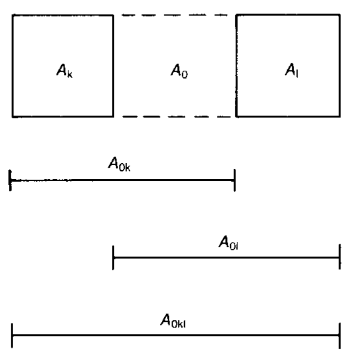

Fig. 1. Notation for areas in equation (8)

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