

DISCUSSION

Strain localization and periodic fluctuations in granular flow processes from hoppers

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The Author has presented an interesting analysis of the shear bands/rupture surfaces that form during mass flow and a rationale for their re-formation.

Analysis of displacements of lead markers in the flow of Leighton Buzzard sand in a Perspex parallel-sided mass flow bunker (Blair-Fish, 1973; Bransby & Blair-Fish, 1975) showed that strain was localized in rupture surfaces after an initial phase of radial flow. Strains were computed from measured displacements of the lead markers on radiographs. During initial flow, volumetric strains in dense sand were significant only close to the outlet. Rupture surfaces were found to form along the directions of zero extension of the initial phase of deformation. They formed in both initially dense and initially loose sand.

Could the Author comment on the theoretical shape of the rupture surfaces?

Author's reply

Dr Blair-Fish points out that in his research the rupture surfaces formed along the directions of zero extension lines and were present in the flow field of both initially dense and initially loose sand. The Author's observations are in accord with these findings.

The particular mechanism of flow presented was aimed at finding a tool for predicting the periodic reappearing of localized strain zones (rupture surfaces). The shape of the rupture surfaces was approximated to facilitate the calculations of energy dissipation during flow. No effort was directed at finding a realistic shape of those surfaces theoretically, using the approach presented.

The shape of the rupture surfaces, however, was studied theoretically using the perfectly plastic model of the granular material, where the yield condition was described by the Mohr–Coulomb function and the non-associative flow rule was used. A theoretical solution was obtained using the method of characteristics. As the solution to the velocity field can be obtained only if the directions of the principal strains are known, the stress problem was solved first and the principal stress directions were then taken as

principal strain trajectories (isotropy, co-axial model). These solutions were presented earlier for a plane converging hopper (pseudo-steady flow) and a hopper-bin bunker (steady-state flow) (Michalowski, 1984, 1987).

It was found out that using the flow rule associated with the Mohr–Coulomb yield condition leads to an unrealistic dilatancy in sand. Therefore, the non-associative flow rule was used with the plastic potential in a form similar to the Mohr–Coulomb yield condition, but with the internal friction angle replaced by the dilatancy angle (smaller than that of internal friction). As the differential equations describing the velocity field are of a hyperbolic type, they allow for strong discontinuities. The discontinuities coincide with the characteristics of the hyperbolic equations, and are inextensible lines. These discontinuities were then interpreted as rupture surfaces.

The kinematic boundary conditions determine which characteristics (if any) become velocity discontinuities. In the case of the hopper-bin con-

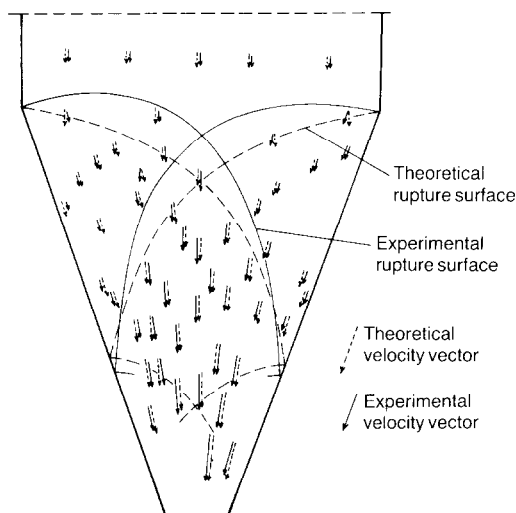


Fig. 14. Rupture surface and velocity vectors in the advanced phase of sand flow through a plane hopper-bin container with half-angle 20° and wall friction angle 11°; the theoretical (steady-state) solution is based on the rigid, perfectly plastic model of sand (internal friction angle 32° and dilatancy angle 0°)

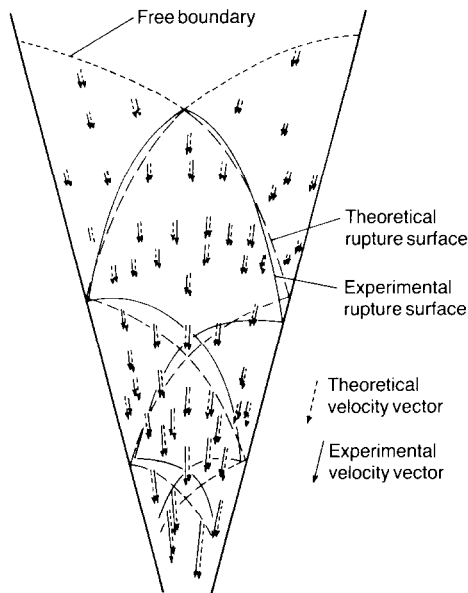


Fig. 15. Comparison of theoretical and experimental rupture surfaces and velocity vectors during the advanced flow of sand through a converging hopper (half-angle 15°, wall friction 11°, internal friction angle 32°, dilatancy angle 0°)

tainer (Fig. 14), the boundary condition is the vertical velocity of the traction-free boundary in the bin section. Material in the bin then moves as a rigid body and, because below the transition points the material at the wall has to move parallel to those walls (mass flow), flowing particles must be subjected to a velocity change ('jump') and the velocity discontinuities must originate at the transition points. The subsequent discontinuities are found from the requirements of kinematical admissibility of the velocity field (Michalowski, 1987).

For converging hoppers, the two triangular areas in the top part of the hopper (Fig. 15) were found to move parallel to the walls, approximately as rigid bodies, and the discontinuities originating from point A (Fig. 15) assure kinematical admissibility of the velocity field (Michalowski, 1984). The theoretical shape of the discontinuities was found to match the experimentally observed rupture surfaces when the dilatancy angle used in the solution was close to zero. Broken lines in Figs 14 and 15 represent the theoretical shape of the rupture surfaces for a dilatancy angle equal to zero. The theoretical solution is sensitive to the hopper angle and the wall roughness, and the tendency of the discontinuities' theoretical shape to change with variation of the

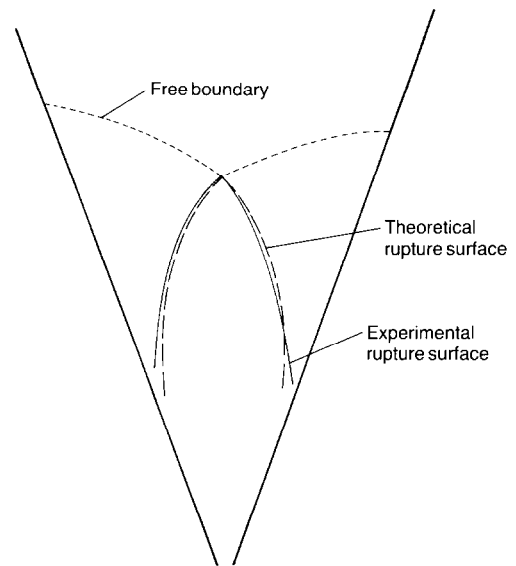


Fig. 16. Experimental and theoretical location of the shear surfaces during the advanced phase of flow through a converging hopper with half-angle 20° and wall friction angle 25°

wall roughness closely follows changes found experimentally (Fig. 16).

The dilatancy angle equal to zero describes an incompressible material. Whereas the shape of the rupture surfaces is well predicted with such an assumption, the density field is not. Theoretical velocity vectors best matched experimentally measured ones in flowing sand when the dilatancy angle in the theoretical solution was about 5–10°. The shape of theoretical rupture surfaces does not differ significantly from the shapes shown in Figs 14–16 when the dilatancy angle is increased to 5–10°.

The dilatancy angle of the Leighton Buzzard sand determined from experimental data was found by Bransby and Blair-Fish (1975) to be between 19° and 26°. The Author's experiments were performed on similar material; however, the measurements of the velocity field were taken at an advanced flow phase when a significant amount of the material had already left the container. The material that was fed into the converging part of the container was likely to have a decreased density (relative to the initial one) due to the rarefaction wave propagating upwards in the bin in the initial stage of the process (Michalowski, 1987). A dilatancy angle lower than that found by Bransby and Blair-Fish (1975) is therefore not surprising. It also may be pointed out that, in a non-stationary flow field (such as at the beginning of the flow process, or in the

advanced phase, as suggested in the Paper), the determination of the dilatancy angle based on the displacement field at one instant (for one time increment) may lead to unrealistic results.

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