Been, K., Jefferies, M. G. & Hachey, J. (1992). *Géotechnique* **42**, No. 4, 655–663

DISCUSSION

The critical state of sands

K. BEEN, M. G. JEFFERIES and J. HACHEY (1991). *Géotechnique* **41**, No. 3, 365–381

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We would like to ask whether the critical state parameter of dense sand can be measured by a drained or an undrained triaxial test.

For drained $d\sigma_3' = 0$ tests there is evidence to indicate that the observed strain softening (as in Fig. 6(h)) is not a material behaviour, but the result of non-homogeneous deformation (Hettler & Vardoulakis, 1984; Chu, Lo & Lee, 1992a, 1992b). It is therefore highly questionable to measure the critical state parameter of a dense sand by a drained test. For undrained tests, the Authors observed a steady state for a dense sample (Fig. 6(c)). We wonder whether this is also due to non-homogeneous deformation, just as in a drained test. We have conducted a number of undrained tests for dense Sydney sand, which is a quartz sand with mean grain diameter of 0·3 mm. None of these tests exhibited a steady state. It was observed that an undrained test led to a stress path approaching a constant stress ratio line (Chu, 1991). This agrees with the t versus S' curve in Fig. 6(c). If a steady state can be assumed to occur in an undrained test, the stress ratio at the steady state will be the same as the slope of the resultant stress path. The principal stress ratio determined from an undrained test is 3·55. This is larger than the stress ratio at critical state—3·28—which was measured for loose sand along a drained $d\sigma_3' = 0$ test at a similar effective confining stress. The Author's data indicate the same fact. Unless there is reliable and sufficient data to show that the same sand but with different initial densities will approach a different critical state, the critical state parameter of dense sand measured in undrained tests will not be suitable for verification purposes. Fig. 16 may therefore need to be re-examined.

Is the quasi-steady state a result of non-homogeneous deformation?

The Authors indicate that the quasi-steady state may be due to the non-homogeneous deformation of samples. However, this is not generally true. Quasi-steady behaviour in an undrained test can be a material property. Fig. 18 presents the effective stress paths of three undrained tests for loose Sydney sand. These tests were conducted with deformation controlled loading. The void ratios after consolidation for the three tests are 0·8764, 0·8493 and 0·8380. Test U3 showed quasi-steady behaviour. When quasi-steady flow occurred, no sign of non-homogeneous deformation was observed. A comparison of tests U2 and U3 indicates that whether a sample will manifest a steady or a quasi-steady behaviour in an undrained test depends on the void ratio of the sample. Fig. 18 also shows that all the tests end at an ultimate line, irrespective of whether the behaviour is steady or quasi-steady. This line is the so-called steady state line in q–p' space.

For loose sand, the critical state is the failure state measured in a drained test and the steady (or quasi-steady) state is the ultimate state in an undrained test. The equivalence of the critical state and the steady state reflects the fact that the failure stress ratio measured from a drained test is the same as the ultimate stress ratio measured from an undrained test for loose sand. However, for dense sand, the critical state is different from the failure state. The relationship becomes more complicated.

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The Authors found that for a wide range of pressure, the representation of the steady state line in terms of void ratio and log p' is almost bi-linear. This can be a simple and good approximation in some cases. However, the physical interpretation given by the Authors to the break point seems to be obscured by the use of a semi-logarithmic plot.

The steady state line of Toyoura sand evaluated by Verdugo & Ishihara (1991) is indicated in Fig. 19, together with the steady state line of Erksak 330/0·7 sand using the Authors' data. This semi-logarithmic plot shows that both sands have relatively linear and flat steady state lines for low pressure. For a medium level of pressure the steady state lines are highly non-linear and steeper, and for a high level of pressure they are nearly linear and much steeper than before. This trend is consistent with results given by Castro, Poulos, France & Enos (1982) and Seed, Seed,

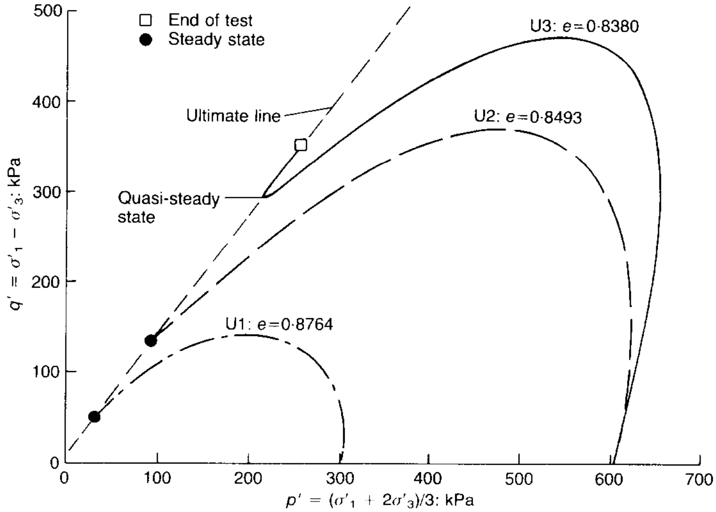

Fig. 18

Harder & Jong (1988). The results shown in Fig. 19 are replotted in Fig. 20 using an arithmetic scale. For Toyoura sand the steady state line is curved from $p' = 0$ to about $p' = 500$ kPa and thereafter is linear up to approximately $p' = 3200$ kPa, the extent of my data. In spite of the scatter at high stress, a similar pattern is observed for Erksak 330/0·7 sand using the Authors' data, but in this case the linear behaviour starts earlier from $p' \approx 200$ kPa. It is evident that the break point at $p' = 1000$ kPa found by the Authors using a semi-logarithmic scale is not manifested in the arithmetic plot. This fact probably indicates that the same mechanism of shearing exists before and after the appearance of the break point in the semi-logarithmic plot. Also in the arithmetic plot, a kind of break point where the steady state line changes from a non-linear to a linear relationship is observed, but it is located at low stress level, typically 100–500 kPa (Verdugo, 1989), and probably it is not related with particle breakage due to the low level of pressure involved. Perhaps, the particle breakage caused by the usage of high pressure can explain the scatter observed in the steady state line on Erksak 330/0·7 sand. If the particle breakage is important, the grain size distribution of the material would be modified, and because the

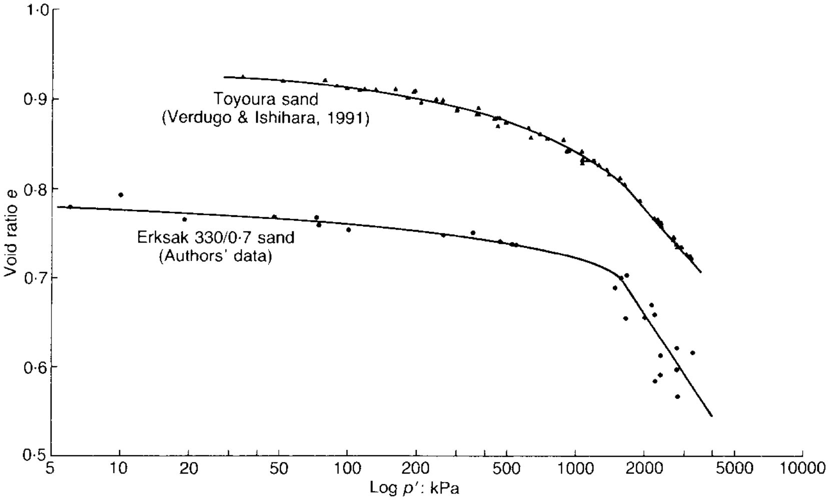

Fig. 19. Steady state lines on semi-logarithmic scale

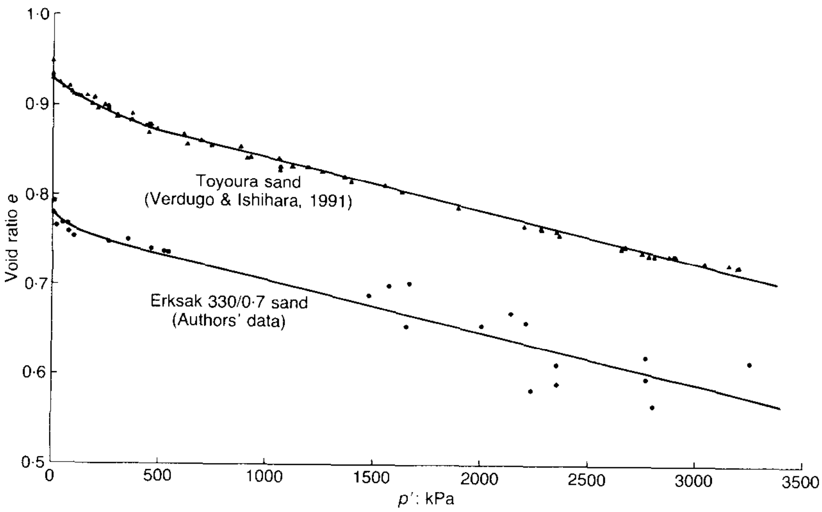

Fig. 20. Steady state lines on arithmetic scale

steady state condition is sensitive to grain size (Poulos, Castro & France, 1985) it would be affected.

I believe that when a small range of pressure is under study, the steady state line can be represented by a linear relationship in e–log p' space, but when the range of pressure under consideration is not small a better representation can be obtained by a bi-linear relationship in $e - p'$ space (Verdugo, Ishihara & Towhata, 1989).

There is a difference between the fabric and structure of a soil mass which can help the interpretation of experimental results with regard to the effect of sample preparation. For this discussion the fabric may be defined as the microstructure of the granular mass that involves only the orientation of individual particles and the distribution of the normal contacts between particles; the structure of a soil mass includes the particle size, distribution of particle size and voids throughout the soil mass and interparticle forces. The definition of fabric given by the Authors includes cementation at contacts, whereas in my definition this is included in the structure. Using this distinction, the bond between grains created by ageing, for example, should be reflected in the structure of the soil, but not in its fabric. Sample preparation methods that generate homogeneous samples, such as moist tamping, air dry pluviation and wet pluviation, affect only the fabric of the soil mass, but methods such as water pluviation can also affect the structure of the soil mass because they create a different distribution of the particle size throughout the soil mass, especially when the soil contains fines. In my opinion the initial fabric of the soil does not affect the location of the steady state line because the level of strain required to achieve the steady state condition erases the initial fabric. In this sense I agree with the Authors' conclusion regarding the effect of moist tamping and wet pluviation methods of sample preparation on the location of the steady state line. However, I believe that the initial structure of the soil affects the position of the steady state line. For example, Fig. 21 (Vasquez & Dobry, 1988) shows the steady state lines for samples prepared by moist tamping and by sedimentation under water. Sample preparation has a clear effect on the location of the steady state line. This is because the sedimentation under water creates a heterogeneous specimen with stratification which cannot be erased even at large deformation.

Further evidence is given in Fig. 22 (Verdugo, Ishihara, Daud & Towhata, 1989) concerning the effect of ageing on the steady state line. Reconstituted and undisturbed specimens taken from an archaeological excavation with clear evidence of boiling sands were tested. The samples were taken from the trace left by the boiling materials. According to archaeological studies the liquefaction of the underlying layers that created the trace of boiling sand probably took place about 400 years ago. The undisturbed samples had an average fines content of 8%. The reconstituted samples were prepared from two batches of soil: the original sandy soil with 8% of fines and a clean sand free of fines obtained by the original soil being washed through a number 200 sieve. The undisturbed specimen developed the steady state condition above and to the right of the steady state line of reconstituted specimens with

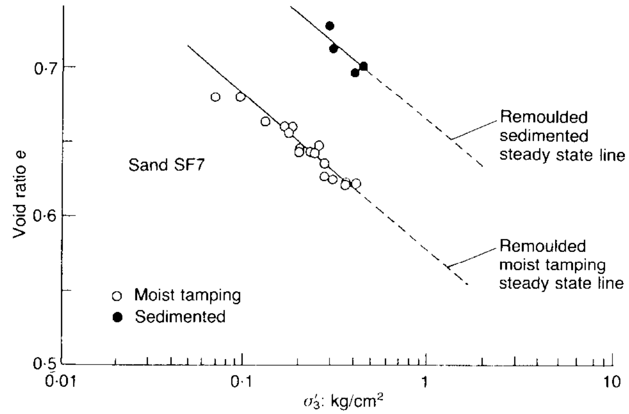

Fig. 21. Steady state lines for sedimented and moist tamped specimens

8% of fines. Moreover, one undisturbed sample developed a steady state condition above the steady state line for clean sand. This indicates that the structure developed after 400 years clearly affects the steady state condition. This pattern was consistently observed in four additional sandy soil deposits (Verdugo, 1989).

To evaluate the friction angle ϕ_{cv} from undrained tests on very loose specimens is complicated because of the membrane compliance. My experience is that usually in these tests, after some relatively small level of axial deformation, the membrane is crumpled and I am not aware of a reliable membrane force correction under this condition of the membrane. If the ordinary membrane force correction is used, the actual axial load on the specimen is underestimated and therefore ϕ_{cv} would also be underestimated. However, experimental evidence (Tatsuoka, Sakamoto, Kawamura & Fukushima, 1986), indicates that drained tests on initial medium-dense specimens under small confining pressure do not show any significant drop in the internal friction angle mobilized at large deformation close to the steady state condition. This observation, together with the Authors' conclusion that the same steady state line can be obtained from undrained and drained tests, and the experimental complications, lead one to question the dependence of ϕ_{cv} on void ratio and stress level observed by the Authors. Furthermore, for extremely loose specimens tested under undrained conditions, I have not found ϕ_{cv} to be dependent on the initial state of the soil (Verdugo & Ishihara, 1991; Verdugo, Ishihara & Towhata, 1991).

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We would like to present experimental and conceptual arguments contrary to the Authors'

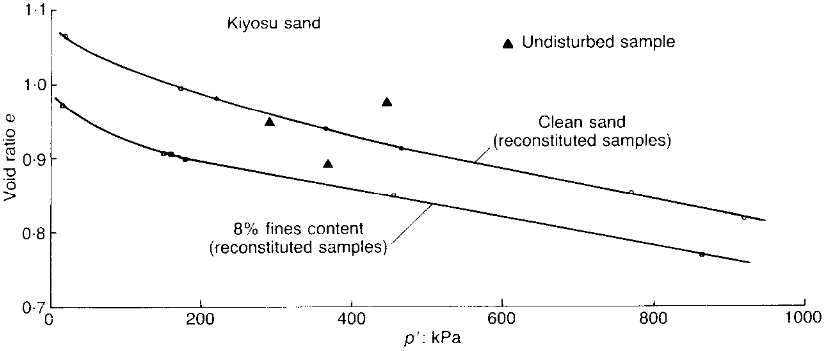

Fig. 22. Steady state lines for reconstituted and undisturbed samples

conclusions that the steady/critical state is unique for a sand (specifically that it does not depend on stress path or initial state), and that the friction angle at the critical state decreases with increasing initial void ratio. We would also like to point out that most steady state concepts have been developed using void ratio states of sands that are inaccessible to them in the water pluviated state, and hence are of questionable relevance to the design of hydraulic fill earth structures.

The loosest void ratio for Erksak sand reported is 0·735 (Table 1), but the flatter section of the steady state line (Fig. 9) corresponds to void ratios which are looser than the loosest. Clearly, such void ratio states are not possible for sands that are water/air pluviated, and hence are not relevant to the definition of steady state in practical design. Recent studies on Duncan Dam foundation sand that was sampled after freezing of the ground, and which is considered very loose (D_r = 30%, $(N_1)_{60} = 10$), show in situ void ratios nowhere greater than the ASTM loosest condition (with suitable account taken of densification under the current overburden stress). Similar deposition void ratio states in relation to the ASTM loosest should apply to other water deposited hydraulic fill and fluvial sands.

Loose water deposited sands have been shown to be anisotropic (e.g. Symes, Shibuya, Hight & Gens, 1985; Arthur & Menzies, 1972). Their undrained response should therefore depend on the direction of σ_1 during loading in relation to the deposition direction. A typical example of such anisotropic undrained behaviour of a tailings sand water pluviated in the loosest state in triaxial compression and extension loading is shown in Fig. 23 (Kuerbis & Vaid, 1989). Whereas contractive behaviour is noted in extension with clear evidence of approach to the steady state, compression response is dilative at the same initial void ratios. Similar evidence for other sands has been presented by Bishop (1971), Miura & Toki (1982), Hanzawa (1980) and Kuerbis & Vaid (1989). Thus, at a given void ratio the steady state may be reached in extension through contractive deformation. However, no such state may occur in compression except through an intense dilative process with continued shearing. This will cause σ_3' to increase to very high values by a continued decrease in pore pressure so that the sand gets transformed into a contractive one. Behaviour of this type has been shown by Poulos (1981) and Negussey, Wijewickreme & Vaid (1988), and it clearly implies non-uniqueness of steady state in e–p' or e–σ_3' space.

The Authors' conclusions regarding the uniqueness of the steady state line in e–p' space of Erksak sand are based on compression and extension tests on moist tamped samples only. It is likely that moist tamping, notwithstanding yielding void ratios which are inaccessible to pluviated materials, does not give rise to pronounced inherent anisotropy as it occurs on pluviation, and thus masks the differences in compression and extension response.

Sample set	Compression		Extension		σ'_{3c}: kPa
	e_c	Dr_c: %	e_c	Dr_c: %	
5	1·027	40	1·025	40	200
6	1·044	35	1·056	33	350
7	1·038	37	1·033	38	500

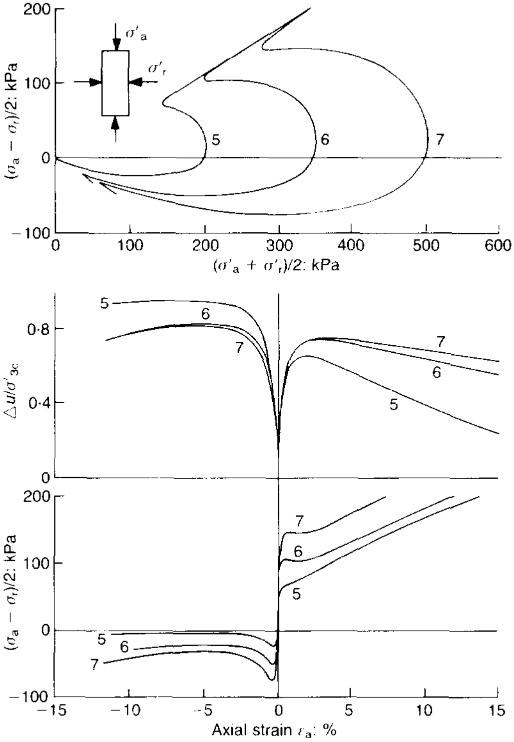

Fig. 23. Undrained compression and extension behaviour of a uniform tailings sand (after Kuerbis & Vaid, 1989)

Moist dumped sands are, in fact, particularly prone to liquefaction due to their metastable honeycomb structure (Casagrande, 1976). Modelling loose water deposited sands by moist tamping might unjustifiably condemn them as being potentially liquefiable in triaxial compression. However, they could be liquefiable under other loading modes, such as extension.

The Authors conclude uniqueness of the steady state line of Erksak sand by looking at its trace in e–p' space only. Uniqueness cannot be accepted unless its trace in p'–q space is also unique. This is clearly not the case, as evidenced by there not being a unique value of ϕ' at the critical/steady state. A large range of ϕ_{cs}'/ϕ_{ss}' (between 13° and 34°) is shown in Table 2 and Fig. 16. This is clearly in conflict with critical state concepts. Uniqueness of the critical state in both stress–void ratio and stress spaces has been demonstrated by Wroth (1958) in simple shear tests, and Castro, Poulos, France & Enos (1982) have shown uniqueness of the steady state in stress space—contrary to the Authors' findings.

Authors' reply

Two different issues are raised in the discussion: practical design using sands and a more fundamental exploration of the critical state concept for sands. The intention of the Paper was to examine the critical state of sands, but both issues are addressed here.

Much of the Paper and discussion revolves around the question of the uniqueness of the critical state line, as measured in the triaxial apparatus. It is interesting to invert the problem, and pose the question as follows: If a unique critical state line for a sand is assumed to exist, and this line is assumed to be the ultimate steady state of all distortional processes to the soil under monotonic loads, what would be measured in the triaxial test under different conditions? An answer to this question requires a constitutive model for the triaxial test based on these assumptions to predict the result of tests. Such a model, called Nor-Sand, has been developed by Jefferies (1992) for drained loading and extended to undrained loading by Jefferies & Been (1992).

Nor-Sand was used to determine the location of the S line that would be determined from conditions in drained triaxial tests on samples initially denser than critical and the F line from samples looser than critical in undrained tests. Fig. 24 shows the results of a series of drained and undrained Nor-Sand simulations, using identical material parameters, but starting from different initial states. In real triaxial tests the maximum axial strain is limited to about 20%, at which point the sample has undergone large deformations. Therefore, for these simulations, the 'critical state' was assumed to occur at the end of the test, i.e. 20% axial strain. There is a clear distinction between the S and F lines derived in this way, despite the fact that the model explicitly includes a unique critical state locus. The discrepancy is due to the fact that at 20% strain the dense samples have not yet reached an ultimate steady state. Fig. 24 also shows that the F line, determined from undrained tests on loose samples, lies close to the CSL and may therefore be used in practice. In contrast, the S line is below the CSL. This numerical example illustrates that the interpretation of laboratory experiments to determine the critical state of sands requires care and may be problematical for dense sand, even leaving aside questions such as non-homogeneous samples and strain localization.

Chu and Lo suggest that strain softening of dense sand is not a material behaviour but is the result of non-homogeneous deformation. This suggestion can interpreted in two ways, and the difference is important. First, non-homogeneous deformation could be caused by limitations of the triaxial apparatus (e.g. end platen restraint); in this case the softening is an artefact of the experiment. The tests in this study used the conventional 2 : 1 sample height to diameter ratio as well as

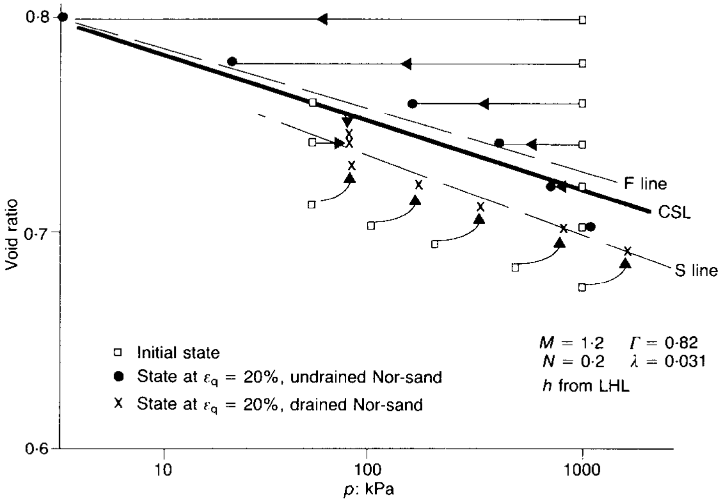

Fig. 24. State diagram showing results of Nor-Sand simulations for drained and undrained tests with S and F line criteria from limits of triaxial apparatus

lubricated end platens and it appears unlikely that there was anything particular to the Erksak test programme to cause unrepresentative behaviour. The alternative interpretation is that non-homogeneous deformation could be a result of strain softening; this is predicted by plasticity theory (Drucker's stability postulate) under work softening conditions. We therefore regard non-homogeneous deformation as a fundamental aspect of the sand behaviour.

The proposition that ϕ_{cs} may be a function of density has been questioned. As was pointed out in the Paper and by Verdugo, the experimental evidence is not convincing as there are extreme problems with measuring very small deviatoric stresses in loose samples. Although there may be intellectual grounds for a reduced ϕ_{cs} on very loose sands, for practical purposes it is likely to be sufficient to use a single value of ϕ_{cs}.

Vaid and Pillai assert that non-uniqueness of the CSL follows from a range in ϕ_{cs} quoted in the Paper. Notwithstanding that ϕ_{cs} may be constant, a variable ϕ_{cs} does not conflict with a critical state framework for soil provided that ϕ_{cs} is a single valued function of void ratio (which is what was suggested in the Paper). In general, the CSL is unique provided it forms a single unfolded surface in four-dimensional σ_1, σ_2, σ_3, e space. There is no requirement for this surface to have any particular shape, or indeed a linear projection of transformed conical form in simplified plots.

A view that does emerge from the questioning of q–p' uniqueness, but which was not discussed in the Paper, is the mapping between compression and extension tests in q–p' space. In general, ϕ_{cs} is approximately the same under conditions of triaxial extension and compression, and therefore $M = q_{cs}/p_{cs}'$ on the CSL in extension and compression space must be necessarily be different (Wood, 1990). Specifically in the case of Erksak sand, for all but the loosest samples, $M = 1{\cdot}2$ in compression and $M = 0{\cdot}85$ in extension. It follows that the CSL uniqueness in the e–log (p') plane implies quite different critical state shear strengths depending on stress path (as suggested by Vaid and Pillai).

Vaid and Pillai argue that sand samples are almost always anisotropic and therefore behaviour will be different in extension and compression. We agree with this proposition; in fact the data in the Paper strongly support this. However, this proposition cannot be extended to imply non-uniqueness of the CSL. The Erksak test programme explicitly included several undrained extension tests and Fig. 13 compares the CSL of undrained extension and compression tests in the e–log (p') plane. No substantial difference was found. The apparent difference between our tests and the results of Vaid, Chung & Keuris (1990) may stem from the quasi-steady state being taken as the critical state (which it is not) and limitations of the triaxial tests to determine the CSL for dense sands.

Alternatively, the difference could revolve around sample preparation technique. Vaid and Pillai suggest that moist tamped samples do not give rise to pronounced anisotropy as do pluviated samples. This argument is rather speculative, and we are unaware of any physical data on particle packing arrangements to support this assertion. As noted in the Paper, there is an urgent need for a technique to quantify inherent anisotropy properly so that this question can be examined.

Until such time as soil structure can be measured, Casagrande's concept of a metastable structure cannot be accepted as fact and used to support the existence of a particular packing arrangement in sands. In fact, the term metastable is contentious and may even convey an incorrect concept. The stress strain behaviour of a drained test (D682) is shown in Fig. 6(b). This sample was as loose as samples which liquefied under undrained loading. The drained sample does not show a metastable collapse but moves monotonically and steadily from the initial to the final states. The behaviour of contractive or liquefiable sands can be explained adequately in terms of either the Nor-Sand critical state model (Jefferies & Been, 1992) or a double hardening model (Molenkamp, 1981) without invoking contentious concepts such as metastable particle arrangements.

It is interesting to note that Verdugo has a different view from Vaid and Pillai, and possibly an explanation for differences between pluviated and moist tamped samples. He presents evidence that sample non-uniformity in terms of the distribution of grain sizes (defined by him as 'structure') causes very different behaviours. This is also consistent with the well-known fact that small changes of grain size distribution can alter the CSL of sands significantly. It would be interesting to see more detailed results of grain size distributions in test samples, and how shear strains within the samples relate to the grain size distribution.

Verdugo also raises the point that the curvature of the CSL is dependent on the axis chosen to plot the line, and therefore little importance should be placed on the point where the curvature changes. We agree that the approximation used to describe the critical state (e.g. linear, bilinear or curved) should be selected based on the engineering problem to be solved. However, the reason for using a semi-logarithmic plot is rooted in the empirical observation that soil stiffness is generally proportional to pressure. In fact, a log–log plot may be more appropriate in avoiding certain mathematical ambiguities (Butterfield, 1979). Further, crushed or sheared grains were observed in samples sheared at stresses greater than 2 MPa and no such crushed grains were found in samples tested at 500 kPa or less. There is, therefore, some justification for a mechanistic change in behaviour of sands in the stress range of 1 MPa as suggested by the conventional semi-logarithmic plot.

Whether or not the critical state concept is applicable to sands, and whether or not it can be used in practice, have been the subject of discussion for some years. Our experience is that it is very useful in design practice. However, some modifications to the original concepts are required when dealing with sands.

The CSL forms a stable reference condition for the quantification of sand behaviour and this is useful for characterizing sand behaviour regardless of initial density (e.g. Been & Jefferies, 1985). A general theory of sand behaviour predicated on the CSL has also been developed by Jefferies (1992). Whether the void ratio is greater or less than e_{max} determined by an arbitrary laboratory procedure is irrelevant. However, practical design should take into account the major differences of behaviour observed in sands deposited under different conditions and whether loading is in extension, compression or any other stress path. Use of the CSL as a reference condition allows a more precise definition of the state of the sand so that other important aspects of sand behaviour, such as stress path and anisotropy, can be compared on a rational basis.

Proceeding from a concern of localization, Chu and Lo imply that unless it can be shown that a dense drained sand approaches the same CSL as encountered with loose undrained tests, then the undrained CSL is inappropriate for characterizing sand behaviour. This concern is not born out by experimental data. The critical state is characterized by a sustained zero dilation rate. The closer the sample approaches the critical state, the smaller the dilatancy. If one plots the peak dilatancy with offset from the undrained CSL when this dilatancy is measured (ψ_m), then a plot such as that in Fig. 25 is obtained. Clearly, the trend of the drained tests is for zero dilatancy at the undrained CSL ($\psi_m = 0$). This implies that the drained and undrained CSL are the same.

The irrelevance of the CSL to real sand behaviour is argued by Vaid and Pillai on the basis that naturally pluviated sands are always denser than the CSL measured by undrained compression tests. This raises another question about the application of critical state principles to sands: what constitutes normal, or virgin, consolidation of sands and where is the normal consolidation line in relation to the CSL? This issue is not discussed in detail here; field evidence shows that a wide range of states of the same sand is possible when different pluviation techniques are used during the construction of underwater sand fills (Jefferies, Rogers, Stewart, Shinde, James & Williams-Fitzpatrick, 1988). The implied one-to-one mapping between a particular laboratory procedure and full-scale construction experience does not exist and therefore definitive statements may not be made on the unstated hypothesis that

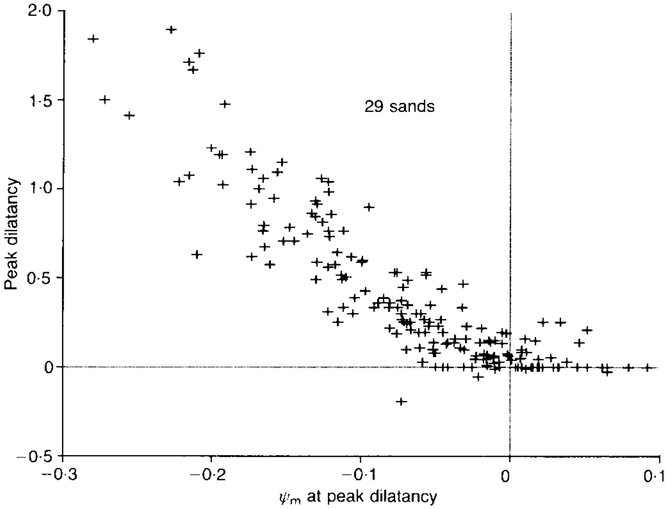

Fig. 25. Dilatancy ($d\varepsilon_v/d\varepsilon_\gamma$) in drained triaxial tests plotted as a function of state (void ratio difference relative to CSL, ψ_m) from undrained tests on loose samples

such a mapping exists and controls sand behaviour.

We note that the critical state (or steady state, or critical void ratio) of sands remains an important area of soil mechanics. However, further work may be handicapped by

(*a*) the limitations of the triaxial apparatus to determine the ultimate state of samples which frequently occurs at strains greater than 20%
(*b*) the absence of a practical method to quantify the packing arrangement (fabric) of particles in a sand mass.

It seems that critical state concepts have much to offer to real civil engineering projects. Significant progress has been made in the development of critical state concepts for sands, but more work needs to be done to clarify some of the remaining issues discussed.

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