Budhu, M. & Al-Karni, A. (1994). *Géotechnique* **44**, No. 4, 185–188

DISCUSSION

Seismic bearing capacity of soils

M. BUDHU and A. AL-KARNI (1993). *Géotechnique* **43**, No. 1, 181–187

A. H. Soubra and P. Lemonnier, *Ecole Nationale Supérieure des Arts et Industries de Strasbourg*

The Authors present a limit equilibrium method to calculate the reduction in the bearing capacity coefficients in the case when seismic loadings are present. Seismic bearing capacity has been investigated by Soubra & Reynolds (1992) who used a variational limit equilibrium method to calculate the seismic bearing capacity of strip footings on slopes, and Soubra (1992) who used an upper-bound method in limit analysis and gave upper-bound solutions for the seismic bearing capacity problem. Soubra & Reynolds showed that the variational limit equilibrium method is equivalent to the upper-bound method in limit analysis for a rotational log-spiral mechanism. However, the mechanism considered by Soubra (1992) is a translational one: it consists of a triangular active wedge and a log-spiral radial shear zone. By applying the upper-bound theorem of limit analysis, the bearing capacity of strip foundations in seismic areas was found for both these mechanisms. In our opinion, a rigorous method for calculating the seismic bearing capacity coefficients is one which gives directly the critical seismic factors and not one using static factors which will be affected by reduction coefficients determined by certain methods such as those presented by the Authors. The model used to calculate the seismic coefficients must give directly acceptable seismic bearing capacity factors and be in good agreement with the results of other authors in both the static and seismic cases. Our results for values of N_γ are now compared with those of the Authors, Sarma & Iossifelis (1990) and Richards, Elms & Budhu (1990b).

Static case

The Authors have presented the reduction in bearing capacity coefficients and not real seismic coefficients. Could they give their real static and seismic coefficients? Fig. 12 shows the variation of N_γ with ϕ for the mechanisms given by Soubra & Reynolds (1992), Soubra (1992) and Sarma & Iossifelis (1990) for the case of no seismic loadings.

Both solutions given by Soubra & Reynolds (1992) and Soubra (1992) are upper bounds to the exact solution for an associated flow rule material obeying Hill's maximal work principle in the framework of the limit analysis theory. The approach used by Sarma & Iossifelis (1990) is a limit equilibrium method based on an a priori assumption concerning the distribution of the inter-slice forces. This category of methods gives solutions which cannot be classified as upper or lower bound solutions when compared with the exact solution. However, the solution of Soubra (1992) using the translational mechanism is better than that of Soubra & Reynolds (1992) using the rotational log-spiral mechanism because the upper-bound solutions are smaller. Also the solution given by Sarma & Iossifelis (1990) shows good agreement with that of Soubra (1992) as the maximum difference does not exceed 14% when $\phi = 40°$.

Seismic case

Figure 13 shows the variation of the seismic $N_{\gamma E}$ value with the horizontal seismic coefficient K_h when $\phi = 35°$, as given by Soubra & Reynolds (1992), Soubra (1992) and Sarma & Iossifelis (1990). As mentioned for the static case, the rotational log-spiral mechanism used by Soubra & Reynolds (1992) continues to give uninteresting $N_{\gamma E}$ values in the seismic case because the results obtained using this mechanism are higher than

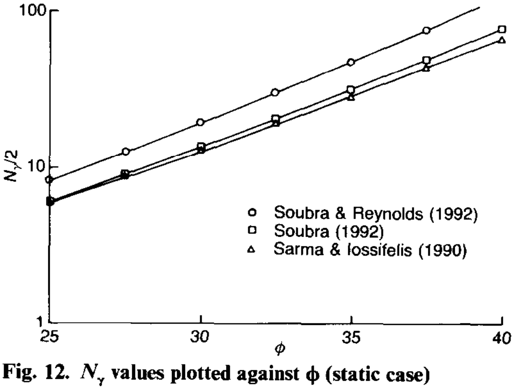

Fig. 12. N_γ **values plotted against** ϕ **(static case)**

those given using the translational mechanism. For example, the difference is about 50% when $\phi = 35°$ and $K_h = 0{\cdot}3$. However, the results obtained by Sarma & Iossifelis (1990) and those given by the upper-bound method in limit analysis for a translational mechanism (Soubra, 1992) show good agreement, as in the static case. The maximum difference does not exceed 9%. Could the Authors show the variation of their values of $N_{\gamma E}$?

Figure 14 shows the variation of $N_{\gamma E}/N_{\gamma S}$ with K_h for values of ϕ according to Soubra (1992). The curves corresponding to $\phi = 30$–$45°$ are close to each other. However, for smaller values of ϕ, the $N_{\gamma E}/N_{\gamma S}$ values are different. Fig. 15 shows the variation of $N_{\gamma E}/N_{\gamma S}$ with K_h for $\phi = 35°$ according to Soubra & Reynolds (1992) and Soubra (1992), and the values proposed by the Authors, Sarma & Iossifelis (1990) and Richards *et al.* (1990b).

The solution recommended by Soubra (1992), which is the smallest upper-bound solution, is close to that given by Sarma & Iossifelis (1990) and lies between the solutions given by Richards *et al.* (1990b) and the Authors.

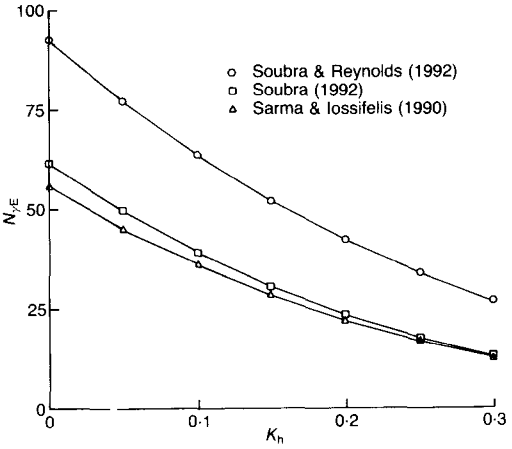

Fig. 13. Seismic $N_{\gamma E}$ values plotted against K_h

For a non-symmetrical mechanism, where a slip along the soil–structure interface may occur, the friction at this interface is mobilized and has a significant effect on the bearing capacity coefficients (Soubra, 1992; Yu & Sloan, 1992). The Authors assume non-zero friction at the soil–structure interface.

Authors' reply

Soubra and Lemonnier infer that our solution for calculating seismic bearing capacity is not rigorous and state that the real seismic (bearing capacity) coefficients were not presented. In fact our solution directly gives the real seismic bearing capacity factors N_{cE}, N_{qE} and $N_{\gamma E}$ for different values of angle of friction, horizontal and vertical accelerations, and the stability factor D. The static bearing capacity factors N_{cS}, N_{qS} and $N_{\gamma S}$ are obtained simply by setting k_h and k_v equal to zero and following the steps shown in Appendices 1 and 2 of the Paper. For example, the static bearing capacity factor $N_{\gamma S}$ obtained accordingly is compared with those of Vesic (1973) and Sarma & Iossifelis (1990) for $\phi = 25$–$40°$ in Fig. 16. The agreement is indeed very good. The seismic bearing capacity factors were normalized to the static bearing capacity factors to show the reduction in bearing capacity as the acceleration is increased and to develop reduction factors that can be applied to the static bearing capacity factors. The latter is preferable because, rather than dealing with a whole new set of bearing capacity factors, practising engineers, familiar with static bearing capacity factors, can calculate the seismic bearing capacity of soils by including seismic reduction factors in the static bearing capacity equations. For example, Vesic's (1973)

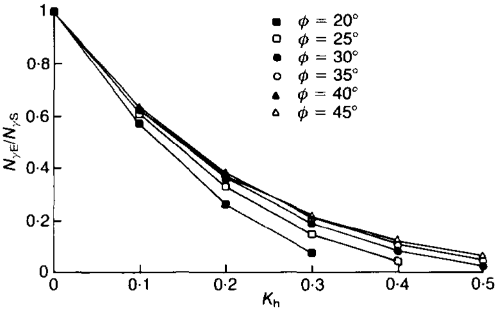

Fig. 14. Effect of K_h on $N_{\gamma E}/N_{\gamma S}$ for values of ϕ according to Soubra (1992)

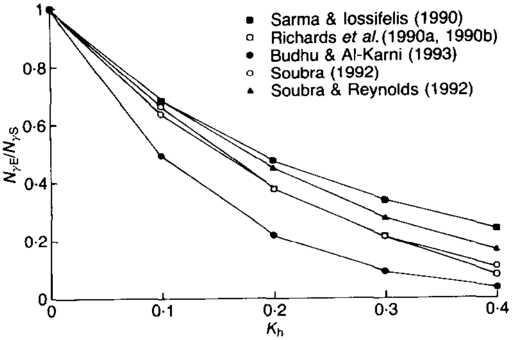

Fig. 15. Effect of K_h on $N_{\gamma E}/N_{\gamma S}$ for $\phi = 35°$

bearing capacity equation is modified by including our seismic reduction factors as follows

$$q_{\mathrm{us}} = CN_{\mathrm{c}}\zeta_{\mathrm{c}}\zeta_{\mathrm{cd}}e_{\mathrm{c}} + q_{\mathrm{f}}N_{q}\zeta_{q}\zeta_{\mathrm{qd}}e_{\mathrm{q}} + \gamma BN_{\gamma}\zeta_{\gamma}\zeta_{\gamma\mathrm{d}}e_{\gamma}/2 \tag{34}$$

where the seismic reduction factors e_{c}, e_{q} and e_{γ} are

$$e_{\mathrm{c}} = e^{-\beta_{\mathrm{c}}} \tag{35}$$

$$e_{\mathrm{q}} = (1 - k_{\mathrm{v}})e^{-\beta_{\mathrm{q}}} \tag{36}$$

$$e_{\gamma} = (1 - 2k_{\mathrm{v}}/3)e^{-\beta_{\gamma}} \tag{37}$$

and $e^{-\beta_{\mathrm{c}}}$; $e^{-\beta_{\mathrm{q}}}$ and $e^{-\beta_{\gamma}}$ are as given in the Paper, and ζ_{c}, ζ_{cd}, ζ_{q}, ζ_{qd}, ζ_{γ} and $\zeta_{\gamma d}$ are shape and depth factors given by Vesic (1973). Similar modifications can be made to the bearing capacity equations of Meyerhof (1963) and Hansen (1970).

Figure 15 is incorrect: the corrected figure is shown in Fig. 17. Our solution is shown for two cases. In the first case—shown by the full line—the horizontal acceleration (motion towards the left in Fig. 5) tends to increase the active earth pressure and reduce the passive earth pressure (Neelakantan, Budhu & Richards, 1991). Thus, the dynamic passive earth pressure component acts in the direction shown in Fig. 5. In the second case—shown by the dotted line—the dynamic passive component acts in the direction opposite to that shown in Fig. 5, i.e. the passive earth pressure increases. For this latter case, the agreement between our results and the others shown in Fig. 17 is very good. However, in our opinion, the second case is incorrect. A direct comparison of our values of $N_{\gamma\mathrm{E}}$, as preferred by Soubra & Lemonnier, with those of Sarma (1990) and Soubra (1992) is shown in Fig. 18. Our solution, which gives the lowest values, seems to be a better solution than that of Soubra (1992).

We did not assume zero friction at the soil-structure interface. In our solution, friction is assumed to be fully mobilized, i.e. the friction coefficient at the base of the footing is unity. For static bearing capacity, footing roughness has been shown by Ko & Davidson (1973), Ko & Scott (1973) and Vesic (1973) to be insignificant. For seismic loading, the effect of footing roughness is likely to be significant because of the non-symmetrical failure surface. However, our solution is the limiting case because the solution

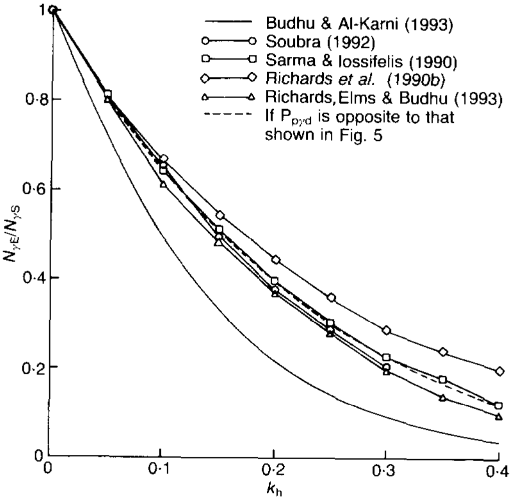

Fig. 17

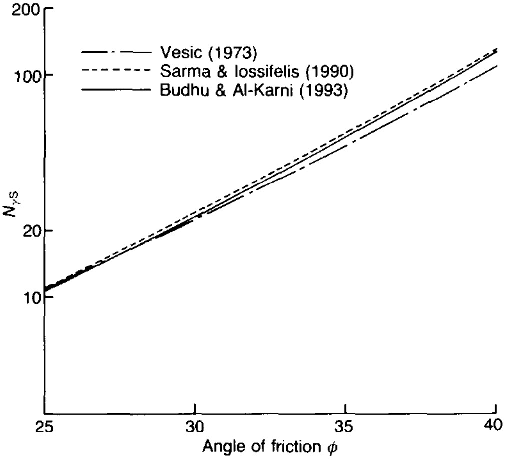

Fig. 16

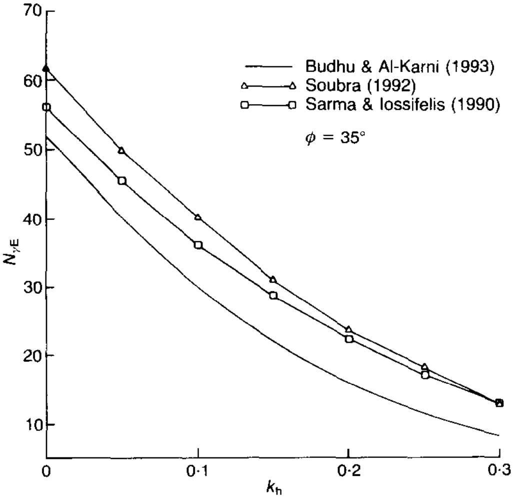

Fig. 18

for friction coefficients of less than unity will result in increased bearing capacity.

REFERENCES

Budhu, M. & Al-Karni, A. (1993). Seismic bearing capacity of soils. *Géotechnique* **43,** No. 1, 181–187.

Hansen, J. B. (1970). A revised and extended formula for bearing capacity. *Dan. Geotech. Inst. Bull.*, No. 28, 21.

Ko, H. & Davidson, L. (1973). Bearing capacity of footings in plane strain. *J. Soil Mech. Fdns Div. Am. Soc. Civ. Engrs* **99,** SM 1, 1–24.

Ko, H. & Scott, R. F. (1973). Bearing capacities by plasticity theory. *J. Soil Mech. Fdns Div. Am. Soc. Civ. Engrs* **99,** SM 1, 25–44.

Meyerhof, G. G. (1963). Some recent research on the bearing capacity of foundations. *Can. Geotech. J.* **1,** No. 1, Sept., 16–26.

Neelakantan, G., Budhu, M. & Richards, R. (1990). Mechanics and performance of a tied-back wall under seismic loads. *Earthquake Engng Struct. Dyn.* **19,** Apr., 315–331.

Richards, R., Elms, D. G. & Budhu, M. (1990a). Dynamic fluidization of soils. *J. Geotech. Engng Div. Am. Soc. Civ. Engrs* **116,** No. 5, 740–759.

Richards, R., Elms, D. G. & Budhu, M. (1990b). Soil fluidization and foundation behavior. *Proc. 2nd Int. Conf. Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, Rolla* **1,** 719–723.

Richards, R., Elms, D. G. & Budhu, M. (1993). Seismic bearing capacity and settlement of foundations. *J. Geotech. Engng Div. Am. Soc. Civ. Engrs* **119,** No. 4, 662–674.

Sarma, S. K. & Iossifelis, I. S. (1990). Seismic bearing capacity factors of shallow strip footings. *Géotechnique* **40,** No. 2, 265–273.

Soubra, A. H. & Reynolds, F. (1992). Design charts for the seismic bearing capacity of strip footings on slopes. *Proc. Fr.–It. Conf. Slope Stability in Seismic Areas* (eds E. Faccioolі & A. Pecker), pp. 273–283. Bordighera: Ouest éditions.

Soubra, A. H. (1992). Seismic bearing capacity of strip footings. *Proc. 3rd Int. Conf. Computational Plasticity* (ed. D. R. J. Owen), pp. 995–1006. Barcelona: Pineridge.

Vesic, A. (1973). Analysis of ultimate loads of shallow foundations. *J. Soil Mech. Fdns Div. Am. Soc. Civ. Engrs* **99,** SM 1, 45–74.

Yu, H. S. & Sloan, S. W. (1992). A finite element formulation of the upper bound theorem for rigid-plastic soils generalized to include coulomb friction. *Proc. 3rd Int. Conf. Computational Plasticity* (ed. D. R. J. Owen), pp. 983–993. Barcelona: Pineridge.