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TECHNICAL NOTE

Stability of infinite slopes

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INTRODUCTION

In a research programme on the stability of overtopping dykes in the Netherlands, attention is focused on the inner slope. Within the inner slope a parallel flow develops which affects the geomechanical stability. One of the aims of this research is to derive simple design rules, in particular for the inner slope gradient for this loading condition.

A one-dimensional approach establishes the traditional formula for the stability of infinite slopes (Taylor, 1948; Heafeli, 1948; Skempton & DeLory, 1957). The critical stress ratio of a slope determines the stability of material sliding downhill with a slip surface parallel to the ground surface (see Fig. 1). This idea incorporates ground-water flow. The critical angle for the slope, or angle of natural slope, follows from the ratio of the shear stress to the vertical stress.

The shear stress and normal effective stress components (where compression is taken as positive) within the soil layer at depth are determined from

$$\sigma_{\mathrm{nt}} = \gamma h \sin \alpha \tag{1}$$

$$\sigma_{\mathrm{nn}} = (\gamma - \gamma_{\mathrm{w}}) h \cos \alpha \tag{2}$$

where α is the angle of the slope, γ is the unit weight of the soil, γ_{w} is the unit weight of water and h is the depth of the layer. The ratio of the shear stress and normal stress follows from

$$\left(\frac{\gamma}{\gamma - \gamma_{\mathrm{w}}}\right) \tan \alpha = \frac{\sigma_{\mathrm{nt}}}{\sigma_{\mathrm{nn}}} \tag{3}$$

For the stress state on the parallel failure plane, the Coulomb yield criterion for a cohesionless material is

$$\sigma_{\mathrm{nt}} = \tan \phi \sigma_{\mathrm{nn}} \tag{4}$$

where ϕ is the friction angle. For a cohesionless material, the critical slope angle is hence given by

$$\left(\frac{\gamma}{\gamma - \gamma_{\mathrm{w}}}\right) \tan \alpha = \tan \phi \tag{5}$$

This is a necessary condition for the stability of slopes, but is not sufficient. It is also necessary to prove that such a stress state can exist within the soil mass.

STRESS STATE IN A COHESIONLESS INFINITE SLOPE

The Mohr–Coulomb yield criterion describes the critical stress state within the soil layer

$$\tau_{\mathrm{m}} = \sigma_{\mathrm{m}} \sin \phi + c \cos \phi \tag{6}$$

where τ_{m} is the radius of the Mohr circle and σ_{m} is the centre of the Mohr circle. This criterion implies that, in addition to the stress components already mentioned, the normal stress parallel to the slope is important, because in the soil all stress tensor components need to be considered. The tangential stress along the slope σ_{tt} is related the normal stress σ_{nn}

$$\sigma_{\mathrm{tt}} = C_0 \sigma_{\mathrm{nn}} \tag{7}$$

where C_0 is the stress ratio between normal and tangential stress. The stress ratio C_0 is similar to K_0, but with respect to the normal and parallel axes of the slope (see Fig. 1). The stresses at failure that conform to the Mohr–Coulomb yield

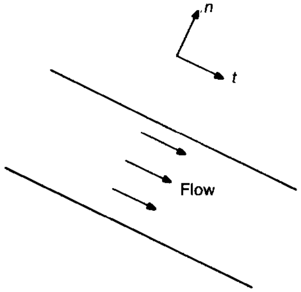

Fig. 1. Stability of single layer with parallel flow

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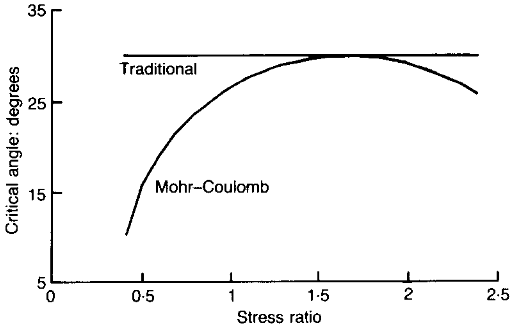

Fig. 2. Critical slope angles for a dry analysis: $\phi = 30°$

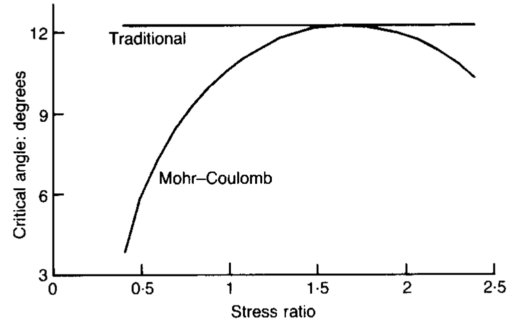

Fig. 3. Critical slope angles for parallel flow: $\gamma = 16$ kN/m^3, $\gamma_w = 10$ kN/m^3 and $\phi = 30°$

criterion are described by the expression

$$\left(\frac{\gamma}{\gamma-\gamma_w}\right)\tan\alpha = \sqrt{\left[(\sin\phi)^2\left(\frac{1+C_0}{2}\right)^2-\left(\frac{1-C_0}{2}\right)^2\right]} \qquad (8)$$

which differs from the traditional formula for a wide range of C_0. Fig. 2 shows the result for a dry slope; Fig. 3 shows the result for a slope with parallel flow. The tangential stress σ_{tt} is important in addition to the stress ratio as described in equation (3).

The critical angles in expressions (5) and (8) become equal for the stress state

$$\sigma_{tt} = \frac{1+(\sin\phi)^2}{1-(\sin\phi)^2}\sigma_{nn} \qquad (9)$$

The traditional formula for the critical slope is optimistic compared with the Mohr–Coulomb yield criterion. Only for a relatively high tangential normal stress σ_{tt} do equations (5) and (8) become identical. In this example for a friction angle of 30°, this means a stress ratio of 5/3, which is rather high. Moreover, these results indicate that the failure condition within the layer itself is more critical. No definite conclusions can be drawn from the evaluation of the stress state itself.

SIMPLE SHEAR MECHANISM IN AN INFINITE SLOPE

The plastic strain rates determine the value for the critical slope angle. Apart from the stress state, it is necessary that the plastic strains generate a kinematically admissible velocity field. The plastic potential describes the plastic strains and has the same form as the Mohr–Coulomb yield function (equation (6)), with the friction angle replaced by the dilatancy angle ψ

$$g = \tau_m - \sigma_m \sin\psi \qquad (10)$$

Plane sliding parallel to the slope describes the failure mechanism. This situation is identical with simple shear. The condition for the velocity field in such a case is

$$\dot{\varepsilon}_{tt} = 0 \quad \text{or} \quad \partial g/\partial\sigma_{tt} = 0 \qquad (11)$$

The condition that there be no tangential strain is the additional condition that determines the stress rate. From this condition, the expression for the slope angle follows

$$\left(\frac{\gamma}{\gamma-\gamma_w}\right)\tan\alpha = \frac{\sin\phi\cos\psi}{1-\sin\phi\sin\psi} \qquad (12)$$

This condition is identical, for the resulting coaxial model, with the limit stress ratio for simple shear conditions (Teunissen & Vermeer, 1988). Davis (1968) published the same result for velocity discontinuities based on the method of characteristics. The conditions for simple shear and for velocity discontinuities have a strong resemblance. Drescher & Detournay (1993) incorporate the results from Davis (1968) for limit load calculations for different materials.

For an associative material with identical dilatancy and friction angles, equation (12) reduces to the traditional formula (equation (5)). Equation (12) is more powerful because it is valid for a wider range of materials; it also shows that the dilatancy angle contributes to the limit value. If the dilatancy is zero, the tangent in equation (5) should be made a sine, resulting in a lower gradient. The range of critical slope angles becomes much wider by inclusion of non-coaxial plastic deformation terms (Teunissen & Vermeer, 1988).

The friction angle is a function of the dilatancy angle in equation (12). The stress–dilatancy relationship as formulated by Bolton (1986) decomposes the friction angle in terms of the friction angle for constant volume ϕ_{cv} and the dilatancy angle

$$\left(\frac{\gamma}{\gamma-\gamma_w}\right)\tan\alpha = \frac{\sin(\phi_{cv}+0{\cdot}8\psi)\cos\psi}{1-\sin(\phi_{cv}+0{\cdot}8\psi)\sin\psi} \quad (13)$$

The dilatancy angle contributes to the critical slope angle. For incorporating the resulting form in design codes, the magnitude of the dilatancy angle is important. The friction angle for constant volume is a reasonable characteristic material parameter. The dilatancy angle is strongly density-dependent and is difficult to estimate. The most conservative approach is to assume no dilatancy at all. This leads to

$$\left(\frac{\gamma}{\gamma-\gamma_w}\right)\tan\alpha = \sin\phi_{cv} \quad (14)$$

This expression gives a lower bound for the angle of natural slope.

STABILITY OF COHESIVE INFINITE SLOPES

The concept of deriving the critical angle of a slope is easily extended to materials that include cohesion. The critical slope angle for dry soil becomes

$$\tan\alpha = \frac{\cos\psi\left(\sin\phi + \dfrac{c}{\gamma h\cos\alpha}\cos\phi\right)}{1-\sin\phi\sin\psi} \quad (15)$$

This equation includes the stability number ($c/\gamma h \cos\alpha$). For an associative material, equation (15) reduces to the traditional expression for cohesive infinite slopes (Taylor, 1948). From this expression it is possible to calculate the required cohesion of a slope. Reduced influence of the cohesion with depth follows from equation (15).

To obtain similar results from slip circle analysis requires a reinterpretation of the constitutive law applied, and the law consistent with such an analysis is

$$\tau = \frac{\cos\psi(\sin\phi\sigma_n + c\cos\phi)}{1-\sin\phi\sin\psi} \quad (16)$$

This implies that the dilatancy angle now becomes a multiplier of the normal stress component and the cohesion. For sands the dilatancy angle also partly determines the friction angle due to the stress–dilatancy relationship.

CONCLUSIONS

The stability condition just within the layer has been proved more critical than the stability condition considering the shear stress and normal stress ratio at the failure surface only. The resulting expression is identical with the traditional expression assuming associative behaviour. Equation (14) describes the lower bound of the natural slope angle.

For infinite slopes the simple shear condition is generally a more critical condition for failure than interface behaviour for materials with identical strength parameters.

The classical solutions for infinite slopes generate the relationships for the shear stress ratio in a slip circle analysis. This implies that the constitutive parameters should be transformed in accordance with equation (16). In the design of embankments the dilatancy angle is important for the allowable strength of frictional materials, and also has a distinct effect on the natural slope angle.

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