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DISCUSSION

Fractal analysis of granular materials

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The author has presented an interesting technical note about the potential use of fractal theory to characterize the two-dimensional shape of granular materials. The discussers fully agree that fractal dimension appears to be promising for characterizing particle shape. This approach was also proposed for quantifying aggregate shapes (Carr *et al.*, 1990).

The author has indicated that at least a four-digit precision past the decimal point is required to express the fractal dimension of two-dimensional close-form profiles since very small differences in fractal dimension (of the order of ten thousandths) can exist, as indicated in the technical note, for example, between glass beads and Ottawa sand. He presents estimates of the fractal dimension (population mean, μ) for three different types of granular material based on the average value of 25 measurements (sample mean, $\bar{x}$). When conducting experiments with limited size samples, the resulting measurements are estimates and the precision of the estimates actually depend on sample size among other factors. The precision of such estimates can be measured by the width of the confidence interval. For a normal distribution with known variance (σ^2), the error $E = |\bar{x} - \mu|$ is less than $Z_{\alpha/2}\sigma/\sqrt{n}$ with a confidence of $100(1 - \alpha)$ (Hines & Montgomery, 1990). When sample size can be controlled, n can be chosen with $100(1 - \alpha)$ per cent confidence that the error in estimating μ is less than a specified error E. The appropriate sample size is then $n = (Z_{\alpha/2}\sigma/E)^2$.

The author has reported that the standard deviation ranged from 0·0085–0·0113 (Table 2). For a 90% confidence level, ($\alpha = 0{\cdot}1$ and $Z_{\alpha/2} = Z_{0{\cdot}05} = 1{\cdot}645$), then for a specified error $E = 0{\cdot}0001$ and a standard deviation of 0·01, the appropriate sample size would be approximately 27 000 instead of 25 as used by the author. For a sample size of 25, the precision (specified error E) is only about 0·0033.

This technical note as well as the earlier study (Carr *et al.*, 1990) indicate a small range of fractal dimension (1 – 1·3) when quantifying particle shape. For the fractal dimension to be useful for characterizing particle shape, the discussers suggest that a precision level of at least 0·001, corresponding to measurements of about 300 particles as a minimum, is required.

Author's reply

The author agrees with the discussers that the estimation of the fractal dimension improves in accuracy as the number of grains measured increases. The use of an image analyzer would expedite this.

The purpose of the technical note was to indicate the usefulness of the fractal dimension concept as a shape indicator for fine granular materials. Of course, the procedure presented could be optimized in the way suggested by the discussers.

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