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DISCUSSION

Sedimentation and self-weight consolidation: general unifying theory

E. A. TOORMAN (1996). *Géotechnique* **46**, No. 1, 103–113.

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The paper is certainly a step towards unification and clarification, but some points need more discussion.

The mass balance equation (1) is only sufficient if there is no exchange between the constituents. This is not the case for the bound part of the pore liquid in diffuse layers around the grains and at the grain contacts. This was introduced as the pectic constituent (Gudehus, 1996); it corresponds to the famous water II proposed by Deryaguin and dating back to Terzaghi's (1920) observations. The pectic volume fraction, which has a higher mass density, increases during densification.

The momentum conservation equation (2) is also not beyond doubt. First, the presence of ions in a liquid having a non-uniform distribution, and also the possibility of surface charges on the particles, leads to electrostatic terms. Second, the terms with p and τ are not sufficiently explained; if the drag on a constituent is properly understood, it should imply p and τ. The electrokinetic drag between water and ions, and ions and solid particles, should be incorporated. One should be aware that the conservation of momentum can be expressed in different ways, which makes the theory of mixtures and the related constitutive equations rather difficult.

The osmotic pressure, which is introduced later in the paper, is related to the 'chemical potential of the suspension'. However, the fluid is not homogeneous: it is partly bound and has a spatially variable ion concentration. One may, at best, speak of the chemical potential of the water molecules, and possibly of the molecules and ions in the Stern layer and at the solid particle contacts. Due to the non-uniform ion concentration there is an osmotic pressure among the grain contacts, which tends to expand unloaded skeletons under water. The related partial pressure was introduced as the pectic pressure (Gudehus, 1996), and may be understood as swelling pressure. It is by no means negligible for clays, and in particular in the case of changing salinity.

The formation of a granular skeleton, also called gelation, is a complicated matter. Physicists (e.g. Dijkstra *et al.*, 1995), chemists, and chemical, geotechnical and hydraulic engineers can develop different views of it. The appearance of a skeleton or gel should be understood as percolation, or a critical phenomenon, which involves a mathematical singularity described by power laws (Stauffer & Ahorny, 1992). The continuum theory of mixtures with spatial averages as state variables cannot suffice for percolation points. The critical remarks in the paper concerning the relationship between the effective stress and the solid fraction are justified, but not sufficient therefore.

In summary, attempts at unification should be continued, and the paper certainly provides some progress in this direction and enhances the discussion of the topic.

Author's reply

Gudehus discusses some details of the physico-chemical properties of cohesive sediments that play an important role in the complex sedimentation and consolidation behaviour of cohesive particles in water. There is a conflict between desiring a more detailed physical description and obtaining a simple model that is accurate enough for engineering applications. Depending on the aim of developing a model, one has to make a choice about the desired complexity and accuracy. Three of the possible approaches are briefly described by Buscall (1990): a microscopic theory (the category to which Gudehus' comments belong), a chemical approximation, and a continuum or engineering approach.

I am looking at the problem from the point of view of a hydraulic engineer, who is developing a model suitable for the prediction of sedimentation and consolidation of sediments in rivers, estuaries, coastal areas, harbours, waterways, dredged material dumping sites, reservoirs, etc. The sedimentation model to be developed is a module in a complex three-dimensional transport model. The model must satisfy the following criteria: be simple (a minimum number of empirical parameters), easy to calibrate, sufficiently accurate, CPU cost-effective, and compatible with the other modules.

Hence, it is clear that the engineering approach is not aiming at a detailed microscopic description of all the processes. The force balance is performed on a control volume of suspension rather than on a single particle. Attention is focused on

the net effect of the processes, which is described by the flow resistance (the inverse permeability) and the stress transmitted by the particles throughout the soil matrix (the effective stress). All the interactions are assumed to be accounted for by these two empirical bulk parameters. The complexity of the model is then assumed to be determined by the formulation of the constitutive equations for these two parameters. Therefore, the theory developed in my paper stops at this point, without presenting closure equations, which will be the subject of a forthcoming paper.

Nevertheless, Gudehus' comments are a valuable addition to the paper. Within the cohesive sediment transport research community the exchange of fluid between the constituents is acknowledged. In this field one speaks of *immobilized* pore water, which makes an integral part of the floc or aggregate, consisting of many (clusters of) primary particles (mainly clays). Besides the pectic constituent, it also incorporates fluid that is caught within the open spaces of the floc during aggregation, and can be released when the floc breaks up due to shear or normal forces. This immobilized pore water determines the floc density and thus its actual terminal fall velocity (e.g. Van Leussen, 1994). This point also raises the question of whether the volumetric concentration of the solids ϕ_s is the proper parameter to describe density effects, particularly for the rheological behaviour of gels in general. Parker (1986) argues that it is better to work with the *effective volume concentration*, defined as the fraction of the volume 'occupied by aggregates and occluded water'. The problem, however, is its measurement.

The explanation of the terms of the two-phase momentum equation (2) is somewhat incomplete: the interparticle stress τ for the solid phase should also include the normal components, while p is the fluid pressure. The force and stress terms, which cover all the interactions, are not developed further because this form of momentum equation is not used, but is replaced by the conservation equation of the mixture and the sediment force balance. No attempt was made to present a complete list of all the possible interaction forces (although the electrostatic contribution is mentioned), because in the engineering approach only the net effect of all the forces is considered. Pressure gradients opposite to the direction of the flow through a swarm of particles are, of course, the result of viscous losses τ and possibly other losses when the particles are not neutral. The net effect of the flow resistance, either through hydrodynamic or electrokinetic drag, is assumed to be incorporated in the empirical drag coefficient, which is replaced by the inverse (net) permeability.

The introduction of the osmotic pressure gradient term finds its origin in the so-called chemical approximation (Buscall, 1990), which is based on a small Péclet number approximation (diffusion rate much larger than sedimentation rate) following Batchelor's (1972, 1976) approximation that on a local scale L, which is much larger than the particle diameter but much smaller than the length scale over which the concentration might vary considerably, the suspension can be considered uniform. This condition is not fulfilled in the case of natural cohesive sediment. Nevertheless, in the engineering approach this term is maintained, but refers to a more general interparticle stress (Buscall, 1990). The net effect of the swelling pressure is assumed to be included in the effective stress.

Even if all the different effects on a microscopic level could be described in separate terms, the practical problem arises of how the large number of model parameters can be calibrated through experiments. For engineering purposes the model parameters of permeability and effective stress are obtained from sedimentation and self-weight consolidation experiments, where density and pore pressure profiles are measured at regular time intervals. The interpretation of these data and the determination of general constitutive equations is not straightforward. Recently, more attention has been paid to time-dependent effects (e.g. Sills, 1995), which can be related to structural changes in the aggregates (thixotropy). For instance, the non-uniqueness of the permeability–concentration relationship for cohesive sediments, depending on the bed formation history, is attributed to structural changes of the floc aggregates with subsequent exchange of pore fluid. This will be discussed in a forthcoming paper. Another major challenge is the polydispersity of natural sediments (Toorman & Berlamont, 1993b).

A strong argument in favour of the engineering approach is its ability to predict density profiles corresponding to the formation of a soil skeleton on the bottom of a suspension. The easiest way to demonstrate this is by assuming that the permeability and the effective stress *gradient* can be written as a simple empirical function of the concentration. In this case, the net settling rate $w_s = w_0(1 - \partial\sigma'/\partial\sigma_0)$, where σ_0 is the buoyant stress (= total stress − hydrostatic pressure), is a function of the concentration alone, and an analytical solution method, based on the theory by Kynch (1952), can be applied (Toorman, 1992). This has been applied to the simulation of the settling of spherical glass beads (Toorman, unpublished). The computed density profiles show the distinct interface between the settling suspension and the bed in formation (Fig. 1). This interface is indeed a discontinuity in the density profile. This is a singularity that cannot be handled by solving the partial differential equation numerically. The only way to

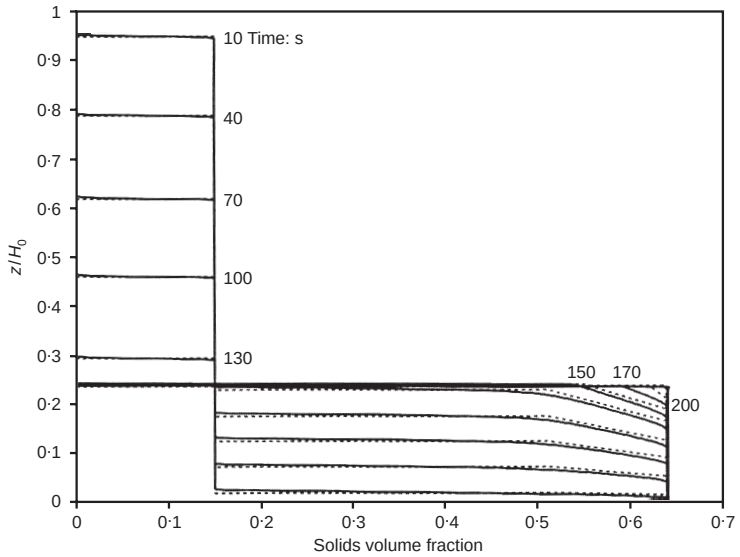

Fig. 1. Computed density profiles for 67-μm spherical glass beads ($\rho_s = 2450$ kg/m^3): (– – – –) analytical solution (Kynch's sedimentation theory); (————) finite element solution on a fixed Eulerian grid (1001 nodes). Initial height $H_0 = 0{\cdot}312$ m; initial solids concentration $\phi_0 = 0{\cdot}15$

overcome it is by adding diffusion and using a very fine grid spacing around the location of discontinuity, which is then approximated by a very steep concentration gradient. Toorman (unpublished) has shown that this indeed is possible and the resulting numerically computed density profiles are very close to the analytical solution (Fig. 1). In the case of compressible, cohesive sediment, terms with a diffusive character appear in the equations (Auzerais *et al.*, 1988). Therefore, the numerical solution is possible, even on a fixed Eulerian grid (e.g. Toorman, 1992; Toorman & Berlamont, 1993a). The difference in behaviour between the suspension and the soil matrix is described completely by the constitutive equations.

Whether the transition between the suspension and the consolidating bed is discontinuous, smooth or even indistinguishable depends on the initial concentration and the shape of the flux curve (the sediment flux as a function of concentration). A detailed discussion of these discontinuities and the conditions under which they occur is presented by Auzerais *et al.* (1988).

Subdividing the numerical domain into separate subdomains for the suspension and the consolidating bed may seem attractive. Different constitutive equations can be defined for each subdomain. A major advantage is that the grid does not need to be refined near the interface (except when there are large gradients), and even a true discontinuity can be dealt with. The only condition that needs to be fulfilled at the interface is continuity of the sediment flux (not of the concentration) (see Kynch, 1952; Auzerais *et al.*, 1988). In the case of a true discontinuity, the location of the interface must be computed as (Kynch, 1952):

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\Delta S}{\Delta \phi_s} = \frac{\phi_{s1} v_1 - \phi_{s2} v_2}{\phi_{s1} - \phi_{s2}}$$

where S is the sediment flux and the indices 1 and 2 refer to the suspension and the bed sides of the interface, respectively. When the density variation is continuous, the finite difference sign Δ should be replaced by the derivative d.

A problem with the domain division approach is the choice of coordinate frame. Material coordinates cannot be used because mass is transferred from the suspension to the bed. The suspension domain shrinks to disappear at a given time, while the bed domain initially grows until all the suspended particles have settled, and then shrinks due to consolidation. Material coordinates can be used only when one domain is considered, but then the same problem as for the Eulerian approach (discussed above) arises that a true discontinuity should be approximated by a high density gradient, because the interface location is not bound to a fixed material coordinate. The only solution is the use of mixed Euler–Lagrange coordinates, where the nodal displacements are defined as a fraction of the displacements of the boundaries (as used for the numerical simulation of free surface flow) (e.g. Toorman, 1993).

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ERRATA

On page 105, in equation (1), $v\varphi j$ should be $v_{\varphi j}$.

On page 110, above equation (23), the text should read: ‘Gibson *et al.* (1967) … use the notation z and ξ (not a) for ζ and z respectively’.

On page 113, the title in the reference to Toorman (1996) should read: ‘Sedimentation and self-weight consolidation: constitutive equations and numerical modelling’.