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DISCUSSION

Surface settlements due to deformation of a tunnel in an elastic half plane

A. VERRUIJT and J. R. BOOKER (1996). *Géotechnique* **46**, No. 4, 753–756.

C. Sagaseta, *University of Cantabria*

The authors of this technical note have presented an elegant and useful solution for displacements and stresses around excavated tunnels at moderate depth in an elastic soil. It is an extension of the discusser's work (Sagaseta, 1987), where the case of incompressible soil was considered, and the tunnel deformation was modelled as a pure ground loss problem, concentrated at the tunnel axis. This lead to a solution in closed form, or requiring only minor numerical integration, even for three-dimensional geometry.

In both analyses, the tunnel in an infinite space is taken as a basic case, and the effect of the surface is considered by the addition of virtual images and corrective surface forces. In the discusser's work, the resulting deformation of the tunnel is an almost uniform radial movement, the symmetry being altered only by the influence of the soil surface. The tunnel undergoes a relatively small ovalization, and the lateral displacements at the sides are of the same order as the vertical downwards movements at the crown. As a result, the surface transversal settlement troughs are wider than usually measured in actual cases (Schmidt, 1988; Sagaseta, 1988).

This effect was further studied, and the results were presented by Uriel & Sagaseta (1989). Some factors which could reduce the lateral extent of soil movements were analysed: soil elastic anisotropy, effect of k_0, and soil compressibility. The second of these factors is related to the authors' analysis.

For the case of uneven initial stresses ($k_0 \neq 1$), the basic Kirsch solution was used. The displacements due to the excavation of a circular tunnel in an elastic medium (G, μ), with initial stresses ($\sigma_{v0} = p_0$; $\sigma_{h0} = k_0 \cdot p_0$) are given by Pender (1980). Using polar coordinates (r, θ), the solution consists of three parts: (*a*) a uniform radial displacement, $O(1/r)$; (*b*) an ovalization, with radial displacements $u = O(1/r)\cos 2\theta$ and circumferential displacements $v = O(1/r)\sin 2\theta$; and (*c*) a third-order ovalization, $u = O(1/r^3)\cos 2\theta$, $v = O(1/r^3)\sin 2\theta$.

The solution used by the authors coincides with the above, neglecting the third-order terms (*c*), which decrease very rapidly with the distance to the tunnel. By comparing the elastic solution with the authors' equations (1) and (2), separating the contributions of the tunnel and its image, and transforming to polar coordinates, one gets

$$\varepsilon = \frac{p_0}{2G}\frac{1+k_0}{2}$$
$$\delta = \frac{p_0}{2G}\frac{1-k_0}{2}4(1-\mu) \qquad (15)$$
$$\mathrm{k} = \frac{\mu}{1-\mu}$$

These expressions show the physical significance of the parameters for radial displacement (ε) and ovalization (δ), and their relations with the soil elastic parameters and initial stresses. However, in the discusser's opinion, the main merit of the authors' approach is to consider both magnitudes (ε, δ) as input parameters, regardless of their origin. ε is the radial convergence, $\Delta R/R$ (relative ground loss, $\Delta V/V_0 = 2\varepsilon$). The ovalization, δ, can be due to uneven stresses, as in equations (15), or to any other reason, such as different support conditions at the crown and at the sides, or to plastic deformations that imply maximum movements at the crown (where the soil moves in the direction of gravity forces).

The second contribution of the authors' formulation is the derivation of closed-form expressions for the displacements for any value of Poisson's ratio and at any point of the soil (equations (1), (2), (10) and (11)), and not only at the surface, as in Uriel & Sagaseta (1989). This increases the practical applicability of the solution, as commented below.

On the other hand, these expressions are derived on the basis of linear elasticity, as opposed to the basic solution for uniform radial ground loss (Sagaseta, 1987), which is valid for any soil behaviour (with the condition of incompressibility only). This is an unavoidable drawback inherent to any extension of the basic solution: the consideration of additional factors implies the assumption of restrictive conditions to keep the possibility of analytical solutions.

In order to check the possibility of practical application, a typical case has been analysed of a tunnel at a given depth h. The transverse settlement trough at the surface is given by equation (12), which can be put in the form

$$\frac{s}{s_{\max}} = \frac{1}{1+\rho}\frac{1}{1+\bar{x}^2}\left(1+\rho\frac{1-\bar{x}^2}{1+\bar{x}^2}\right) \tag{16}$$

where $s_{\max}$ is the maximum (centre-line) settlement, and the distances are scaled by the tunnel depth ($\bar{x} = x/h$). The parameter $\rho = [1/2(1-\mu)]$ (δ/ε) is the relative tunnel ovalization. For $\rho = 0$, the tunnel cavity contracts horizontally and vertically by the same amount. For $\rho > 0$ the horizontal convergence decreases with respect to the vertical one, and becomes negative (the tunnel expands horizontally) for $\rho > 1$.

For $\rho = 0$ (no ovalization), expression (16) coincides with the discusser's initial result for pure ground loss. In order to get a better agreement with actual observations, the discusser proposed the introduction of an exponent (Sagaseta, 1988; Oteo & Sagaseta, 1996) (this was based on the fact that in a non-elastic medium the displacements attenuate with a power of the distance, $O(1/r^{\alpha})$, $\alpha > 1$):

$$\frac{s}{s_{\max}} = \frac{1}{(1+\bar{x}^2)^{\alpha}} \tag{17}$$

Both expressions (16) and (17) can be compared with the widely used error curve (Peck, 1969):

$$\frac{s}{s_{\max}} = \mathrm{e}^{\bar{x}^2/2\bar{i}^2} \tag{18}$$

where $\bar{i} = i/h$, and i is the abscissa of the inflection point, for which $s/s_{\max} = 0{\cdot}61$.

As shown in Fig. 3, the three expressions give very similar results when appropriate values are taken for the parameters $\bar{i}$, α and ρ. The range for $\bar{i}$ in actual cases is from 0·4 to 0·5, which corresponds to values of α from 2 to 4 and ρ from 0·5 to 4.

The authors' formulation has been applied by the writer to a well-documented case of shield tunnelling in clay (Attewell & Farmer, 1974). The

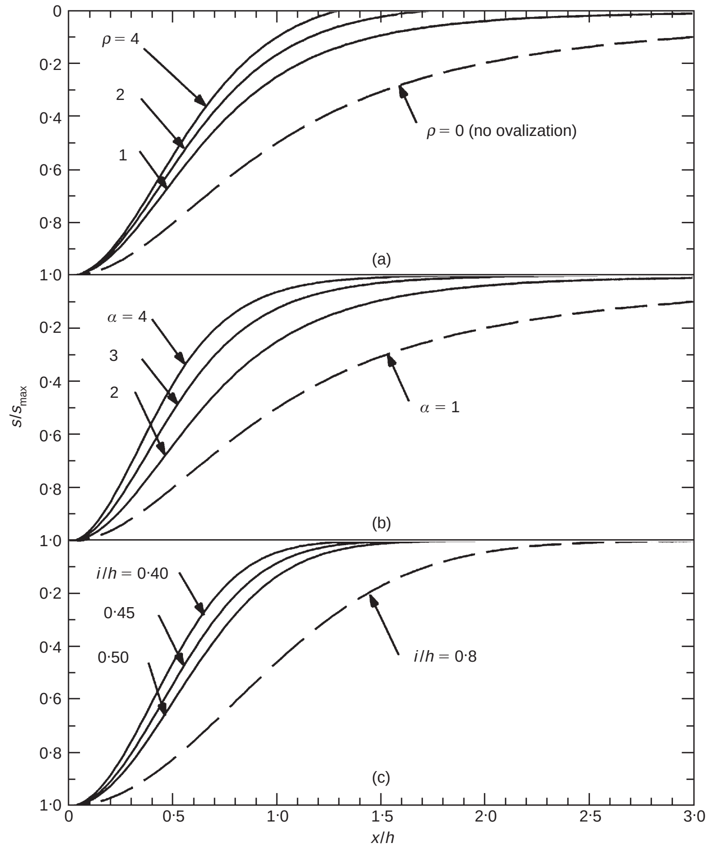

Fig. 3. Comparison of expressions for transverse settlement profiles: (a) equation (2); (b) equation (17); (c) equation (18), error curve

tunnel has a diameter of 4·10 m at a depth of 29 m. Fig. 4 shows inner soil movements: vertical displacements above the tunnel, between the crown and the surface, δ_v, and horizontal displacements at the tunnel sides, δ_h. These data were analysed by Mair & Taylor (1992) to illustrate the applicability of cavity contraction solutions. As can be seen, the theoretical solutions closely match the observations, for a convergence, ε, between 0·010 and 0·015 (relative ground loss 2–3%), and a relative ovalization, ρ, of 0·5. With these parameters, Fig. 5 shows the predictions for surface settlements, together with measured values. The agreement is satisfactory, with some tendency to overestimate the settlements in the far field ($x > 20$ m).

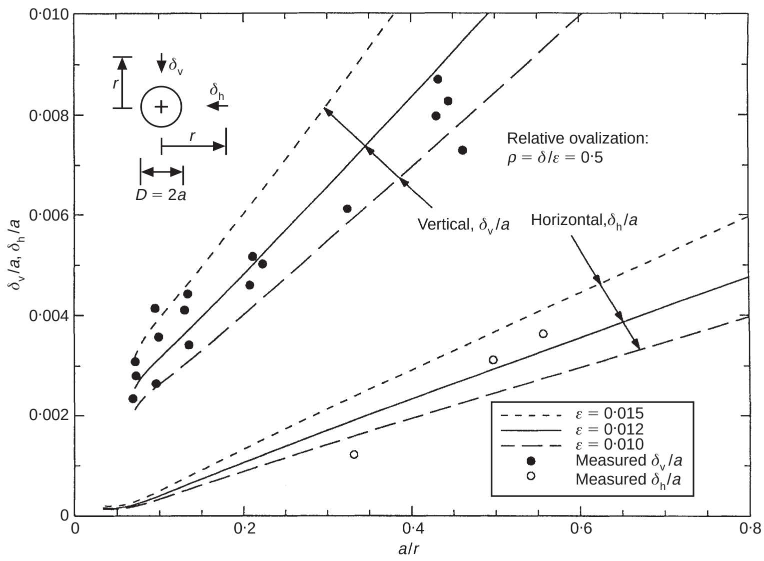

Fig. 4. Shield tunnel in London Clay. Inner soil displacements

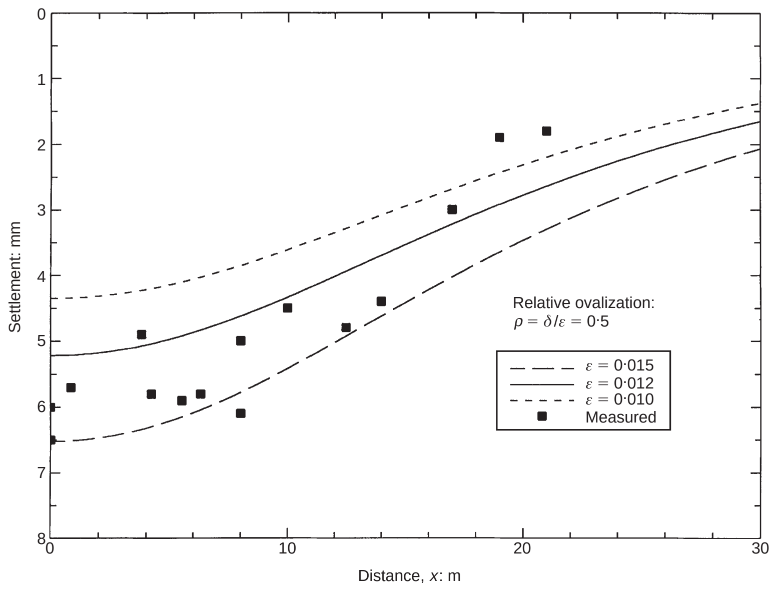

Fig. 5. Shield tunnel in London Clay. Surface settlements

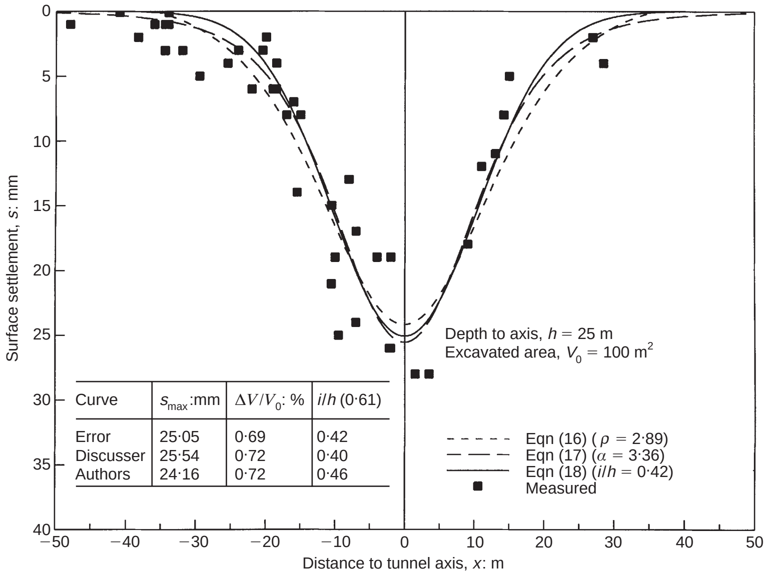

Fig. 6. NATAM tunnel in sand. Surface settlements

There are not as many documented cases for soils other than clay. In Fig. 6 a case is shown of a tunnel excavated by the new Austrian tunnelling method (NATM) in Lisbon, Portugal, with a cross-section area of 100 m^2, at a depth of 25 m. The soil above the tunnel is dense, partially cemented Miocene sand. The surface settlement profile has been fitted using the above three equations (16)–(18). The three curves are almost identical, with a volume of settlements of about 0·7% of the excavated area, a maximum settlement of about 25 mm and i/h (defined for $s/s_{\max} = 0{\cdot}61$), in the range 0·40–0·46. For the authors' expression (2), the fitting parameters are $\varepsilon = 0{\cdot}0024$ and $\rho = 2{\cdot}89$. The total integral of the settlement profile is actually 0·48% of $V_0(2\varepsilon)$ with a volume of settlements for $x < 35$ m of 0·72%, and an outside heave of 0·24%. The relative ovalization is abnormally high, possibly due to the influence of non-elastic deformations (shear strain concentrations, dilatancy, volume changes) that require non-realistic values of the involved parameters to be reproduced with the simple assumptions adopted.

Author's reply

We are grateful to Professor Sagaseta for his kind remarks and the additional references that he provides for the problem of surface deformations due to tunnelling, and especially for the field measurements that he shows. We all seem to agree that despite the natural drawbacks of a linear elastic analysis, it has the merit of being a complete and consistent solution, and may well be capable of showing all the major effects, with a minimum of parameters. And it also gives a useful reference for the validation of more realistic numerical solutions, of course.

Since the appearance of our paper some more work has been done, and it has been found that a rigorous analytical solution for problems of a circular tunnel in an infinite half-plane can be obtained using complex variable methods. For the case of a uniform radial displacement at the tunnel boundary (the ground loss problem) the solution can be expressed as an infinite power series in the complex plane (Verruijt, 1997). The solution gives the displacements and the stresses throughout the entire half-plane, without any approximation. One of the most interesting results is perhaps that the total volume of the settlement trough at the surface in general is larger than the total ground loss at the tunnel boundary. For an incompressible material these two quantities are equal, of course, but for values of Poisson's ratio smaller than 0·5 the volume change at the surface may be considerably larger than the total ground loss, up to a factor 2 for a relatively small or deep tunnel if Poisson's ratio is 0. This confirms a result already obtained in the paper discussed here.

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