

## DISCUSSION

# A unique relationship for $\chi$ for the determination of the shear strength of unsaturated soils

N. KHALILI AND M. H. KABBABZ (1998). *Géotechnique* 48, No. 5, 681–687.

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After decades of unsuccessful attempts to find an expression for effective stress in unsaturated soils, scientists have resorted to an independent stress variables approach. Khalili and Kabbaz have gone back to show that the shear strength of unsaturated soils can be described by a single effective stress parameter. Of course, this work pertains only to the shear strength of the soil and hence is still not free from the difficulty faced by earlier researchers, that the  $\chi$  parameter is different for compression and shear strength. However, it may be a useful method for describing the shear strength of the soil, but we have some comments on the concepts presented.

In this paper it has been shown that the  $\chi$  parameter bears a unique relation to the normalised suction: that is, suction normalised air entry value,  $(u_a - u_w)/(u_a - u_w)_b$ . Thus the air entry value of the soil is an important factor in this relation. Certain aspects are not clear about this air entry value.

The authors have used data from several published works in the literature to arrive at the above relation, and have listed the air entry value for all these soils in Table 1. But as far as we are aware, the air entry values have not been reported in the original papers for any of these soils. Thus these are not experimentally determined values except for the two soils tested by the authors in the present paper. Even for these two soils it is mentioned only that a filter paper technique has been used to determine the air entry value, but details of the testing are not given. For the other soils listed in the table, this air entry value has been calculated as the suction value at which the bilinear plot of shear strength against matric suction shows a sudden change in slope, as shown by the authors, for example, in Fig. 1 for the data of Gan *et al.* (1988). It is our opinion that this does not or need not correspond to the air entry value of the soil.

By definition, the air entry value is the suction at which air can displace water from the pores, and is related to the size of the pores. But for a clayey soil the initial size of the water pores will not be the criterion, unlike the situation in sandy soils, because the pores keep on shrinking with increase in suction. For a saturated soil, the air entry value can be identified as the suction at which desaturation begins, and this happens only at the shrinkage limit. The corresponding suction can be a few kPa for sandy or coarse-grained soils, but it will be hundreds or even thousands of kPa for a clayey soil. (The plasticity of the soil—LL or PI, or even OMC—may be a rough indication of this parameter: the higher the plasticity, the higher the air entry value.) Even for an initially unsaturated clayey soil such as the compacted soil, the suction value at air entry is likely to be the same, although the void ratio at which this happens may be different, provided there is continuity of water phase initially. There will be a continuity of water phase for the usual water contents that we work with for compacted soils. From this basis, the reported values of air entry for different soils in Table 1, especially the clayey soils, appear to be too low (62 kPa for soil 1 with OMC = 22%, 38 kPa for soil 7 with PL = 32%, 94 kPa for soil 3 with OMC = 21.5%, 86 kPa for soil 11 with OMC = 29%, etc.). When the authors' sandy soil itself has measured a value of 105 kPa, these clayey soils should be expected to have even higher values. Thus we feel that the break point in the strength plot does not represent the air entry value.

Our experience shows that the transition point depends

mainly on the level of initial compaction stress, and perhaps to some extent on the initial mixing water content and also the applied confining stress during shearing. The higher the initial compactive stress, the higher will be the suction induced in the soil. As long as the applied suction during shearing is less than the initial suction induced, and the stress levels are low so that the combined stress state is still less than the prestress during compaction, the applied suction will be fully effective for strength mobilisation because the soil has been already compressed to a higher effective stress level. For higher suction values the applied suction may be less effective because suction causes compression of water pores only, whereas an applied stress causes compression of both water and air pores. For example, in the soils examined above, the authors' clayey soil was compacted to standard compaction, which according to Cui & Delage (1995) corresponds to a static stress of  $\sim 850$  kPa, and hence the transition stress was quite high:  $\sim 400$  kPa, whereas kaolinite soil described by Wheeler & Siva kumar (1995) was statically compacted to a lower stress of 400 kPa, and hence the transition point was lower:  $\sim 82$  kPa. Of course, if the air entry value of the soil is really low as may be true for the authors' sandy soil, higher applied suctions may be less effective for strength mobilisation, for other reasons discussed later.

Thus one needs to examine the response of shear strength to matric suction from different angles, such as the effect of initial compactive stress level, or the magnitude of the applied stress, and it may not be related to the air entry value as proposed in the paper.

Another question concerns the meaning of suction beyond the air entry value. In most of the cases reported in this paper, suction is applied or measured during shear tests, using the axis translation technique. One can apply or measure water (or air) pressure only when the water (or air) phase is continuous. Beyond the air entry value, since air displaces water from some of the pores, continuity of the water phase may be lost. In such a case the applied water pressure may not be the same as that transmitted to different points within the sample, and it is therefore meaningless to talk about suction. The axis translation technique can be used only as long as there is continuity of both water and air phases, and hence it cannot be used beyond the air entry value.

There are some other minor points. For example, the assumption that  $\phi$  of the soil is the same for both saturated and unsaturated states is not true (with which the authors also agree); seems to vary with the initial state of the soil, sometimes quite significantly.

At high suction values and low confining pressures, the soil shows a pronounced peak and then softening behaviour, as shown by the data of Cui & Delage (1995). Then the question arises as to whether the authors' relations correspond to peak strengths or ultimate strengths at large strains.

### Authors' reply

The authors wish to thank the discussers for their interest in the paper. The main points raised in the discussion relate to the determination and the role of the air entry value in the behaviour of unsaturated soils, and the choice of the shear strength parameters.

The air entry values reported in Table 1 were obtained using

the points of intersection between the trend lines of  $\chi$  against suction and the saturation ordinate in Fig. 2 of the paper. This approach, although unconventional, accords fully with the definition of the air entry value, which is the demarcation point between saturated and unsaturated states for a material. The values of the air entry for the compacted kaolin and compacted sand-kaolin mixture were obtained using the filter paper technique, and confirmed using the pressure plate technique for the sand-clay mixture. As indicated in the paper, in both cases the values of air entry corresponded very closely to the break point in the plots of shear strength against matric suction plots. The reason for this correspondence is very simple, and relates to the fundamental role of the air entry value in the mechanics of unsaturated soils. At suction values below the air entry value the soil is saturated, and suction contributes directly to the effective stress and thus to the shear strength. This trend continues until the point of air entry, where the process of desaturation commences. After desaturation, suction becomes less effective in increasing the shear strength, leading to the observed break in the plots of shear strength against suction. Of course, this aspect of the behaviour of the unsaturated soils is not new, and has been recognised by several investigators in the past (e.g. Vanapalli et al., 1996).

We do not subscribe to the notion that the break point in the shear strength data is related to the maximum stress or the preconsolidation pressure that the soil has experienced in the past, as this would imply that: (a) similar break points should also occur in saturated soils past the preconsolidation pressure; and (b) no break points should occur in the plots of shear strength against suction for normally consolidated soils. Neither of these inferences is supported by the current experimental evidence.

With regard to the appropriateness of the air entry values reported in Table 1, it is important to note that, in addition to the soil type, the value of air entry, for a given soil, is influenced by void ratio, dry density and the way the sample is prepared, amongst other factors. Even for pure kaolin, depending on the way the sample is prepared (i.e. compacted statically or dynamically, dry or wet of the optimum, or tested as a slurry) air entry values ranging from 50 to 2000 kPa can be obtained (e.g. Vicol, 1990; Fleureau et al., 1993; Khalili & Khabbaz, 1999). In our experience, it would be misleading, if not incorrect, to infer air entry values based solely on index properties such as the Atterberg limits or the grain size distribution. If we were to accept the assertion by the discussers that the air entry value for clayey soils should be on the order of hundreds to thousands of kPa, irrespective of their sample preparation technique, then a vast majority of the test data reported in the literature for clayey soils, including those by Wheeler & Sivakumar (1995), would have been obtained in a saturated state, which seems highly implausible.

Concerning the continuity of the water phase beyond the point of air entry, the following desaturation states may be identified (Bear, 1972). Immediately after the air entry the air phase will be discontinuous, with the water phase filling most of the void space (insular air saturation). With increasing

suction both air and water phases become continuous (funicular saturation), and it is only at the residual water content that the water phase becomes discontinuous (pendular saturation). Even at this state, some investigators believe that the continuity of the water phase is maintained through the water retained in menisci at the contact grain points and the water attracted to the grain surface in films (Nitao & Bear, 1996). Therefore the suggestion by the discussers that the water phase becomes discontinuous after the air entry is incorrect.

An important aspect of the behaviour of unsaturated soils is that suction increases not only the effective stress but also the preconsolidation pressure. This means that the value of peak friction angle for a given soil will be a function of suction as well as of the effective stress state. However, as indicated in the paper, the current experimental evidence indicates that, for most practical problems, the value of peak friction angle may be considered as constant for suction values less than 600 kPa. This is particularly the case if the peak shear strength parameters are determined for the effective stress range of interest. Note that the effective stress relationship proposed is general in nature, and can be applied to quantify both peak and large strain strengths provided the appropriate peak and large strain strength parameters are adopted in the calculations.

Finally, we wish to stress that, to be valid, the effective stress must be the same for shear strength and volumetric change. Since submission of the paper we have conducted an extensive programme of laboratory testing to investigate the application of the relationship proposed for  $\chi$  in predicting volumetric change in unsaturated soils. The results have been very encouraging, with some partial results reported in Khalili & Khabbaz (1999).

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