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DISCUSSION

Critical state parameters for an unsaturated residual sandy soil

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E. J. Murray, *Murray Rix Geotechnical*
V. Sivakumar, *Queen's University of Belfast*

The paper deals with the critical state of a residual soil, Jurong soil, but also includes previously published data for another residual soil, Kiunyu gravel (Toll, 1990). The interpretation of the data is based on the premise that there is a smooth transition between unsaturated and saturated behaviour. This may be true, and seems a reasonable hypothesis at first consideration, but it relies on the progressive breakdown in the aggregated structure associated with unsaturated conditions to the more dispersed condition associated with saturated soils. However, an alternative philosophy might argue that, if the suction in an unsaturated soil is sufficient to maintain the structure at critical state, then it is likely that it will hold everywhere. Only at relatively low suctions, when the structure can no longer be maintained under shearing, will it break down, and at the critical state it is likely to break down everywhere. A smooth transition may thus not occur. This is important in that, as rightly identified by the authors and Toll (1990), the behaviour under a suction-controlled aggregate structure can be expected to differ from that of a dispersed structure.

Murray (2002) and Murray *et al.* (2002) describe the derivation and use of the average volumetric 'coupling' stress p'_c for unsaturated soils as given below:

$$p'_c = (p - u_a) + s\frac{v_w}{v} \tag{10}$$

where p is the mean total stress, $s = (u_a - u_w)$, v_w is specific water volume, v is specific volume, and u_a and u_w are pore water pressure

Normalised plots of q/s against p'_c/s for the Jurong soil and Kiunyu gravel are presented in Figs 18 and 20. The data fall on relatively straight lines with an intercept on the q_c/s axis at $p'_c/s = 1$ in both cases close to 0·6. As shown by Murray (2002), if the critical state data of Wheeler & Sivakumar (1995) for kaolin are plotted in a similar manner, this also results in a straight-line relationship with a similar intercept close to 0·60 on the q/s axis.

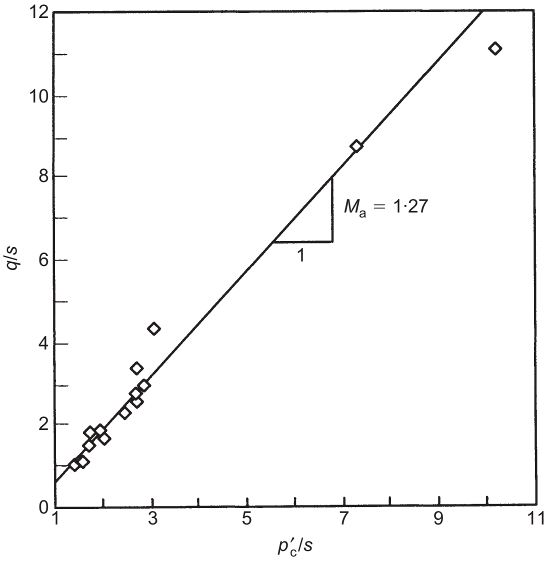

Fig. 18. Plot of *q*/*s* against p'_c/s at critical state for Jurong soil (data from Toll & Ong, 2003)

Adopting the nomenclature of Toll & Ong, the equation of the straight line is given by

$$q = M_a\left(\frac{p'_c}{s} - 1\right) + \Lambda \tag{11}$$

where M_a is the total stress ratio, and Λ is the intercept on the q/s axis at $p'_c/s = 1$.

Using equation (10), this may be written in a similar form to the deviator stress equation (4) in the paper of Toll & Ong as

$$q = M_a(p - u_a) + M_b s \tag{12}$$

where M_b is the suction ratio, given by

$$M_b = M_a\left(\frac{v_w}{v} - 1\right) + \Lambda \tag{13}$$

Figures 19 and 21 show plots of M_b against v_w/v for the Jurong soil and Kiunyu gravel using equations (12) and (13) and taking $\Lambda = 0{\cdot}6$. The plots again suggest a linear relationship, as has been found for kaolin (Murray, 2002).

Figures 18–21 indicate a consistent interpretation of the data with M_a constant and M_b decreasing with decreasing v_w/v (increasing suction).

For a saturated soil at critical state $q = Mp'$ (where M is the critical state stress ratio and p' is the effective stress). There is no obvious indication of a smooth transition between unsaturated and saturated conditions. For the Jurong soil $M = 1{\cdot}23$ and M_a is interpreted as approximately 1·27. For the Kiunyu gravel $M = 1{\cdot}62$ and M_a is 1·76. From the results of Wheeler & Sivakumar (1995) on kaolin $M = 0{\cdot}82$ and $M_a = 0{\cdot}86$. $M_a > M$ is interpreted as implying the

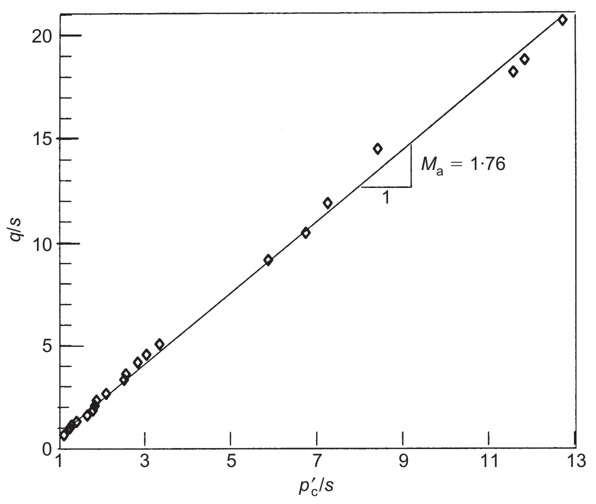

Fig. 19. Plot of *q*/*s* against p'_c/s at critical state (data from Toll, 1990)

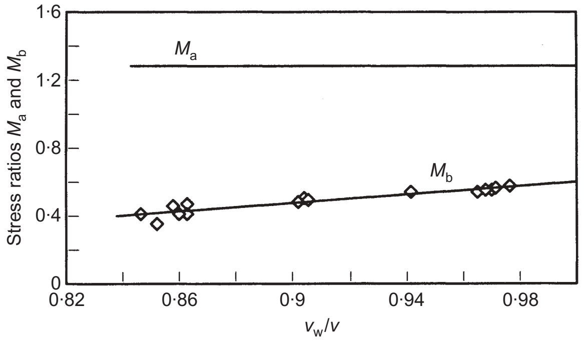

Fig. 20. Plots of M_a and M_b against v_w/v from the experimental results of Toll & Ong (2003)

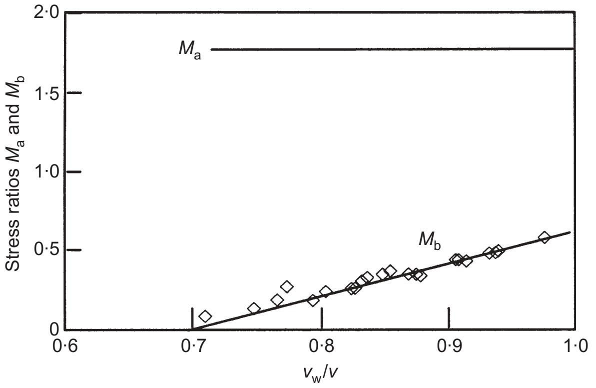

Fig. 21. Plots of M_a and M_b against v_w/v from the experimental results of Toll (1990)

creation of aggregated packets of particles that act as 'large particles' (Toll, 1990). Once they are created, however, the experimental evidence suggests that the modified soil structure results in a relatively constant M_a unaffected by changes in suction. The change in suction influences M_b. The formulation thus separates the influence of suction from the influence of soil structure. M_b decreases with decreasing v_w/v, consistent with equation (13) and the water phase being drawn back into the finer pores within the soil packets and having a reducing influence on the interpacket contacts where shearing is taking place (Toll, 1990).

Murray (2002) also shows that the normal consolidation lines for kaolin (Wheeler & Sivakumar, 1995) may be represented by the following equation:

$$v = N_t - \lambda_t \ln\left[(p - u_a) + s\frac{v_w}{v}\right] \tag{14}$$

where N_t is the extrapolated value of p'_c at 1·0 kPa, and λ_t is the slope of the straight-line portion of the consolidation plots.

The equivalent critical state lines would be given by

$$v = \Gamma_{ab} - \lambda_a \ln\left[(p - u_a) + s\frac{v_w}{v}\right] \tag{15}$$

where Γ_{ab} is the extrapolated value of p'_c at 1·0 kPa, and λ_a is the slope of the critical state line.

Toll & Ong indicate the following five variables as being necessary to describe the critical state for unsaturated soils: q, $p - u_a$, s, v and S_r, where S_r is the degree of saturation. They also indicate that five parameters are required: M_a, M_b, Γ_{ab}, λ_a and λ_b.

However, the foregoing formulation suggests that v_w or v_w/v is probably a more appropriate volumetric variable than S_r, and it may be possible to reduce the number of parameters from five to four. The suggested changes arise from equation (13), which indicates $M_b = f\,(M_a, v_w/v, \Lambda)$, and equation (15), which suggests that λ_b is unnecessary. The significance of v_w/v is that it represents the volume of the saturated 'packets' per unit volume of soil.

That is, the following five variables may prove more fundamental in an understanding and interpretation of the critical state for unsaturated soils: q, $p - u_a$, s, v and v_w(or v_w/v).

The following four parameters also emerge from the analysis: M_a, Λ, Γ_{ab} and λ_a.

It may be possible to further reduce the parameters to three if, as for the Jurong soil, Kiunyu gravel and kaolin, Λ can be shown to be constant at 0·6 for other soils. In addition, importantly, M_a is shown to be relatively constant for a given soil. This means that, in accordance with equations (12) and (13), if a relationship between s and v_w/v at critical state is established for a soil, only a limited number of strength determinations are necessary to determine M_a and Λ and facilitate strength assessments under other conditions. There is, however, the possible discontinuity between the behaviour in the saturated and unsaturated states as a result of soil structural changes that needs further investigation.

Authors' reply

Murray (2002) and Murray *et al.* (2002) have suggested using an average volumetric *coupling stress* in which they combine the contributions from the mean net stress ($p - u_a$) and suction, s, by weighting the suction by the ratio of water volume to the entire volume. This gives a different weighting function from that used by Bolzon *et al.* (1996), where the degree of saturation was used. This latter combined stress is referred to as *Bishop's stress* by Bolzon *et al.* or the *average soil skeleton stress* by Jommi (2000). Murray's approach averages over the entire volume, whereas using degree of saturation averages over the pore volume. There is a rationale for adopting Murray's suggestion, as we treat the total stress and pore water pressure as averaged over the entire area (or volume) in the effective stress law for saturated soils.

Such combined stresses (using either Murray's coupling stress or the average soil skeleton stress) have to be used in combination with suction (Toll, 2003), as they do not take account of the surface tension component imparted by suction (Burland, 1965). However, there is a trend to adopt a combined stress together with suction to explain unsaturated soil behaviour. Examples of this approach can be found in the same *Symposium in Print* (Gallipoli *et al.*, 2003; Wheeler *et al.*, 2003).

The re-analysis of our data using Murray's approach does raise some concerns. As is made clear by Murray & Sivakumar, there is a sharp and significant change as the degree of saturation drops below 100% (M_b drops to 0·6 and M_a increases to 1·27, from the saturated M_s value of 1·23). A more significant transition is seen for the Kiunyu gravel data (Toll, 1990), where M_b drops to 0·6 and M_a jumps to 1·76, from the saturated M_s value of 1·62. Murray & Sivakumar argue that the transition from the saturated state to the unsaturated state could involve a sharp change in M_a and M_b owing to the change in fabric, but in reality the transition would be expected to be gradual. At high degrees of saturation (about 90% and greater) the pore air will be present as occluded bubbles, and there will not be a surface tension component to affect strength. Even when the aggregated fabric is supported by suction the process will be gradual, as a very small suction is unlikely to be able to prevent the fabric from being destroyed by shear. The interpretation given in our paper shows M_a and M_b equal to M_s for high degrees of saturation, and then a gradual divergence between M_a and M_b as the suction becomes large enough to support the fabric and prevent breakdown. This seems to fit with what would be expected.

The fact that Murray's parameter Λ shows the same value of 0·6 for three different soil types seems to be coincidence. At least, until we have a theoretical explanation for this, or more data to justify it, we can only assume it is coincidence. However, we look forward to developing such interpretations for more data sets as they emerge. We can only hope that our fundamental understanding of unsaturated soils continues to improve so we can see whether it is best to adopt the stress variables of net stress and suction, average soil skeleton stress and suction or Murray's coupling stress and suction.

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