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DISCUSSION

On the applicability of cross-anisotropic elasticity to granular materials at very small strains

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This work offers an impressive set of data on the anisotropic characteristics of small-strain stiffness of granular materials. The trends revealed by the paper are, overall, coherent with other data on anisotropic soil stiffness (Arroyo & Muir Wood, 2003). This discussion focuses on one admittedly minor but nevertheless intriguing result included in the paper: the observed differences between S_{vh} and S_{hv}.

Commenting on Fig. 4, the authors state that 'Waves propagating through any homogeneous continuum should develop identical V_{hv} and V_{vh} values (to satisfy moment equilibrium), and yet these two velocities showed similarly divergent trends in many tests.' They then offer a micromechanically based explanation for the observed divergence. We shall first show that, even for an homogeneous continuum, identity of these two velocities is not always necessary, and then point to some reasons why the testing system employed may have introduced such a difference as an artefact.

PHASE VELOCITY SYMMETRIES

Plane wave propagation in an anisotropic elastic continuum is ruled by the elastodynamic equilibrium equation

$$\rho \ddot{u}_m = f_m + C_{mnpq} u_{p,qn} \tag{23}$$

In a well-known development (e.g. Udías, 1999) the phase velocity, v, and polarisation, p, of its plane wave homogeneous solutions are related to the elastic tensor, C, to obtain the characteristic equation

$$[\boldsymbol{\Gamma} - \rho v^2 \mathbf{1}]\boldsymbol{d} = \mathbf{0}$$

$$\Gamma_{ij} = C_{iljk} p_l p_k \tag{24}$$

This is an eigenvalue problem on the acoustic tensor Γ whose symmetry—a consequence of the major symmetry $C_{iljk} = C_{jkil}$—ensures that three real-valued solutions are possible. Three surfaces are therefore obtained, one for each solution, plotting the roots of the characteristic equation for every possible direction, p. Those surfaces are known as *phase-velocity surfaces*.

Some algebra (Arroyo, 2001) shows that the symmetries of the phase velocity surfaces are equal to those of the elastic tensor. A generally anisotropic material would not show $V_{\mathrm{hv}} = V_{\mathrm{vh}}$, nor would, for instance, a monoclinic material, whose elastic stiffness in principal axes may be written as

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{45} & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{bmatrix} \tag{25}$$

It is simple to form the acoustic tensor for two principal directions, [1,0,0] and [0,1,0], say vertical and horizontal:

$$\Gamma_{\mathrm{v}} = \begin{bmatrix} c_{11} & c_{16} & 0 \\ c_{16} & c_{66} & 0 \\ 0 & 0 & c_{55} \end{bmatrix} \quad \Gamma_{\mathrm{h}} = \begin{bmatrix} c_{66} & c_{26} & 0 \\ c_{26} & c_{22} & 0 \\ 0 & 0 & c_{44} \end{bmatrix} \tag{26}$$

It is apparent that they do not share any eigenvalue, and therefore the condition $V_{\mathrm{hv}} = V_{\mathrm{vh}}$ does not hold for this kind of elastic symmetry. This example has the added interest that an apparent monoclinic symmetry is what is observed when a transverse isotropic material is tested off axis. This will happen, for instance, in a triaxial transverse isotropic sample if its axis of symmetry does not coincide with the vertical axis.

Therefore, at least in principle, such a result may be attributed to a different kind of symmetry—apparent or real—from that assumed beforehand. In general it is possible to expect from soils any kind of elastic symmetry. Although often cloaked in the classic language of induced and inherent anisotropy, there is ample agreement that the small-strain stiffness of soils depends, at least, on two variables of non-scalar nature. This language can be made more precise (Zheng, 1995; Arroyo, 2001). In the case discussed, if these two variables are symmetric tensors (say stress and plastic strain), the recovery of a monoclinic or completely anisotropic elastic matrix would suggest non-coaxiality between those two variables.

Having said so, it is true that in a sample formed by vertical dry pluviation and loaded by a triaxial apparatus the most parsimonious assumption seems to be that of maintained cylindrical symmetry. We should then perhaps refrain from speculating on its meaning and consider the measurements.

MODELLING PROBLEMS

The discussion above assumes that a plane wave propagation model is adequate for interpretation of the test set-up described in the paper, and that phase velocities are unequivocally measured. This assumption may not be easily granted for bender transducer systems.

It is now clear that the accuracy of bender-based stiffness measurements is related not so much to the resolution of the data acquisition system as to the adequacy of the chosen inversion model (Arroyo *et al.*, 2003). One of the main concerns relates to sample size effects; the wavelengths quoted in Table 6 of the paper range between 35 and 58 mm, values that are commensurate with the sample diameter and of the same order as the sample height. Numerical results (Arroyo *et al.*, 2002) indicate that associated size effects may seriously mislead signal interpretation even when a single axial testing direction is involved. It is not at all clear that those size-induced errors would be the same when testing across the sample. This may also explain why differences between V_{hv} and V_{vh} appear when bender-testing Bothkennar clay (Greening *et al.*, 2003), a material with surely a different microstructure from those tested by the authors.

Authors' reply

The discussers propose alternative explanations for the discrepancies observed between the S_{vh} and S_{hv} wave velocities reported from some of the authors' tests, which had been interpreted as resulting from the particulate nature of the granular materials investigated. The discussers show that such discrepancies might be expected theoretically under conditions in which the assumed transverse isotropy did not hold. However, all the specimens discussed were vertical cylinders formed by vertical dry pluviation, to which transverse isotropy should apply. Samples whose bedding is inclined with respect to the horizontal, or specimens that have been subjected to principal stress (or strain) axis rotation, could exhibit tilted axes of anisotropy; the discussers' treatment would indeed be relevant to such cases.

The discussers' second proposal may be more applicable to the authors' tests. The interpretation of bender element data is made difficult by several factors, and previous studies have stressed the importance of properly identifying shear wave arrival and minimising any confusion caused by near field effects (e.g. Sanchez-Salinero *et al.*, 1986; Viggiani & Atkinson, 1995; Brignoli *et al.*, 1996; Jovicic *et al.*, 1996). The recent work at the University of Bristol and elsewhere by Arroyo and his colleagues has explored the causes of uncertainty further, although in our experience shear wave velocity measurement by bender techniques remains to some extent ambiguous. The discussers are right to speculate that different ratios of wavelength to travel path length could have affected the apparent velocities of the vertically and horizontally travelling waves. The different boundary conditions applying to the vertical and horizontal measurements might also have had effects that were not fully captured through the authors' (conventional) interpretive approach.

However, Kuwano (1999) and Jardine *et al.* (2001) show that the apparent discrepancies between S_{vh} and S_{hv} varied systematically with effective stress state and with particle shape. The differences were minimal for samples tested under initially isotropic conditions. With the sub-angular to sub-rounded Ham River and Dunkerque sands $S_{vh} > S_{hv}$ when σ'_v was elevated above σ'_h; similar trends were seen with more spherical glass ballotini, but with far smaller differences between S_{vh} and S_{hv}. In general, the apparent S_{vh} and S_{hv} velocities measured on the sand samples depended more on the normal effective stress acting in the direction of wave propagation than on that acting in the direction of polarisation. The micromechanical hypothesis matches the experiments well.

The paper noted the discrepancy before going on to develop sets of elastic continuum parameters based on the mean of the S_{vh} and S_{hv} measurements. However, it is natural to expect a particulate assembly containing a well-developed structure of concentrated force chains to show discrepancies between the two velocities that depend on effective stress states. While the discussers make some interesting points they have not shown why the velocity discrepancy should vary with stress state or particle shape. For the time being the micromechanics argument remains the authors' preferred explanation.

REFERENCES

Arroyo, M. (2001). *Pulse tests in soil samples*. PhD thesis, University of Bristol, UK.

Arroyo, M. & Muir Wood, D. (2003). Simplifications related to dynamic measurements of anisotropic G_0. *Proc. Int. Symp. Deformation Characteristics of Geomaterials, ISLYON'03* (in press).

Arroyo, M., Medina, L. & Wood, D. M. (2002). Numerical modelling of scale effects in bender-based pulse tests. *Proc. 8th Int. Symp. Numerical Models in Geomechanics, Rome* 589–594.

Arroyo, M., Greening, P. & Muir Wood, D. (2003). An estimate of uncertainty in current laboratory pulse test practice. *Riv. Ital. Geotec.* **37**, No. 1, 38–56.

Arroyo, M., Muir Wood, D. & Greening, P. D. (2003). Source near-field effects and pulse tests in soil samples. *Géotechnique* **53**, No. 3, 337–345.

Greening, P., Nash, D. F. T., Benahmed, N., Ferreira, C. & Viana da Fonseca, A. (2003). Comparison of shear wave velocity in different materials using time and frequency domain techniques. *Proc. Int. Symp. Deformation Characteristics of Geomaterials, ISLYON'03* (in press).

Sanchez-Salinero, I., Roesset, J. M. & Stokoe, K. H. (1986). *Analytical studies of body wave propagation and attenuation*, Geotechnical Engineering Report GR86–15. Austin: University of Texas at Austin.

Udías, A. (1999). *Principles of seismology.* Cambridge: Cambridge University Press.

Viggiani & Atkinson (1995). Cited in text.

Zheng, Q.-S. (1995). On the roles of initial and induced anisotropies. In *IUTAM symposium on anisotropy, inhomogeneity and nonlinearity in solid mechanics* (eds D. F. Parker and A. H. England), pp. 57–62. Dordrecht: Kluwer Academic.