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DISCUSSION

Contributions to *Géotechnique* 1948–2008: Dynamics

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In their vivid and eloquent overview of 60 years of contributions to *Géotechnique* in the field of dynamics, the authors restricted their coverage to papers covering mainly earthquake engineering topics such as the seismic design of earth structures and retaining walls, including the related use of centrifuge testing, and developments of so-called 'seismic' techniques for soil investigation. Of course, there is nothing wrong with their choice, especially in view of the enormous societal significance of earthquake engineering in the world.

Yet, no matter how interesting and significant these topics may be, they do not encompass the field of foundation dynamics and dynamic soil–foundation interaction. Indeed, as will be shown in this discussion, *Géotechnique* has hosted (and, effectively, nurtured) developments relating to shallow and pile foundation dynamics through groups of papers that sustained a few distinct (and sometimes unique) themes over a number of years. This contribution highlights some of these themes, and hence complements the presentation of the authors.

THE FIRST EARLY CONTRIBUTIONS

The very first article on dynamics appeared in 1954 by George Housner, titled 'Geotechnical problems of destructive earthquakes' (Housner, 1954). Among other topics the paper introduced the concept of ground–structure interaction, and explained three of its possible manifestations.

(*a*) The elasticity of the ground affects the natural period of vibration of the structure.
(*b*) The energy loss into the ground affects the intensity of the vibrations induced in the structure by the earthquake.
(*c*) The footings settle, owing to oscillating loads produced by vibrations of the structure.

He elaborated on the first two phenomena. Regarding the effect of soil on natural period he noted that

> A soft soil may permit deflexions of the footings and, thus, allow the structure to rotate about its base ... This will increase the period of vibration of the structure which, in general, is beneficial during an earthquake ... This is significant for very soft soils and relatively narrow buildings.

Regarding the second effect he noted that

> When a structure vibrates during an earthquake, it imposes an oscillating force on the ground, which sends out stress waves through the ground. If an appreciable amount of energy is dissipated in this fashion, there will be a beneficial effect on the behaviour of the structure.

He also addressed issues relating to pile-supported structures and to very long structures.

The writer would like to think that this paper, literally in the dawn of seismic engineering history, has had an appreciable effect in practice in later years.

On a different note, Jones (1958) outlined the wave propagation aspects of in situ measurement of dynamic soil moduli. By placing significant emphasis on footing vibrations he described the method, subsequently widely used, for obtaining the soil shear modulus from the resonant frequency, which he then verified with suitable experiments.

A similar methodology, based primarily on model experiments and the (homogeneous) halfspace theory, was later presented by Gupta (1972) for the resonant frequencies of embedded foundations.

DYNAMICS OF SHALLOW FOUNDATIONS

At least one aspect of foundation dynamics, that of foundation vibrations to externally applied loads, is closely related to the problem of static foundation response to loading. Indeed, when the frequency of a harmonically oscillating foundation becomes increasingly small, the stiffnesses of the foundation (i.e. the force–displacement and moment–rotation ratios) reduce to the static stiffnesses (translational and rotational respectively). Thus there has been a substantial crossover of results between classical elasto-static and elastodynamic solutions, as will become apparent.

Early elasto-static pioneering work by Gibson (1967, 1968, 1974) explored the importance of soil inhomogeneity and soil anisotropy in ground deformation and foundation settlement. Particularly striking was his discovery that an incompressible halfspace (i.e. with Poisson's ratio $\nu = 0{\cdot}5$) with shear modulus proportional to depth responds to surface stresses as a perfect Winkler spring medium. The finding, later called Gibson's law, had a significant theoretical and practical impact, as it bridged the gulf between the continuum (halfspace) and the Winkler-spring models.

Gibson's seminal work opened up the way for a fundamental new understanding in the analysis of soil–foundation interaction. It motivated researchers studying the dynamic and static response of foundations. Most significant was the sequence of theoretical articles published in *Géotechnique* in the early 1970s by Awojobi (1972, 1973a, 1973b, 1974a, 1974b), in which he explored the applicability and extended the meaning of Gibson's law under dynamic vertical and moment loading.

For a vertically vibrating rigid circular foundation, Awojobi (1972) showed that the Winkler model obeyed by the inhomogeneous halfspace with $G = mz$ and $\nu = 0{\cdot}5$ ('Gibson soil') is still approximately maintained at low frequencies. He also showed that radiation damping is much less than in an 'equivalent' homogeneous halfspace, whose (constant) modulus G is the same as that of Gibson soil at a depth equal to the foundation radius: $G = mR$.

Studying the harmonic steady-state rocking vibrations of a rigid strip foundation, Awojobi (1973a) found that: (a) Gibson's law is still maintained precisely under static moment loading, thus establishing the perfect coincidence of the two ground models (elastic continuum and Winkler) for Gibson soil; and (b) the resonant frequency of foundation rocking on Gibson soil is the same as on a homogeneous halfspace, if their moduli coincide at a depth equal to

$4/3\pi \approx 0{\cdot}42$ of the semi-width of the foundation. For what he named 'generalised' Gibson soil, that is, an inhomogeneous incompressible continuum with shear modulus $G = G_0 + mz$ (in other words, with a non-vanishing surface modulus), he showed that at asymptotically large frequencies or large inertia forces the resonant frequency of the system is equal to that on a homogeneous halfspace having $G = G_0$. This implied a kind of extreme 'short-sightedness' of the foundation on the inhomogeneous soil; only the surface modulus could be 'seen' by the foundation (i.e. only G_0 influences the response).

The response of several types of generalised Gibson soil under torsional loading was examined analytically by Gazetas (1981c). His conclusion was that Winkler-type behaviour can apply only qualitatively in this general case: with increasing soil inhomogeneity, the surface torsional displacements (u_θ) tend to become proportional to the applied local shear stress ($\tau_{z\theta}$), and to decay very rapidly away from the torsionally loaded area.

With the advent of semi-analytical and numerical methods, papers in the 1980s presented more complete solutions, beyond resonant frequencies, and for all modes of vibration. For instance, Gazetas (1980, 1981b) presented dimensionless parametric results for an inhomogeneous halfspace and a stratum-on-rock, in which the shear modulus increased or even decreased with depth (the latter idealising soft clay deposits overlain by a weathered crust, a soil profile that is all too frequently encountered).

In addition to, and in fact irrespective of, the inhomogeneity, soils are inherently anisotropic materials. London clay, for example, has been known to be strongly anisotropic: thus *Géotechnique* had published over the years numerous papers dealing with anisotropic soil behaviour, its modelling, and its consequences (e.g. Ward *et al.*, 1959; Barden, 1963, 1971, 1972; Pickering, 1970; Uriel & Cañizo, 1971; Atkinson, 1975; Milovic, 1972; Hooper, 1975).

The dynamic consequences of anisotropy, assumed to be of the form of cross-anisotropy with a vertical axis of material symmetry, has been explored by Gazetas (1981a). The study referred to the dynamic steady-state harmonic response of strip foundations, subjected to vertical, horizontal and moment loading, under both drained and undrained conditions. Parametric dimensionless diagrams and simple formulae of direct applicability were given for displacement amplitudes and resonant frequencies. The degree of soil anisotropy, described through the ratio of horizontal to vertical Young's moduli, E_H/E_V, was shown to be of paramount importance: values of E_H/E_V much higher than unity lead to much reduced static and dynamic response compared with the response on an 'equivalent' isotropic soil.

As already mentioned, from studies of harmonic vibrations of foundations, static solutions are recovered by letting the vibrational frequency approach zero. For partially or fully embedded foundations of any (reasonable) solid plan shape, Gazetas *et al.* (1985, 1988), using this methodology, developed closed-form expressions for the settlement, horizontal displacement and rotation due to vertical, horizontal and moment loading respectively. A key to that development was the separation of the problem into a surface foundation problem, corrected for two effects: a so-called 'trench' effect, which refers to the fact that even in a perfectly homogeneous soil an isolated foundation raft, placed at depth D below the ground surface, will displace/rotate less than the same raft on the surface; and the 'side-wall contact' effect, which accounts for the increased foundation stiffness thanks to the transmission of load through vertical and horizontal, normal and shear, tractions from the circumference of the sidewalls that are in contact with the soil.

PILES AND PILE GROUPS

This subject was partially covered in the paper, mainly from the viewpoint of earthquake effects on piles. In the wake of Gibson's seminal work, Randolph (1981) studied the static lateral response of flexible piles in a 'generalised' inhomogeneous soil, offering valuable closed-form expressions and charts for the 'active' (or 'effective') length, the deflexions/rotations, the bending moments, and the pile-to-pile interaction factors.

As a kind of extension of Randolph's work, Krishnan *et al.* (1983) presented parametric non-dimensional results for the dynamic problem of a single pile in Gibson soil under lateral harmonic loading. The convergence of their results for vanishing frequency with those of Randolph confirmed the additional usefulness of the dynamic studies (which, it seems, the profession has not fully appreciated and utilised as yet).

For the pile group problem, Dobry & Gazetas (1988) developed an extremely simple solution, based on fundamental (if not elementary) physics. Hence they explained the significant undulations of the pile-group efficiency factors (defined as group stiffness divided by the sum of the individual pile stiffnesses). It is well known that static efficiency factors are well below unity (as a function of the closest distance between neighbouring piles, their total number and their configuration). Under harmonic oscillatory motion, wave propagation and wave interference effects ('constructive' and 'destructive'), are responsible for pile groups attaining efficiency factors much greater than unity, in all modes of vibration (axial and lateral).

The surprisingly good accord between the simple solution of Dobry & Gazetas (1988) and the rigorous solutions by Kaynia & Kausel (1982) prompted an investigation by Makris & Gazetas (1993) to unveil whether the agreement was coincidental or had a deeper physical cause. They proved the latter, pointing out one of the culprits of the success of the simple solution: the nearly simultaneous emission of waves from the pile circumference due to the small phase differences along the pile.

While all of the solutions cited so far are for elastic soil, the dynamics of piles oscillating vertically in non-linear soil were studied in an approximate, non-linear Winkler-spring fashion by Michaelides *et al.* (1998). The soil yielding near the pile circumference when the vibration amplitude is large was modelled (in the equivalent-linear sense) through a linear but laterally (i.e. radially) inhomogeneous soil model: $G = G(r)$ for every soil slice at a particular depth z. $G(r)$ was a monotonically increasing function of r, reflecting the decreased amplitudes (and thereby increased stiffness) away from the pile.

In the last 10 or 15 years, as the interest of the engineering community has turned increasingly to the earthquake problem, there has not been a comparable effort on foundation dynamics. Some problems, however, such as the effect of soil non-linearity on reducing the magnitude of dynamic pile-to-pile interaction, remain unsolved and are formidable. But the potential consequences of reduced interaction on the stiffness of large pile groups embedded in soft soil can hardly be overstated. Nor can its practical significance. One could only hope for related publications in the not too distant future. Keeping with the tradition, *Géotechnique* is certain to attract and publish some of them.

Authors' reply

As stated in our introductory notes, we deliberately restricted our review, by necessity subjective, to earthquake engineering papers, published in *Géotechnique*, that have been of significant interest to the engineering community. We thank Professor Gazetas for the submitted discussion, as it expands our very brief review of foundation dynamics and highlights the many important contributions hosted by *Géotechnique* on this topic. Interestingly, Professor Gazetas also outlines unsolved problems in dynamic pile-to-pile interaction.

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