Zhang, F. *et al.* (2019). *Géotechnique* **69**, No. 7, 655–658 [https://doi.org/10.1680/jgeot.18.D.003]

DISCUSSION

Upper-bound solutions for face stability of circular tunnels in undrained clays

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This discussion is based on the paper by Zhang *et al.* (2018). The authors presented three-dimensional (3D) upper-bound (UB) solutions for undrained face stability of circular tunnels based on the kinematic approach of limit analysis, where a continuous velocity field with a toric envelope was adopted. The authors validated the derived UB solutions against that of Klar & Klein (2014) for a constant strength profile, and presented the stability charts of the stability number N affected by the dimensionless parameters C/D, and $\rho D/s_{u0}$. Finally, an approximate formula for N_{c0} and $N_{c\rho}$ was proposed in equation (28) in the original paper for a calculation of the critical pressure.

Recently, the discussers published a paper in *Computers and Geotechnics* entitled 'Three-dimensional undrained tunnel face stability in clay with a linearly increasing shear strength with depth' (Ukritchon *et al.*, 2017). Unlike the authors' approach, 3D finite-element analysis (FEA) was employed by the discussers to study only the collapse of the tunnel face. In that paper, a non-linear regression was also employed to statically derive N_{c0} and $N_{c\rho}$ from the finite-element (FE) solutions with a coefficient of determination (R^2) of 99·99%, and an algebraic formula of these was proposed. However, a comparison between the work of the authors and Ukritchon *et al.* (2017) was not given in the paper under discussion. Thus, it seems appropriate to write this discussion that addresses the comparison between them regarding the derived N_{c0} and $N_{c\rho}$ and the proposed approximate formula of these.

Figure 10 shows a comparison of N_{c0} for tunnels in clays with constant strength profile ($\rho = 0$) between the paper under discussion, Ukritchon *et al.* (2017) and Mollon (2012). Note that the latter is based on 3D UB calculations using a continuous velocity field with a toric failure mechanism, and was simulated by Ukritchon *et al.* (2017) using the Matlab toolbox, tunnel face stability software (TFSS) (Mollon, 2012), which represents the most up-to-date of previous studies by Mollon *et al.* (2009, 2010, 2011, 2013). Since the UB analysis with the same failure mechanism of toric shape was employed by both the authors and Mollon (2012) and the UB solution of the former is higher than that of the latter by about 17–24%, the N_{c0} solution in the paper under discussion is moderately inaccurate. For $C/D \leq 1{\cdot}5$, there is excellent agreement in N_{c0} between Mollon (2012) and Ukritchon *et al.* (2017). For $C/D \geq 2{\cdot}0$, these two solutions diverge and the latter predicts a lower N_{c0} ratio. For $C/D = 0{\cdot}5-5$, N_{c0} of Ukritchon *et al.* (2017) is smaller than that of the paper under discussion by approximately 20–36%. Fig. 11 shows a comparison of $N_{c\rho}$ for tunnels in clays with linearly increasing strength profile ($\rho = 0$). Note that there is no solution of Mollon (2012) in this case since TFSS is restricted to an analysis of the tunnel face in homogeneous clays. In general, both the results of Ukritchon *et al.* (2017) and the paper under discussion predict that $N_{c\rho}$ increases non-linearly with C/D, where the latter is always plotted above the former by approximately 32–45%. To justify which $N_{c\rho}$ is more accurate, a lower-bound solution of the problem is needed. However, it can be inferred from the result in Fig. 10 that the $N_{c\rho}$ of the paper under discussion is likely to be inaccurate, as in the case of N_{c0}.

The second issue of the discussion is related to the selected mathematical form of the approximate formula for N_{c0} and $N_{c\rho}$ used in the original paper, where a third-order polynomial function was employed, as shown in equation (28). Unlike that form, Ukritchon *et al.* (2017) employed a power function to regress the FEA results with a coefficient of determination (R^2) of 99·99%, as

$$
\begin{aligned}
N_{c0} &= a_1 \left(\frac{C}{D}\right)^{a_2} \\
N_{c\rho} &= a_3 \left(\frac{C}{D}\right)^{a_4}
\end{aligned}
\qquad (32)
$$

where $a_1 = 7{\cdot}8835$, $a_2 = 0{\cdot}3365$, $a_3 = 7{\cdot}6072$ and $a_4 = 1{\cdot}2489$.

It should be noted that the regression range with $C/D = 0{\cdot}5-5$ and $N_\gamma = C/D + 0{\cdot}5$ were adopted in the regression analyses by both Ukritchon *et al.* (2017) and the authors. Figs 12 and 13 show comparisons of N_{c0} and $N_{c\rho}$ between the two papers, respectively, particularly when C/D is outside the range of regression (i.e. $C/D \geq 5$). It can be observed from Fig. 12 that N_{c0} in the original paper changes its curvature for $C/D \geq 5$. In contrast, the power function employed by Ukritchon *et al.* (2017) gives a gradual increase in N_{c0} for all C/D ratios that are both inside and outside the range of the regression. For example, at $C/D = 10$, the original paper predicts N_{c0} that is 2·7 times higher than that of Ukritchon *et al.* (2017). This result implies that the stability solution calculated from the original paper will be too high, thereby likely predicting an unsafe limiting surcharge (σ_s) for $C/D \geq 5$. As shown in Fig. 13, it is apparent that $N_{c\rho}$ in the original paper is a concave function with a global maximum at C/D of about 13, and hence produces an unrealistic prediction for $C/D \geq 13$. On the contrary, such an unusual result is not observed for $N_{c\rho}$ of

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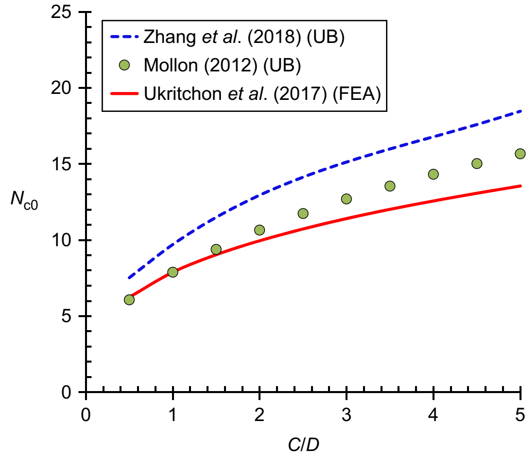

Fig. 10. Comparison of N_{c0} for $C/D = 0{\cdot}5{-}5$

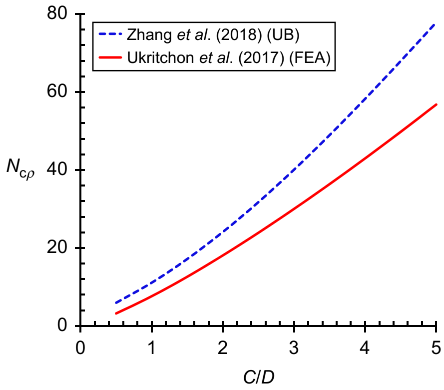

Fig. 11. Comparison of $N_{c\rho}$ for $C/D = 0{\cdot}5{-}5$

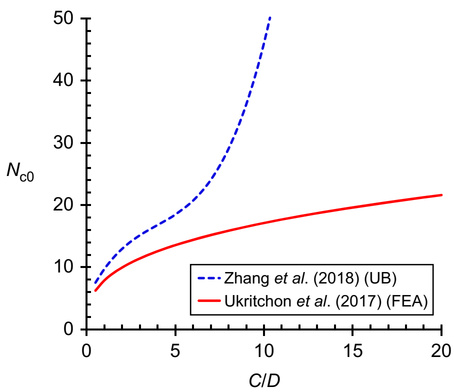

Fig. 12. Comparison of N_{c0} for $C/D = 0{\cdot}5{-}20$

Ukritchon *et al.* (2017). Consequently, it can be concluded that the cubic functions selected by the authors are unsuitable for accurately and realistically describing both N_{c0} and $N_{c\rho}$ for $C/D \geq 5$ that is outside the range of the regression. Thus, practitioners should be aware of this limitation. In contrast, the power function for N_{c0} and $N_{c\rho}$ proposed by Ukritchon *et al.* (2017) in equation (32) above is much simpler and requires fewer constant terms than the cubic function in the original paper, and yields a conservative and reliable prediction on these stability factors as described earlier.

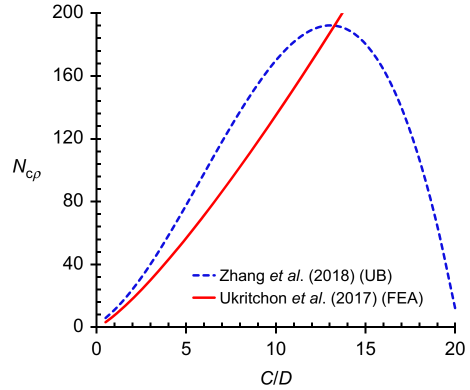

Fig. 13. Comparison of $N_{c\rho}$ for $C/D = 0{\cdot}5{-}20$

Authors' reply

The authors appreciate the comments of the discussers. All the issues raised by the discussers have been addressed as follows.

First of all, this note does not make a comparison of the derived UB solutions with the numerical FE results given by the discussers (Ukritchon *et al.*, 2017). This note (received 10 September 2016; accepted 14 March 2017) was accepted a little earlier than the discussers' paper (received 18 December 2016; accepted 19 March 2017). Unfortunately, the authors could not give the comparisons.

Second, the discussers made the comparisons of the critical support pressure on the face of the circular tunnel in undrained clay. For constant undrained strength with depth, the presented comparison of the UB solutions between Mollon *et al.* (2013) and this study demonstrates that the solutions of this study are less critical than those derived by Mollon *et al.* (2013). The reason for this has been explained by Klar & Klein (2014) as that the non-rigorous definition of the strain rate tensor in Mollon *et al.* (2013) may yield a lower value of the stability number N_{c0}. Therefore, this study adopted the closed-form analytical expression of the curvilinear strain rate tensor presented by Klar & Klein (2014) to calculate the least UB solutions of the support pressure. The comparison between Klar & Klein (2014) and this study has been given to show the good agreement of the calculated results, as shown in Table 1. The obtained UB solutions of this study are definitely accurate.

Next, the discussers compared their numerical FE results with the UB solutions of this study and demonstrate the criticality of support pressure obtained from the FE method. The FE results could be affected by the boundary conditions, the mesh refinement and the selection of the limit state and cannot be regarded as a rigorous solution for a problem in the limit state. The kinematic approach of limit analysis (LA) is based on the UB theorem of plasticity. The obtained UB solution is rigorous but its criticality depends on the constructed kinematically admissible velocity field. For the face stability of a tunnel in undrained clay, the continuous velocity field proposed by Mollon *et al.* (2013) may not yield the least UB solution in LA and can be further improved.

Finally, the discussers extended the application of the presented approximate formula based on the third-order polynomial fitting function to the case for larger ratio of the cover depth to the tunnel diameter ($5 \leq C/D \leq 20$). However, the fitting values of the coefficients P_0, P_1, P_2 and P_3 from the UB solutions for C/D ranging from 0 to 5 are used for the comparisons by the discussers. In fact, the suggested values of these coefficients given in Table 3 are only limited to the case for C/D ranging from 0 to 5. As the discussers stated, such a limitation should be given for clarity in practical use. For the larger ratios C/D, the authors obtained the UB solutions of N_{c0}

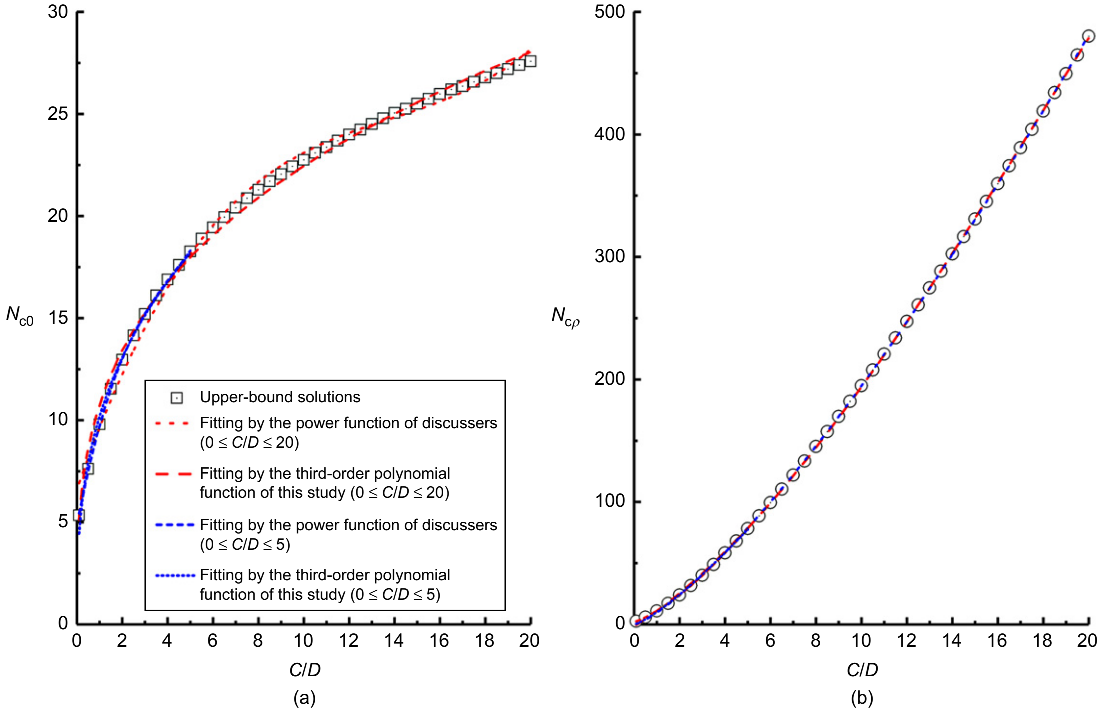

Fig. 14. Comparisons of the fitting UB results between the power function of the discussers and the third-order polynomial function of the study under discussion

Table 4. Results determined by different regression functions

Regression functions	C/D	Fitting coefficients	N_{c0}		$N_{c\rho}$	
			Value	Statistical coefficient R^2	Value	Statistical coefficient R^2
Third-order polynomial function of this study	[0, 5]	P_0	4·8659	0·9996	1·8388	0·9999
		P_1	5·8187		7·0609	
		P_2	−1·0703		2·3339	
		P_3	0·0861		−0·1363	
	[0, 20]	P_0	6·6051	0·9937	−1·5494	0·9999
		P_1	3·2155		11·6699	
		P_2	−0·2063		0·9589	
		P_3	0·0050		−0·0171	
Power function of discussers	[0, 5]	a_1	10·1847	0·9934	10·1622	0·9986
		a_2	0·3605		1·2627	
	[0, 20]	a_1	10·7306	0·9959	9·6396	0·9999
		a_2	0·3206		1·3053	

and $N_{c\rho}$, as shown in Fig. 14. Their trends remain consistent with the FE results given by the discussers. Using the same function to regress the extended results can determine the values of those coefficients, as shown in Table 4. It can be seen that the statistical coefficient R^2 is still 0·9999 for $N_{c\rho}$, but slightly reduces for N_{c0}. Generally, the presented third-order polynomial regression function is in good agreement with the obtained UB solutions, as shown in Fig. 14. In addition, the authors also employ the power function suggested by the discussers to regress the UB results and obtain the values of the coefficients a_1 and a_2 (a_3 and a_4 for $N_{c\rho}$) in equation (32), as shown in Table 4. When the ratio is less than 5, the value of the statistical coefficient R^2 obtained by the power function is less than that derived from the third-order polynomial function. Therefore, the third-order polynomial fitting function is more suitable for the UB solutions of this study. In practice, the cover depth is usually less than five times the tunnel diameter.

In summary, the note aims to determine the UB solutions of the critical support pressure on the circular face of the tunnel in undrained clay in which undrained strength increases linearly with depth. The continuous velocity field proposed by Mollon *et al.* (2013) is adopted here to calculate the UB solutions in LA. However, this postulated velocity field can be further improved for better estimation on the UB solutions. The approximate formula, based on the third-order polynomial regression function fitting to the UB solutions, provides an alternative tool for quick estimation of the critical supporting pressure. It should be noted that the suggested values of the fitting coefficients in Table 3 are limited to the case for C/D ranging from 0 to 5.

Again, the authors of this note thank the discussers for their interest in this study.

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