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Optimal adaptive decision rules in geotechnical construction considering uncertainty

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To design a geotechnical engineering structure optimally, an iterative decision-making process is required due to the prevailing uncertainty of the ground conditions. At present, these decisions are taken based on simple deterministic rules and models. This paper proposes a risk-based decision-theoretic framework to the optimal planning for geotechnical construction. This framework combines geotechnical probabilistic models, cost analysis using Monte Carlo simulation and the observational method. The framework is illustrated on the design of the surcharge for an embankment on soft soil, whereby the optimal preloading sequence of added surcharge is adapted to the observed settlement. The approach balances the cost of surcharge material against financial penalties related to project delays and insufficient overconsolidation, which causes damage due to residual settlement. The result is a preloading strategy that optimally accounts for information obtained from settlement measurements under uncertain ground conditions. The findings highlight the potential of using risk-based decision planning in geotechnical engineering, in particular when combined with the observational method. For the investigated case study, a reduction in the expected cost in the order of 25% is observed.

KEYWORDS: embankments; observational method; planning & scheduling; preloading; risk & probability analysis; sequential decision problem

INTRODUCTION

Design of geotechnical engineering structures implies decision making under uncertainty. The reason is mainly a lack of knowledge about the prevailing ground conditions, but there are also limitations in understanding and predicting the ground–structure interaction or temporal variations. Managing these uncertainties is essential to achieving a design of satisfactory quality without unnecessary delays and at a reasonable cost. One approach to this challenge is to view the geotechnical design and execution as a sequential decision problem, which has been studied in other areas of engineering and decision making (e.g. Rosenstein & Barto, 2001; Straub & Faber, 2005; Memarzadeh *et al*., 2014; Papakonstantinou & Shinozuka, 2014; Malings & Pozzi, 2016; Bismut & Straub, 2021; Wang *et al*., 2022). The engineering challenge lies in finding a cost-effective sequence of design decisions, considering not only the technical requirements at the time of project completion, but also the respective probabilities and costs of potential consequences caused by an unsuccessful design. In the ideal case, the analysis should also consider operational and maintenance costs (Mendoza *et al*., 2021). At present, these decisions are taken based on simple deterministic rules and simplifying model assumptions.

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A typical example of a geotechnical engineer's decision under uncertainty is the design of embankments on soft soil prone to consolidation settlements using a surcharge and prefabricated vertical drains (PVDs) (Fig. 1). The embankment load initiates a consolidation process towards a final long-term settlement, but neither the magnitude of this settlement, nor the time until it is reached, can be well predicted by the engineer; despite geotechnical pre-investigations being performed, there are typically considerable uncertainties regarding the soil's hydraulic conductivity and deformation properties. Unless this uncertainty is carefully managed by a planned sequence of inspection decisions and mitigating actions during design and construction, unwanted costly consequences such as time delays or residual settlements after completion of the superstructure may occur. The engineering challenge therefore essentially lies in finding a cost-effective design solution, considering not only the technical requirements at the time of project completion, but also the respective probabilities and costs of potential consequences caused by an unsuccessful design.

To the authors' knowledge no geotechnical problem has ever been formalised as a sequential decision problem under uncertainty. A few studies have, however, used other, simpler decision theoretical analyses for other geotechnical applications: Einstein *et al*. (1978) showed an early application of decision theoretical principles; Zetterlund *et al*. (2011), Sousa *et al*. (2017), Klerk *et al*. (2019) and Hu *et al*. (2021) performed value of information analyses; and preposterior analyses were performed by Schweckendiek & Vrouwenvelder (2013), Spross & Johansson (2017), van der Krogt *et al*. (2022), Löfman & Korkiala-Tanttu (2022) and Spross *et al*. (2022).

Probabilistic settlement analyses have recently been performed by, for example, Bari *et al*. (2016), Bong & Stuedlein (2018) and Löfman & Korkiala-Tanttu (2022). Addressing the design issue of embankment preloading with PVDs, Spross & Larsson (2021) specifically showed how a probabilistically evaluated initial surcharge height can be used in an observational method to limit the probability of time delay and residual settlement in soft soil. Spross *et al*. (2019)

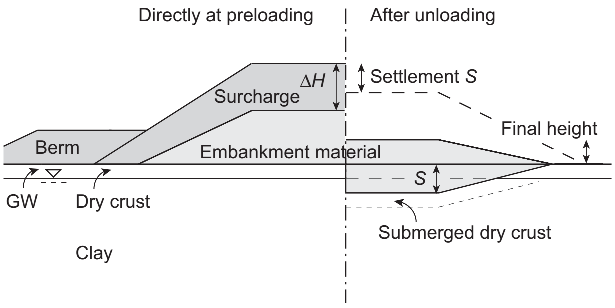

Fig. 1. Preloading of an embankment with a surcharge of total height ΔH to accelerate consolidation. Prefabricated vertical drains are omitted for clarity (GW, ground water)

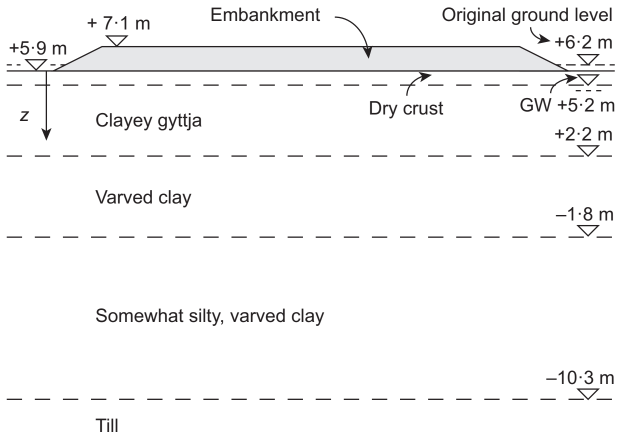

Fig. 2. Cross-section of the soil under the planned embankment (from Spross & Larsson (2021) CC-BY-4.0, https://creativecommons.org/licenses/by/4.0/)

discussed how settlement monitoring can be evaluated as a basis for a decision to increase the surcharge height. The specific decision-theoretical problem was highlighted, but not solved.

In this paper, the authors propose a risk-based decision-theoretic framework to optimal sequential planning in geotechnical construction. This framework combines a geotechnical probabilistic model with models of the observations and the cost models of actions and unwanted consequences. As a methodology to identify optimal decisions, the authors propose – for the first time in geotechnical engineering – the use of heuristics to describe and optimise the sequence of decisions (Bismut & Straub, 2021).

The proposed framework and methodology are illustrated through an embankment preloading problem. The sequence of decisions on initial surcharge height and later additions to the surcharge are optimised such that a desired settlement is achieved at a minimal expected cost, which reflects whether the settlement is achieved within a fixed timeframe. Construction delays as well as insufficient overconsolidation, which is a cause of residual settlement, are explicitly penalised.

The probabilistic preloading model by Spross & Larsson (2021) is used to describe the settlement evolution. Here, this model is extended to allow simulation of soil settlement curves when the surcharge height is adjusted, thereby enabling modelling of the effect of sequential surcharge height decisions on the settlement evolution.

The outcome of the analysis is a preloading strategy, which prescribes how much surcharge to add conditional on settlement measurements through optimised heuristic parameter values.

EXAMPLE APPLICATION

To illustrate the proposed framework, the current authors take the specific example introduced by Spross & Larsson (2021). A section of an embankment built for the construction of a highway in the south of the county of Stockholm, Sweden is considered. A cross-section of the soil is shown in Fig. 2.

The authors consider the planning of the surcharge loading on the embankment during an available preloading time, $t_{\max}$, within which an acceptable soil consolidation is to be reached. The engineering questions can be stated as follows. (*a*) What initial surcharge height should be used? (*b*) When is a load increase warranted during the preloading time, and if so, how much more should be added?

GEOTECHNICAL MODEL AND DESIGN REQUIREMENTS

In this section, the main aspects are presented of the probabilistic model adopted to describe the evolution of soil settlement and resulting overconsolidation ratio (OCR), first under constant load and then under multi-stage loading. This geotechnical model, described in detail in Spross & Larsson (2021), considers (*a*) how primary compression settlement develops with time, due to the weight of the embankment and the surcharge, and (*b*) the effect of the unloading of the surcharge on the OCR. More detailed and complex models of settlement and consolidation behaviour for staged construction are available in the literature (see, e.g. Walker & Indraratna, 2009; Yin *et al.*, 2022), but the effect of the choice of the geotechnical model on the results is outside the scope of this paper.

Settlement evolution

Under a constant load $\Delta\sigma$ and known soil properties, a settlement trajectory with time follows

$$S(t) = U(t) S_{\infty} \tag{1}$$

where

$$U(t) = 1 - [1 - U_{\mathrm{v}}(t)][1 - U_{\mathrm{h}}(t)] \tag{2}$$

is the spatially averaged degree of consolidation at time t, and S_{∞} is the predicted long-term primary compression settlement under load $\Delta\sigma$. The vertical consolidation component $U_{\mathrm{v}}(t)$ is obtained from Terzaghi's consolidation theory. For the horizontal consolidation component $U_{\mathrm{h}}(t)$, Hansbo's well-established analytical PVD model is applied (Hansbo, 1979). Owing to the specific consolidation behaviour of soft clays, S_{∞} is predicted as (Larsson & Sällfors, 1986)

$$S_{\infty}(\Delta\sigma) = \sum_{i=1}^{l} h_{\mathrm{cl},i} \Delta\varepsilon_i(\Delta\sigma) \tag{3}$$

where $h_{\mathrm{cl},i}$ is the thickness of the ith clay layer and $\Delta\varepsilon_i$ is the strain increase caused by the load $\Delta\sigma$. The strain depends on parameters evaluated from constant-rate-of-strain (CRS) tests, including the preconsolidation pressure and soil moduli (Spross & Larsson, 2021).

In the analyses performed, the embankment and surcharge are assumed to be of the same material; hence the load $\Delta\sigma$ is proportional to the material unit weight and to its total height.

If the surcharge is increased by $\Delta\sigma_{\mathrm{add}}$ after some preloading time, t_{add}, the adjusted settlement trajectory is modelled as

$$S(t) = \begin{cases} U(t) S_{\infty}(\Delta\sigma), & \text{for } 0 \leq t < t_{\mathrm{add}} \\ U(t - t_{\mathrm{shift}}) S_{\infty}(\Delta\sigma + \Delta\sigma_{\mathrm{add}}), & \text{for } t \geq t_{\mathrm{add}} \end{cases} \tag{4}$$

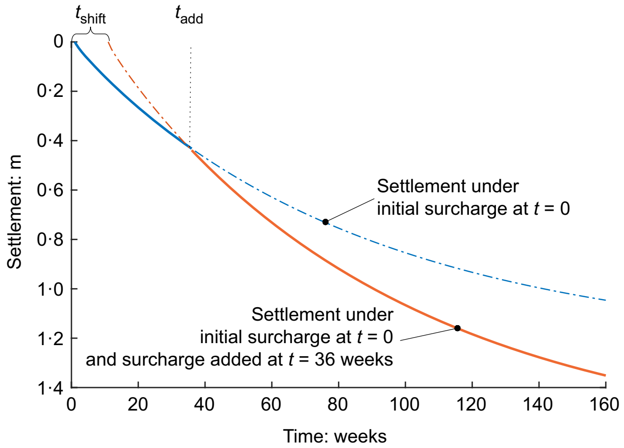

Fig. 3. Effect of the added surcharge at time t_{add} on the settlement, where $t_{shift} = t_{add} - t_0$

Under this staged preloading, the first part of the trajectory is equivalent to equation (1). The second part contains, due to the load increase, a recalculated, larger long-term primary consolidation settlement $S_{2,\infty} = S_\infty(\Delta\sigma + \Delta\sigma_{add})$ following equation (3) and a corresponding degree of consolidation $U(t - t_{shift})$, for which a hypothetical zero degree of consolidation occurs at time $t_{shift} = t_{add} - t_0$. To determine t_0, it is noted that the settlement curve is continuous at t_{add}, which results in the degree of consolidation

$$U(t_0) = \frac{U(t_{add})S_{1,\infty}}{S_{2,\infty}} \tag{5}$$

where $U(t)$ is obtained from equation (2). Fig. 3 illustrates t_{shift} and the resulting settlement curve for staged preloading.

Overconsolidation ratio

As achieving overconsolidation by the removal of the surcharge in practice has been found to reduce residual settlement (Alonso *et al.*, 2000; Han, 2015; Indraratna *et al.*, 2019), the model considers this effect through the OCR in the middle of the clay stratum. Under constant surcharge, this quantity can be obtained as in the paper by Spross & Larsson (2021)

$$\mathrm{OCR}(t) = \frac{\sigma_0' + U(t)\Delta\sigma_{sur}}{\sigma_0' + U(t)\Delta\sigma_{emb}} \tag{6}$$

where σ_0' is the initial vertical stress in the middle of the clay stratum; $\Delta\sigma_{sur}$ is the vertical stress caused by the preloaded embankment (i.e. including the surcharge); and $\Delta\sigma_{emb}$ is the remaining stress increase directly after the unloading of the surcharge (see Fig. 1).

For staged preloading, the effect of the added load on the OCR at unloading depends on the preloading time of both the initial and any added load. To the present authors' knowledge, there are no validated analytical models for this issue. Therefore, the following reformulation of equation (6) is used to capture the effect on the OCR at the unloading at time t, when it occurs after a previous load increase at time t_{add}

$$\mathrm{OCR}(t) = \frac{\sigma_0' + U(t)\Delta\sigma_{sur} + \Delta U(t)\Delta\sigma_{add}}{\sigma_0' + U(t)\Delta\sigma_{emb}} \tag{7}$$

where $\Delta U(t) = U(t - t_{shift}) - U(t_{add} - t_{shift})$. Consequently, the effect on the OCR of the added load will depend on the degree of consolidation achieved along the recalculated settlement trajectory after the load has been added. The OCR for staged preloading is depicted in Fig. 4.

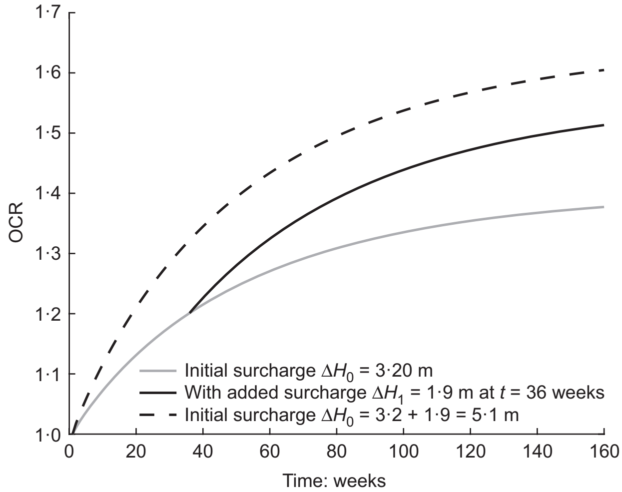

Fig. 4. Effect of the surcharge added at time t_{add} on the OCR, using equation (6) with initial surcharge $\Delta\sigma_{sur}$ corresponding to height ΔH_0 for the first part of the curve until $t = 36$ (weeks) and equation (7) with $\Delta\sigma_{add}$ corresponding to additional surcharge height ΔH_1. The resulting curve is located below the one for the case where the total surcharge (initial and additional) is applied directly at $t = 0$, with equation (6)

Uncertainties in the soil parameters

The presented soil consolidation model depends on numerous parameters for the soil properties and PVD design. These parameters govern the evolution of the vertical and horizontal consolidation, hence the settlement, as per equations (1)–(7). As explained in the paper by Spross & Larsson (2021), these soil properties are modelled as random variables with an associated probability distribution either evaluated from CRS oedometer tests or assigned based on engineering judgement when data on variability were not available. The parameters in Hansbo's PVD model (Hansbo, 1979) are assumed constant. The complete deterministic and probabilistic assumptions are described in detail in tables 2 and 3 of the paper by Spross & Larsson (2021) and are therefore omitted for brevity. Random settlement trajectories obtained by Monte Carlo (MC) simulation are depicted in Fig. 5.

Settlement and OCR requirements

The proposed risk-based planning framework for optimal preloading requires the definition of performance criteria, such that a preloading decision can be assessed in terms of its success to reach the desired goals. These goals are here expressed in terms of sufficient soil consolidation, through targets on the settlement and OCR, s_{target} and OCR_{target}, respectively. These targets are defined in the following paragraphs.

Owing to the uncertainty associated with the ground properties, the long-term settlement S_∞ caused by the load of the completed embankment, $\Delta\sigma_{emb}$, is also uncertain. To ensure an acceptable residual (post-construction) primary consolidation settlement, Spross & Larsson (2021) proposed a target settlement s_{target}, such that the probability that the long-term settlement under the embankment load attains this target is equal to an acceptable, fixed probability, p_{FT}

$$\Pr\left[S_\infty(\Delta\sigma_{emb}) > s_{target}\right] = p_{FT} \tag{8}$$

In the numerical investigations, p_{FT} is set to 5% to represent a serviceability limit state. By generating sample values of $S_\infty(\Delta\sigma_{emb})$ from the defined probabilistic model and equation (3), s_{target} is obtained as the quantile value corresponding to p_{FT} (Fig. 5). The value of s_{target} is thereafter used to define penalty mechanisms.

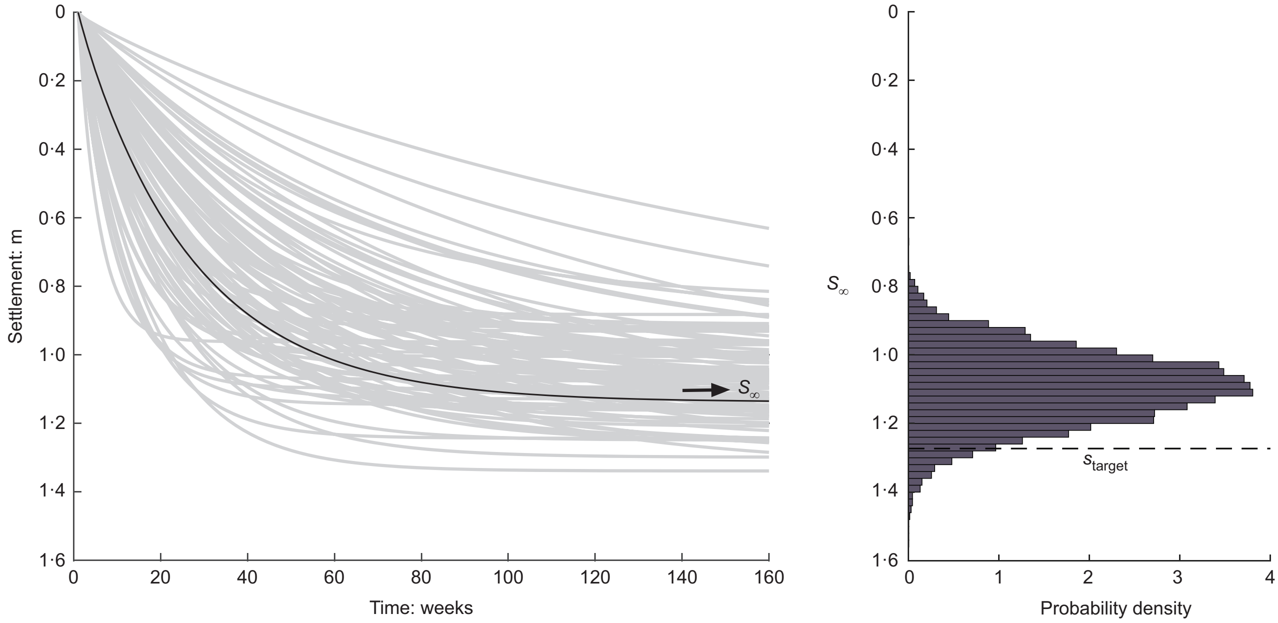

Fig. 5. One hundred sample soil settlement trajectories for an initial surcharge $h_0 = 0$ (m) (no added surcharge). One such trajectory is highlighted in black. For each trajectory, the value of the long-term settlement S_∞ is obtained with equation (3). The histogram on the right shows the resulting distribution of S_∞. The value s_{target} is obtained from the condition Pr $(S_\infty > s_{\text{target}}) = p_{\text{FT}}$

In addition, it is required that the OCR exceeds the threshold $\text{OCR}_{\text{target}} = 1{\cdot}10$ in the middle of the soft soil stratum after unloading of the surcharge. This is in line with the general technical requirements and guidance for geotechnical works issued by the Swedish Transport Administration (Trafikverket, 2013a, 2013b).

In addition to settlement and OCR requirements, a successful embankment design needs to consider the stability of the embankment. This is typically ensured by the berms (Fig. 1), which add to the construction cost. Ideally, the dimensions of the berms should also be evaluated from the geotechnical model based on the undrained shear strength of the clay. For simplicity, the current authors do not carry out this analysis but do consider the berms in the cost model (see equation (16)).

OPTIMAL PRELOADING STRATEGIES

To find the optimal preloading strategy, the authors rely on the decision analysis framework of Raiffa & Schlaifer (1961), which formalised decision problems under uncertainty with varying information. This enables the optimisation of the surcharge decisions, which can be done in a sequential manner based on measurements of the settlement. Further general information on sequential decision making can be found in Kochenderfer (2015).

Elements of the decision analysis

A decision analysis under uncertainty is based on a probabilistic model of the system, a model of the decision alternatives as well as a utility or cost function. These models are summarised in the following.

Probabilistic model. A complete probabilistic model must account for the effects of actions affecting the system (see 'Decision alternatives' below) and reflect the uncertainty in information collection, through a likelihood function (Bismut & Straub, 2022).

In the investigated engineering problem, the soil consolidation model described above is used. Information on the state of the system is obtained as a measurement M_{t_1} of the settlement S_{t_1} at time t_1. The M_{t_1} is related to the true value of the settlement by an additive measurement error, ε

$$M_{t_1} = S_{t_1} + \varepsilon \tag{9}$$

For simplicity, the numerical investigation is restricted to error-free measurement – that is, $\varepsilon = 0$. The proposed methodology can easily be adapted to account for noise in the measurement.

Decision alternatives. The decision alternatives describe the available mitigating actions and must account for operational constraints. The description of the available decision alternatives should also include operational constraints that must be accounted for in the planning process.

Utility and cost. The effects of a decision are evaluated in terms of utility, which reflects the preferences of the decision maker. Ultimately, the optimal decision is selected as the one that maximises the expected utility. Assuming a risk-neutral context, the utility can simply translate to costs associated with the actions and the system performance. In this case, utility is expressed in monetary terms.

For the embankment preloading illustration, three cost components are identified, and these are summarised in Table 1. The total cost C_{tot} incurred at the completion of the preloading operation is obtained as

$$C_{\text{tot}} = \sum_i C_{\text{sur},i} + C_{\text{delay}} + C_{\text{OCR}} \tag{10}$$

If relevant, discounting can be used to reflect the decreasing value of an investment over time; this effect is, however, ignored here.

Decision settings and influence diagrams

With the above elements specified, a decision setting (DS) is defined. A typical compact graphical representation of a DS is the influence diagram (ID) (Jensen *et al.*, 2007). Round nodes represent uncertain outcomes (which are described by the probabilistic model), square nodes are the decisions and lozenge-shaped nodes are the utility. The nodes are connected by directed edges, which represent stochastic, causal and monetary dependence.

The DS is usually determined by operational constraints, as well as the level of complexity of the decision sequence considered. For this study IDs were constructed for three different DSs.

DS #1: surcharge applied at $t=0$. In DS #1, the case is considered where the surcharge is applied only at the time of constructing the embankment – that is, at $t=0$. The only decision variable is the height ΔH_0 of the surcharge. The settlement at time t, S_t, and the achieved OCR if unloaded at time t, OCR_t, are both probabilistic quantities, which depend on the applied surcharge. The overall decision process is summarised by the ID of Fig. 6.

DS #2 and DS #3: surcharge applied at $t=0$ *and adjusted at time* t_1. DS #2 and DS #3 consider that there is an opportunity to add a surcharge of height ΔH_1 at a fixed time t_1, on top of the initial surcharge height ΔH_0. The decision on how much to add is based on a measurement M_{t_1} of the settlement at time t_1 (equation (9)). The overall decision process is summarised by the ID depicted in Fig. 7. In DS #2, the time t_1 is fixed and cannot be influenced by the decision maker, whereas in DS #3, this time can be chosen and optimised.

Table 1. Components of the cost model for the embankment preloading example

Cost component	Description
$C_{\mathrm{sur},i}$	Cost of adding a preloading surcharge of height ΔH_i. Includes material costs, mobilisation costs, material availability at the time of the decision, and additional berms for slope stability
C_{delay}	Cost penalty for project delay – that is, sufficient settlement (s_{target}) has not been reached within a dedicated time period
C_{OCR}	Cost penalty for reduced serviceability of the superstructure, due to residual settlement caused by insufficient overconsolidation at time of unloading

Optimal decision making

The most desirable outcome of the decision process is the one with the lowest cost. Owing to the uncertain nature of the soil parameters, the outcomes of a sequence of decisions are uncertain, hence so is the total cost. The optimal sequence of decisions is therefore that which results in the minimum expected total cost (Raiffa & Schlaifer, 1961). For DS #1, the optimal decision for ΔH_0 is therefore defined as

$$\Delta H_0^* = \arg\ \min\ E[C_{\mathrm{tot}}(\Delta H_0)] \tag{11}$$

where $E[C_{\mathrm{tot}}(\Delta H_0)]$ is the expected value of the total cost evaluated with equation (10), when an initial preloading surcharge of height ΔH_0 is applied. This expected total cost thus accounts for the associated risk $E[C_{\mathrm{delay}}(\Delta H_0)] + E[C_{\mathrm{OCR}}(\Delta H_0)]$ of not achieving the desired settlement or OCR within the available preloading time.

The formulation of the optimisation problem is not as straightforward for DSs that involve one or more opportunities to adjust the surcharge after the initial surcharge is applied – that is, DS #2 and DS #3. In these sequential decision problems, the optimal actions depend on the past observations. Therefore, one must find the optimal function that maps past observations to actions. In general, this type of problem is hard to solve and an exact solution becomes intractable with increasing number of decision or observation steps (Papadimitriou & Tsitsiklis, 1987). Approximate solutions are possible – for example, by way of partially observable Markovian decision processes (POMDPs) or reinforcement learning (Porta *et al.*, 2005; Roy *et al.*, 2005; Silver & Veness, 2010; Mnih *et al.*, 2013; Memarzadeh &

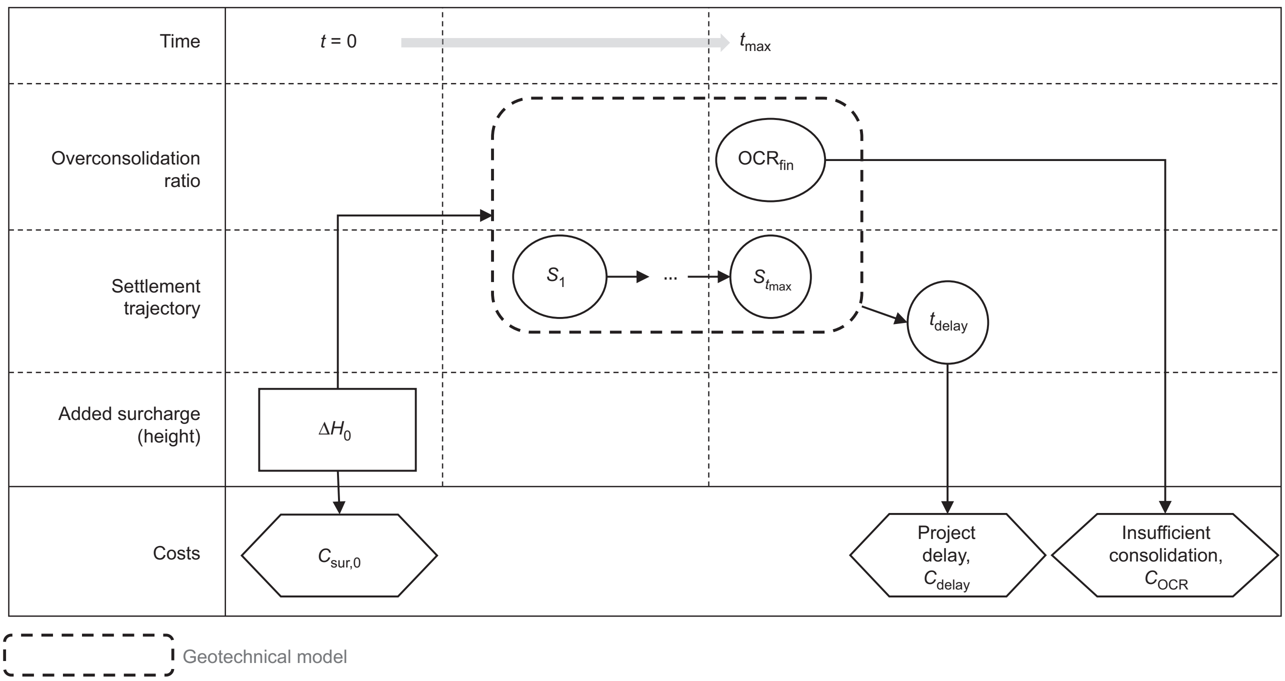

Fig. 6. Influence diagram for DS #1. Optimisation of the initial surcharge. The square node ΔH_0 indicates that first a value of ΔH_0 is chosen, at a cost $C_{\mathrm{sur},0}(\Delta H_0)$. The now fixed ΔH_0 influences the evolution of the settlement S_t and the overconsolidation ratio at unloading $\mathrm{OCR}_{\mathrm{fin}}$ as well as the time t_{target} when the target settlement is reached, defined by $S(t_{\mathrm{target}}) = s_{\mathrm{target}}$. Monetary consequences due to project delay and residual settlement result from these quantities. The interaction between ΔH_0 and the geotechnical model is represented in a simplified manner

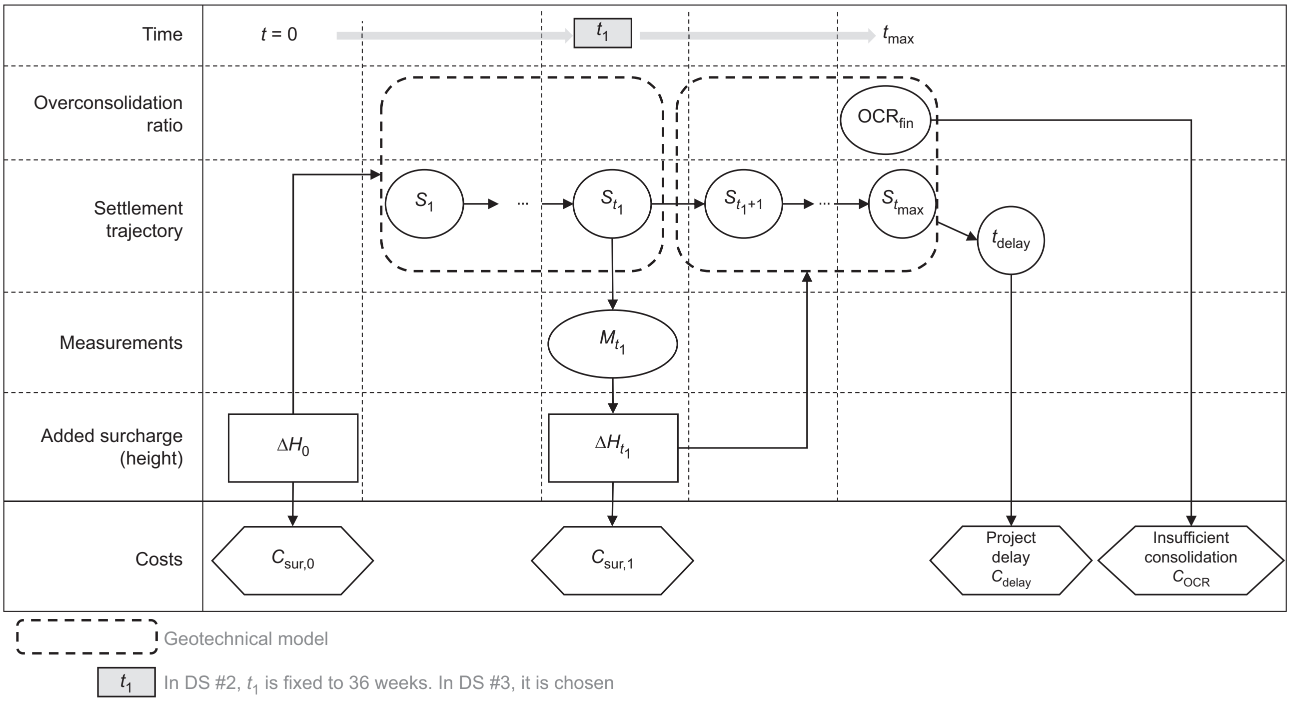

Fig. 7. Influence diagram for DS #2 and DS #3. The interactions between the decisions on the initial and added surcharge heights, ΔH_0, ΔH_1, and the geotechnical model are represented in a simplified manner

Pozzi, 2016; Papakonstantinou *et al.*, 2018; Andriotis & Papakonstantinou, 2019).

To solve the general sequential decision problem, it is convenient to define preloading strategies $\mathcal{S}$, which compactly prescribe the sequence of decisions. A strategy consists of a set of rules that prescribes how much surcharge to add at any time as allowed by the DS. For example, for DS #1, a strategy simply prescribes the surcharge height at time $t = 0$; for DS #2, it prescribes the surcharge height at time $t = 0$ and gives a rule at time t_1, which can be based on settlement measurements, to adjust the surcharge. In DS #3, the strategy additionally prescribes the time $t = t_i$ at which to collect the settlement measurement and adjust the surcharge.

Generalising the notation to any preloading strategy $\mathcal{S}$, the expected total cost associated with a preloading strategy $\mathcal{S}$ is thus evaluated as

$$E[C_{\text{tot}}(\mathcal{S})] = E\left[C_{\text{sur}}(\mathcal{S})\right] + E[C_{\text{delay}}(\mathcal{S})] + E[C_{\text{OCR}}(\mathcal{S})] \tag{12}$$

The optimal preloading problem is equivalent to finding the preloading strategy that minimises the expected total cost

$$\mathcal{S}^* = \arg\min_{\mathcal{S}} E[C_{\text{tot}}(\mathcal{S})] \tag{13}$$

In general, $E[C_{\text{tot}}(\mathcal{S})]$ cannot be evaluated analytically. An MC approximation can instead be obtained using the assumed probabilistic geotechnical model. The latter enables the generation of n_{MC} random settlement trajectories, $S_t^{(k)}$, and OCR at unloading $\text{OCR}_{\text{fin}}^{(k)}$, obtained from surcharge sequences $\Delta H_0^{(k)}$, $\Delta H_1^{(k)}$ and so on, with $1 \leq k \leq n_{\text{MC}}$. A total cost can be computed for each of these trajectories as per equations (10), (16), (18) and (19). The MC approximation of the expected total cost of a preloading strategy $\mathcal{S}$ is therefore

$$E[C_{\text{tot}}(\mathcal{S})] \simeq \frac{1}{n_{\text{MC}}} \sum_{k=1}^{n_{\text{MC}}} C_{\text{tot}}\left(S_t^{(k)}, \text{OCR}_{\text{fin}}^{(k)}\right) \tag{14}$$

The estimate improves with the number of samples n_{MC}.

HEURISTICS FOR OPTIMAL PRELOADING STRATEGIES

The problem of finding the best strategy is equivalent to finding the best sequence of decisions and an exact solution to equation (13) is still intractable in general. To address this challenge, the space of possible strategies that are considered in the optimisation is reduced, following Bismut & Straub (2022). The proposed methodology considers only strategies that can be described by a specific set of rules, so-called heuristics. A heuristic is typically formulated with simple statements (the rules), in which a number of parameters $\mathbf{w} = [w_1; w_2; \ldots; w_n]$ intervene. For example, the following heuristic is defined for DS #2:

- the initial surcharge ΔH_0 is h_0
- the additional surcharge ΔH_1 at time $t_1 = 36$ weeks is h_1 if the measured settlement at this time is lower than a threshold s_{th}.

The parameters $\mathbf{w}$ for this heuristic are h_0, h_1 and s_{th}. In this DS, t_1 is fixed to 36 weeks. An arbitrarily chosen preloading strategy following this heuristic format with parameters $h_0 = 0{\cdot}94$ m, $h_1 = 1{\cdot}04$ m and $s_{\text{th}} = 0{\cdot}77$ m will react to different trajectories, as shown in Fig. 8. The total cost incurred will depend on (*a*) the strategy and (*b*) the settlement occurring. The expected cost of a strategy with fixed parameters can be estimated with equation (14).

For a given heuristic, there is a set of parameter values that optimises the expected cost. The associated strategy is called the optimal heuristic strategy. Thus, for a given heuristic and associated parameters $\mathbf{w} = [w_1; w_2; \ldots; w_n]$, the preloading problem is reduced to finding

$$\mathbf{w}^* = \arg\min\ E[C_{\text{tot}}(\mathcal{S}(\mathbf{w}))] \tag{15}$$

As the heuristic formulation of the optimisation problem operates in a restricted strategy space, it yields a sub-optimal preloading strategy. However, the heuristic parametrisation enables the inclusion of operational constraints (e.g. surcharge can only be added at certain prescribed times) and provides

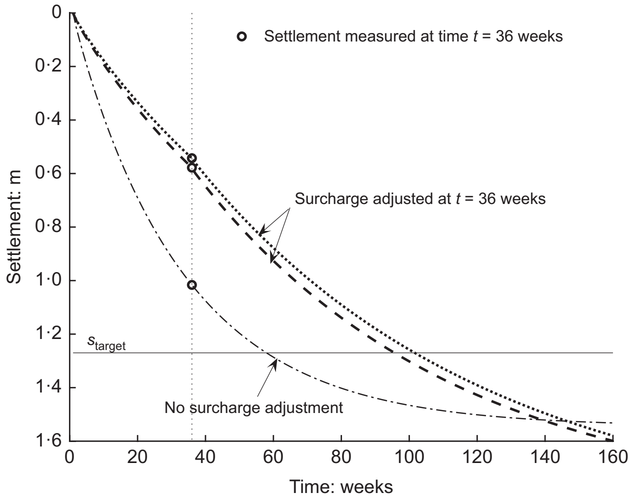

Fig. 8. Three sample trajectories for a strategy parameterised with heuristic 2A (DS #2), with $h_0 = 0{\cdot}95$ m, $h_1 = 1{\cdot}04$ m and $s_{th} = 0{\cdot}77$ m. The time at which the curves intersect with the level s_{target} corresponds to t_{target}. For $t_{max} = 72$ (weeks), it can be seen that only one of these trajectories satisfies $t_{target} < t_{max}$ and does not lead to project delay

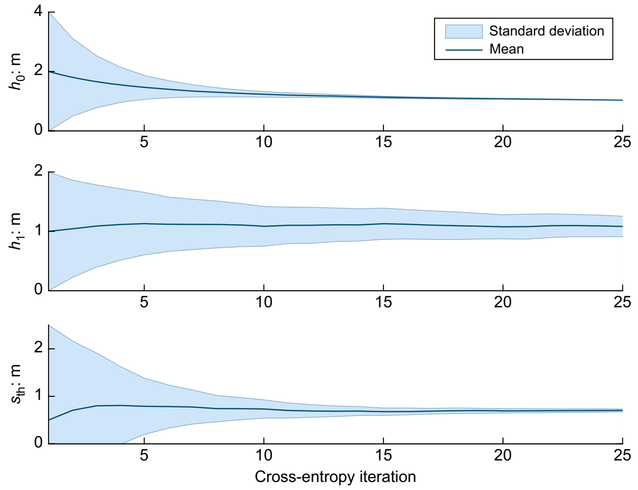

Fig. 9. Convergence of heuristic parameters in the CE optimisation for heuristic 2A defined in DS #2

easily interpretable strategies. Furthermore, the definition of preloading strategies with heuristics makes sense from the point of view of geotechnical engineering practice, as most preloading strategies would indeed be defined with such simple rules. In addition, several heuristics can be compared and the better-performing strategy selected. In the numerical investigations, the impact of different heuristic choices is discussed, in particular the impact of increasing the number of heuristic parameters.

The optimal parameter values $\mathbf{w}^*$ are the solution of a noisy optimisation problem where the objective function is expressed as an expected value (Rubinstein & Kroese, 2004), for which no analytical expression exists. An efficient approach is a sampling-based optimisation, which was previously developed for this purpose in Bismut & Straub (2021) and is based on the cross-entropy (CE) method (Rubinstein & Kroese, 2004). The basic steps are summarised in the Appendix and the convergence to the optimal parameter values is illustrated in Fig. 9. The current authors have previously demonstrated this method on other sequential decision planning problems and discussed details of its implementation and performance in Bismut & Straub (2021) and Bismut *et al.* (2022). The method stands out for the simplicity of its implementation and robustness. However, it can be replaced by any other method suitable for noisy optimisation.

NUMERICAL INVESTIGATIONS

Probabilistic model set-up

As stated above, the probabilistic geotechnical model is described in detail in Spross & Larsson (2021). The settlement target is computed for $p_{FT} = 0{\cdot}05$, and is obtained as $s_{target} = 1{\cdot}27$ (m) (Fig. 5).

Cost model

Refer to the cost components in Table 1. $C_{sur,i}$ corresponds to the cost of adding surcharge of height ΔH_i. It increases with the total surcharge height, and accounts for the cost of berms needed to ensure slope stability (see Fig. 1). It is evaluated from the cost of total surcharge height H_{tot}

$$C_{sur}(H_{tot}) = \begin{cases} H_{tot} c_{sur} & \text{if } H_{tot} \leq 1\text{ m} \\ 1.25 H_{tot} c_{sur} & \text{otherwise} \end{cases} \quad (16)$$

where 1·25 is a cost factor addressing the cost increase related to the construction of berms for embankments higher than 1 m. The cost attributed to each increase ΔH_i of surcharge on top of existing surcharge H_{tot} is computed as

$$C_{sur,i}(\Delta H_i) = [C_{sur}(H_{tot} + \Delta H_i) - C_{sur}(H_{tot})] \times f_{add,i} \quad (17)$$

where the factor $f_{add,i} \geq 1$ accounts for additional costs incurred by increasing the surcharge at a later time $t_i > 0$. Note that the cost of the remaining embankment material after unloading is not included here, as it is the same for all scenarios.

In the model, project delay occurs when the settlement trajectory either does not meet s_{target} within the preloading time allowed by the construction contract, t_{max}, ($t_{target} > t_{max}$) or is unable to meet s_{target} at all ($t_{target} > t_{lim}$) (see Fig. 8). The associated penalty is

$$C_{delay}\left(t_{target}\right) = \begin{cases} 0 & \text{if } t_{target} \leq t_{max} \\ c_{delay} \times \left[\min(t_{lim}, t_{target}) - t_{max}\right] & \text{otherwise} \end{cases} \quad (18)$$

where c_{delay} represents the penalty per week of delay.

Finally, the penalty associated with residual settlement due to insufficient OCR is evaluated with the logistic function

$$C_{OCR}(\mathrm{OCR}_{fin}) = \frac{c_{OCR}}{1 + \exp\left[-(1{\cdot}075 - \mathrm{OCR}_{fin})/(4{\cdot}5 \times 10^{-3})\right]} \quad (19)$$

where OCR_{fin} is the OCR at unloading at time t_{target} or t_{lim} if the settlement target has not been achieved in time. This smoothed step function approaches the maximum penalty c_{OCR} when $\mathrm{OCR}_{fin} < 1{\cdot}05$, and 0 when $\mathrm{OCR}_{fin} > \mathrm{OCR}_{target} = 1{\cdot}1$ – that is, when the OCR requirement is satisfied.

Table 2. Parameters of the cost model

Cost factor	Value
c_{sur}	$3{\cdot}45 \times 10^6$ SEK/m
c_{delay}	3×10^5 SEK/week
c_{OCR}	2×10^7 SEK
$f_{add,0}$	1
$f_{add,1}$	1
t_{max}	72 weeks

Note: SEK, Swedish krona (1 SEK = £0.075)

The cost factors c_{sur}, c_{delay} and c_{OCR} and the available preloading time t_{max} for the initial numerical investigation are given in Table 2.

Heuristic parametrisations

The following heuristics are investigated for the different DSs. The heuristic parameters for each defined heuristic are indicated in bold.

DS #1. As explained above, the optimisation for this setting only consists in optimising the initial surcharge height ΔH_0. Thus the corresponding heuristic, with single heuristic parameter h_0, is simply as follows.

Heuristic 1: $h_0 \geq 0$

(1) $\Delta H_0 = h_0$.

DS #2. For DS #2, the performance of two different heuristics in approximating the optimal preloading strategy is investigated. A preloading strategy described with heuristic 2A specifies the initial surcharge height and adjusts it by adding a surcharge height if the measured settlement is lower than a threshold.

Heuristic 2A: $h_0 \geq 0$, $h_1 \geq 0$, $s_{th} \geq 0$

(1) At time $t = 0$, add surcharge of height $\Delta H_0 = h_0$.
(2) Obtain measurement m_{t_1} at time $t_1 = 36$ (weeks).
(3) If $m_{t_1} < s_{th}$, add surcharge $\Delta H_1 = h_1$. Otherwise $\Delta H_1 = 0$.

With heuristic 2B, the strategy adjusts the height of the added surcharge based on the difference d between the measured settlement and the threshold. This height adjustment is defined by a sigmoid function varying between 0 and maximum added height h_1, characterised by a curve steepness a. When $a = 0$, this sigmoid function is a step function.

Heuristic 2B: $h_0 \geq 0$, $h_1 \geq 0$, $s_{th} \geq 0$, $a \leq 0$

(1) At time $t = 0$, add surcharge of height $\Delta H_0 = h_0$.
(2) Obtain measurement m_{t_1} at time $t_1 = 36$ weeks.
(3) Compute $d = m_{t_1} - s_{th}$.
(4) Add surcharge

$$\Delta H_1 = \begin{cases} 0, & d \leq a \\ 2h_1\left(\dfrac{d-a}{2a}\right)^2, & a \leq d \leq 0 \\ \left[1 - 2\left(\dfrac{d-a}{2a}\right)^2\right]h_1, & 0 \leq d \leq -a \\ h_1, & d \geq -a \end{cases}$$

DS #3. Heuristic 3 is the same as heuristic 2B, with the additional freedom to choose the time t_1 at which the settlement is measured and the surcharge height is adjusted. The t_1 is thus an additional heuristic parameter.

Heuristic 3: $h_0 \geq 0$, $h_1 \geq 0$, $s_{th} \geq 0$, $a \leq 0$, $t_1 \in \{1, 2, 3, \ldots, t_{max}\}$

(1) At time $t = 0$, add surcharge of height $\Delta H_0 = h_0$.
(2) Obtain measurement m_{t_1} at time t_1.
(3) Compute $d = m_{t_1} - s_{th}$.
(4) Add surcharge

$$\Delta H_1 = \begin{cases} 0, & d \leq a \\ 2h_1\left(\dfrac{d-a}{2a}\right)^2, & a \leq d \leq 0 \\ \left[1 - 2\left(\dfrac{d-a}{2a}\right)^2\right]h_1, & 0 \leq d \leq -a \\ h_1, & d \geq -a \end{cases}$$

Computational set-up

For the CE method, the values $n_{CE} = 100$, $n_E = 30$ and $n_{MC} = 10$ are fixed. On an eight-core CPU 3·2 GHz machine, optimising the heuristic parameters for a given heuristic takes around 4 min. The expected cost of the resulting optimised strategy is evaluated with $n_{MC} = 10^4$ samples.

RESULTS

The CE method is applied to obtain the optimal parameter values and associated expected costs for the different DSs and heuristics defined above, assuming the cost model of Table 2. Fig. 10 illustrates the optimisation of the heuristic parameters for DS #1. The results for all DSs are summarised in Table 3.

The expected costs of the optimal heuristic strategies obtained for each of the DSs decrease from DS #1 to DS #3. This is in agreement with the fact that DS #1 is more restrictive in terms of available actions than DS #2, and in turn DS #2 is more restrictive (because the adjustment time is fixed) than DS #3. Table 3 also reports the estimated standard deviation of the total cost. For the investigated heuristics, the coefficient of variation of the total cost for the optimal strategy varies around 95%. The standard error of the MC estimates of the expected costs is therefore 1%, which ensures a sufficient accuracy to rank the heuristics according to the estimated expected cost of their optimal strategies.

The optimal initial surcharge prescribed by heuristic 1 in DS #1 is higher than the initial surcharge prescribed in DS #2 and DS #3. This shows that the heuristics chosen for DS #2 and DS #3 exploit the fact that measurement information enables an optimised adjustment of surcharge.

For DS #2, it is noted that heuristic 2B performs better than heuristic 2A in terms of expected cost; hence the smoothed step function for the selection of the adjusted load is a better heuristic than the simple step function.

Figure 11 depicts the breakdown of the costs for each optimal heuristic strategy. It is observed that heuristic 3 yields a lower risk of delay than heuristics 2A and 2B and a lower expected total cost, even though it applies on average a higher total surcharge. Therefore, the choice of time t_1 to adjust the surcharge plays a significant role in efficiently controlling the settlement. The expected penalty associated with insufficient OCR is here negligible in comparison with the other cost components, for all heuristics.

Figure 12 illustrates the effect of adjusting the surcharge at time $t_1 = 36$ on the settlement trajectory, following the

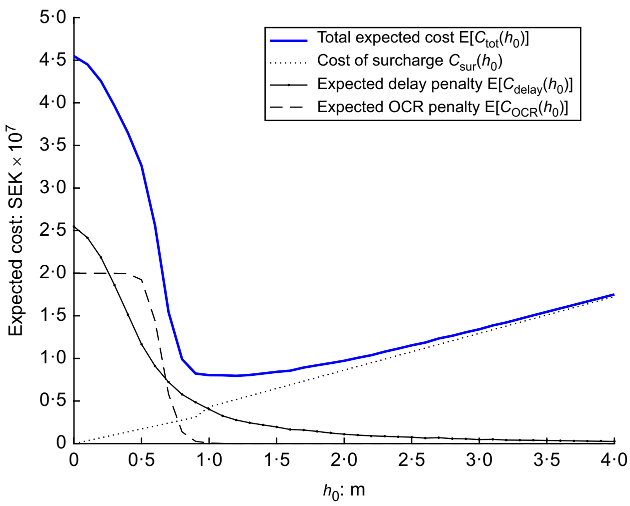

Fig. 10. Expected costs for DS #1 as a function of $\Delta H_0 = h_0$

Table 3. Optimal heuristic parameters and associated expected costs

Parameter	Unit	DS #1	DS #2		DS #3
		Heuristic 1	Heuristic 2A	Heuristic 2B	Heuristic 3
h_0	m	1·05	0·98	0·96	0·95
h_1	m	—	1·06	1·08	1·81
s_{th}	m	—	0·71	0·73	0·37
a	m	—	—	– 0·15	– 0·28
t_1	weeks	—	36*	36*	20
Expected cost	10^6 SEK	8·11	6·54	6·29	6·06
Std dev. cost	10^6 SEK	7·4	6·3	6·0	5·6

*Value is not optimised but fixed.

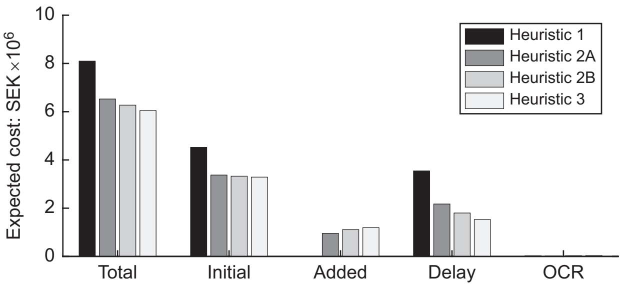

Fig. 11. Breakdown of the expected cost of the optimal strategies for the different DSs and heuristics

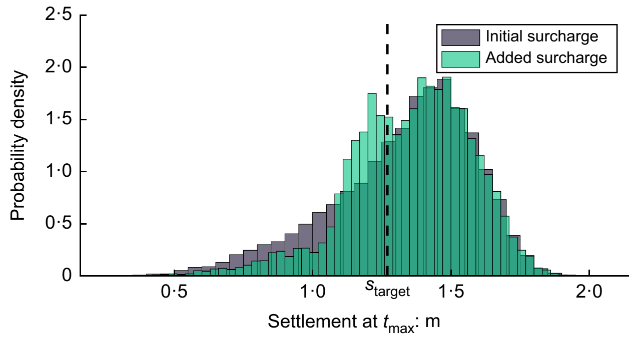

Fig. 12. Distribution of settlement at t_{max} for the optimal strategy for DS #2, heuristic 2A, obtained from 10^4 sample settlement trajectories. The grey histogram represents the distribution of the settlement if only the initial surcharge of height $h_0 = 0{\cdot}95$ m is applied. The bimodal histogram shows the distribution of the settlement obtained by adjusting the surcharge at $t = 36$ weeks, as prescribed by the strategy (see Table 3). A full-colour version of this figure can be found on the ICE Virtual Library (www.icevirtuallibrary.com)

optimal strategy for heuristic 2A. The distribution of the settlement at time t_{max} is obtained from 10^4 sample trajectories for both the case where only the initial surcharge is applied and not adjusted at $t = 36$ weeks and the case where the surcharge is adjusted according to the optimal strategy. With the load adjustment action, the settlement trajectories that already reach the target at t_{max} with the sole initial load are unaffected, while a portion of trajectories which would not have achieved s_{target} at t_{max} are now compliant – that is, the probability Pr $(S_{t_{max}} < S_{target})$ decreases by enabling the adjustment of the surcharge. Most of the corrected trajectories will nevertheless incur a delay penalty, which is optimal under the assumed cost model of Table 2.

The effect of the different heuristics on the final settlement at time t_{max} and on the OCR at unloading is depicted in Figs 13(a) and 13(b). Heuristics 2A, 2B and 3 can be distinguished from heuristic 1, where the preloading is only added at $t = 0$. The uncertainty in the settlement reduces when the surcharge is adjusted based on the measured settlement, and the probability that $S_{t_{max}}$ is larger than s_{target} increases from heuristic 1 to heuristic 3. Notably, the optimal strategies for heuristics 2A, 2B and 3 result in a larger probability that the OCR at unloading is smaller than the critical value 1·1, compared to heuristic 1. Hence these heuristics can balance both penalties associated with insufficient settlement and OCR against the applied surcharge in a more efficient manner.

DISCUSSION

To demonstrate the potential of quantitatively analysing and optimising geotechnical design under sequential information, the design of preloading for an embankment on soft soil is considered. The preloading problem is formulated as a sequential decision problem in different DSs. Preloading strategies are described through heuristics with associated parameters, which are optimised to minimise the total expected cost. Different heuristics are considered and it is observed that – as expected – the more flexibility in decision the heuristic provides, the more cost efficient the resulting optimal heuristic strategy is. For the case study investigated, the authors observe a reduction in the expected cost in the order of 25% between heuristics 1 and 3.

It is noted that – with all investigated heuristics – the coefficient of variation of the total cost is large, around 100%. While this variability depends on the assumed cost model, if the decision maker wanted to prioritise strategies that reduce this variability, a risk-averse behaviour could be included in the objective function of equation (13) by considering a utility function that is non-linear with costs (Straub & Welpe, 2014).

Other heuristics than those proposed can be investigated and might result in lower expected costs. For example, one might replace the sigmoid function of heuristic 2B by another function. As settlement measurements are typically available at weekly intervals, a heuristic could be formulated such that the adjusted surcharge at time t_1 depends on an observed trend. In this case, the processing of the measurements for the purpose of decision making, hence the trend prediction model, is part of the definition of the heuristic. Ultimately, one could define a heuristic to address the setting in which continuous settlement measurements are available, with near-real-time decision support.

The advantage of the heuristic methodology for the planning of preloading decisions is that the resulting strategies are interpretable, because the decision rules are explicitly defined through the chosen heuristic. This also entails that the heuristic can encode geotechnical expertise. The flexibility in the formulation of the DS through the IDs and the cost functions also enables the analyst to integrate additional

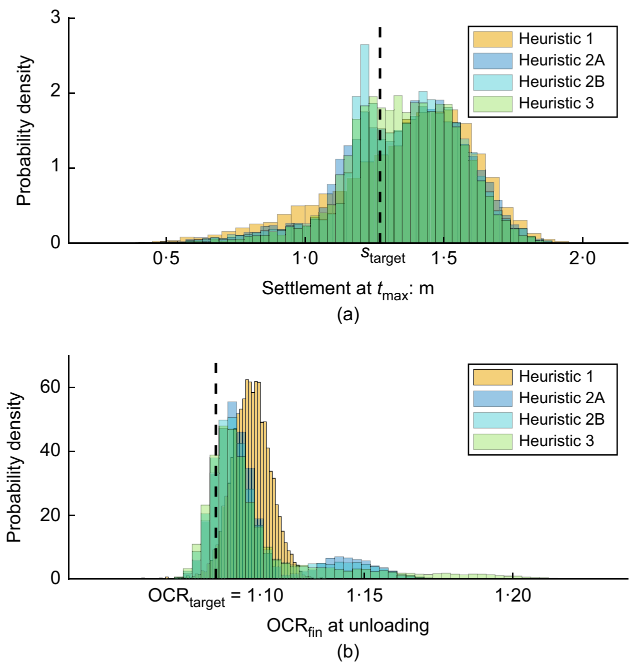

Fig. 13. Distribution of (a) settlement achieved at t_{max} and (b) of the OCR at unloading for the optimal heuristic strategies (see Table 3). The area of the histograms to the left of the dotted line represents for each optimal heuristic strategy, in (a) the probability Pr $(S_{t_{max}} < s_{target})$, and in (b) the probability $\Pr(OCR_{fin} < OCR_{target})$. A full-colour version of this figure can be found on the ICE Virtual Library (www.icevirtuallibrary.com)

constraints. For instance, the uncertainty in the availability of preloading material could be explicitly modelled, such that there is a certain probability of obtaining the requested material at a given point in time.

The decision-theoretical framework described in this paper is suitable to apply in combination with the observational method, which was first defined as a design approach by Peck (1969) and today is accepted into design codes such as Eurocode 7, EN 1997-1:2004 (CEN, 2004). The observational method implies that the geotechnical engineer establishes a monitoring plan with thresholds that trigger prepared design changes specified in an action plan, thereby adjusting the initial design to fit better to the actual ground conditions.

In the context of a sequential decision problem, such thresholds and design changes can be formulated as heuristics, allowing the geotechnical engineer not only to compare conceptually different options of monitoring and action plans, but also to optimise their included threshold values and specified actions. The evaluated DSs in this paper illustrate this clearly: the heuristics 2A, 2B and 3 can be seen as three different options of monitoring and action plans, while Table 3 specifies the optimised heuristics for the plans and also shows their respective expected costs. Such risk-based optimisation of monitoring and action plans is a considerable leap forward from the current practice, where monitoring and action plans usually are defined based on deterministic analyses, although probabilistic approaches are emerging (e.g. Spross & Gasch, 2019).

CONCLUSION

The authors have formalised a geotechnical problem as a sequential decision problem and proposed a methodology based on heuristics to finding optimal strategies. This framework was applied to an embankment preloading problem and highlighted how the DS, chosen heuristics and cost model affect the optimal preloading strategies. This enables a quantitative optimisation of preloading decisions under uncertainty. It was shown that the potential for cost savings is significant. This framework is not limited to embankment design and construction, but is designed as a decision support tool to be extended to a vast range of geotechnical engineering applications, especially those to which the observational method is applied.

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APPENDIX. CROSS-ENTROPY OPTIMISATION ALGORITHM

Algorithm 1 describes the steps of the CE method used for the optimisation of the heuristic parameters. The algorithm also applies a smoothing operation, which is not described here, to prevent convergence to local minima (refer to Kroese *et al.* (2006) for more details). The optimal cost is obtained with equation (14) evaluated in $S(\mathbf{w}^*)$.

The sampling density is here chosen as a truncated normal for positive (or negative) parameters. For integer parameters, the sampled value is rounded to the nearest integer. The updated distribution parameters $\boldsymbol{\lambda}^*$ of the multivariate truncated normal distribution are the mean and covariance of the elite samples.

Algorithm 1. Cross-entropy method applied to noisy optimisation

Input: cross-entropy sampling density $P(\cdot|\boldsymbol{\lambda}^*)$; initial sampling distribution parameter $\boldsymbol{\lambda}^*$; number of CE samples per iteration n_{CE}; number of elite samples n_E; number of sample settlement trajectories n_{MC}; maximum number of iterations n_{max}.

$l \leftarrow 0$;
while $l < n_{max}$ **do**
  for $m \leftarrow 1$ to n_{CE} **do**
    generate random heuristic parameter values $\mathbf{w}^{(m)}$ from sampling density $P(\cdot|\boldsymbol{\lambda}^*)$;
    generate n_{MC} settlement trajectories and measurement following strategy $S\left(\mathbf{w}^{(m)}\right)$;
    evaluate the expected total life-cycle cost q_m with n_{MC} samples (equation (14));
  end
  sort $(\mathbf{w}^{(1)}, \ldots, \mathbf{w}^{(n_{CE})})$ in increasing order of q_m;
  fit the distribution parameter $\boldsymbol{\lambda}^*$ to the n_E elite samples;
  $l \leftarrow l+1$;
end
$\mathbf{w}^* \leftarrow$ mean of $P(\cdot|\boldsymbol{\lambda}^*)$;
return $\mathbf{w}^*$

NOTATION

a	heuristic parameter
C_{delay}	cost penalty for project delay
C_{OCR}	cost penalty for reduced serviceability of the superstructure
$C_{sur,i}$	cost of adding a preloading surcharge of height ΔH_i
C_{tot}	total cost
c_{delay}	cost factor for C_{delay}
c_{OCR}	cost factor for C_{OCR}
c_{sur}	cost factor for C_{sur}
E	expectation operator
$f_{add,i}$	penalty factor for adding surcharge at later time t_i

H_{tot}	total added surcharge height
h_0	heuristic parameter for surcharge height
h_1	heuristic parameter for surcharge height
$h_{\mathrm{cl},i}$	thickness of *i*th clay layer
l	number of layers of clay stratum
M_t	measured settlement at time t
n_{CE}	number of cross-entropy samples per iteration
n_{E}	number of elite samples
$n_{\max}$	maximum number of cross-entropy iterations
n_{MC}	number of sample settlement trajectories
$\mathrm{OCR}_{\mathrm{fin}}$	overconsolidation ratio at unloading
OCR_t	overconsolidation ratio at time t
$\mathrm{OCR}_{\mathrm{target}}$	target overconsolidation ratio
$P(\cdot)$	cross-entropy sampling density
p_{FT}	acceptable target failure probability
S	settlement
S	preloading strategy
S_t	settlement at time t
$S_{t_{\max}}$	settlement at time $t_{\max}$
S_∞	long-term primary consolidation settlement
s_{target}	target settlement
s_{th}	heuristic parameter for settlement threshold
t	time
t_1	heuristic parameter for time of added surcharge
t_{add}	time of addition of surcharge
t_{lim}	maximum possible preloading time
$t_{\max}$	allowed preloading time in contract
t_{shift}	time at which a hypothetical zero degree of consolidation occurs
t_{target}	time at which the settlement reaches s_{target}
U	average degree of consolidation
U_{h}	average degree of horizontal consolidation
U_{v}	average degree of vertical consolidation
$\mathbf{w}$	vector of heuristic parameters w_j
$\mathbf{w}^*$	optimal heuristic parameters
ΔH	height of added surcharge
ΔH_i	height of surcharge added at time t_i
ΔU	difference in degree of consolidation with additional surcharge
$\Delta\varepsilon_i$	strain increase
$\Delta\sigma$	load – that is, stress increase in soil
$\Delta\sigma_{\mathrm{add}}$	stress increase in soil caused by surcharge added at t_{add}
$\Delta\sigma_{\mathrm{emb}}$	remaining stress increase in soil after unloading
$\Delta\sigma_{\mathrm{sur}}$	vertical stress increase caused by preloading, including the surcharge
ε	measurement error
$\boldsymbol{\lambda}$	initial cross-entropy sampling distribution parameter
σ_0'	initial vertical stress

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