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A relief ordering policy for declining demand and realized shortage cost

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Abstract

Purpose – The purpose of this paper is to propose a model for relief inventory ordering considering declining demand and realized shortage (RS) costs. This study is motivated by the overflow of unsolicited items in affected areas, which hinder the urgent relief flow.

Design/methodology/approach – This study proposes a dynamic programing model reflecting the change of demand and urgency during relief operations. A solution algorithm for finding a replenishing point of relief is also introduced here.

Findings – Relief ordering policy based on the RS cost increases the service level (in terms of time). The service level is dependent on the deterioration rate.

Research limitations/implications – Deterioration is not applicable for all relief items. Therefore, the proposed model is applicable for certain relief items. Additionally, computation of temporal urgency is not included.

Practical implications – The study outcomes can be invaluable for resource planning. The model shows a gradual decrease of ordering quantity and a gradual increase of cycle length. This observation can be utilized for resource (e.g. truck) planning and fewer truck can be used in later stages of relief operations.

Originality/value – This study introduces temporal urgency for disaster logistics (DL) and proposes a relief inventory policy by incorporating temporal urgency. This model highlights the importance of considering declining demand for DL.

Keywords Inventory, Deterioration, Disaster logistics, Urgency

Paper type Research paper

1. Introduction

Large-scale disasters such as the 2004 Indian Ocean tsunami, the 2010 Haiti earthquake, and the 2011 Great East Japan Earthquake (GEJE) underscore some challenges and the importance of planning in response to extreme hazards. Because of the poor performance of relief distribution, disaster logistics (DL) has become a central part of improvement efforts. Academics have attempted to propose mathematical models to improve DL performance (some indicators of DL performance might be found in Beamon and Balcik, 2008). According to Holguín-Veras *et al.* (2013), DL mathematical models must be improved by incorporating post-disaster practical scenarios. For example, DL mathematical models examine constant or uniform distributed demand throughout the relief operation period (Beamon and Kotleba, 2006; Das and Hanaoka, 2014b). However, the demand for relief items declines gradually. The relief operation terminates when demand reaches zero. If constant demand is assumed for relief inventory modeling, the logistical cost will be greater than the actual cost. Therefore, the changing of demand should be incorporated in relief modeling and might appear in any functional form, including linear and exponential.

Again, demand and urgency should be considered separately in DL: it makes a difference in DL and commercial logistics (CL). "Urgency" represents the degree of

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effectiveness of a relief item at the time of delivery. If a relief item with lower urgency at a particular time is brought to an affected area, bringing relief items with higher urgency to affected areas may not be possible given the capacity limitations of resources (i.e. fleet, warehouse, or human power). Moreover, the urgency for a particular relief item is not constant over time. The value of urgency depends on the relief item and timing (Section 2 explains the properties of urgency). Existing DL are modeled as similar to CL, thereby engendering flaws. The existing mathematical model cannot represent a real situation of disaster and provides greater logistical costs than the actual cost. We have attempted to overcome the flaws by incorporating declining demand and realized shortage (RS) costs.

Existing mathematical models do not incorporate the previously described issues. Altay and Green (2006) provided a holistic review of the operational research/management science models for disaster operations management until 2004. Subsequently, Galindo and Batta (2013) extensively reviewed progress on disaster operations management after 2004. Both resources show the importance of mathematical models. Fiedrich *et al.* (2000) proposed a dynamic combinatorial optimization model to find the optimal resource rescue schedule with the goal of minimizing the total number of fatalities during a search and rescue period. Özdamar and Demir (2012) proposed a vehicle routing model that is intended to minimize travel time, incorporating the notion of a hierarchical cluster. A few studies have assessed inventory planning models. Adivar and Mert (2010) introduced a fuzzy linear programming for relief collection from international communities after a disaster that minimizes logistics cost while maximizing credibility. Beamon and Kotleba (2006) proposed a relief inventory management model for stochastic demand that is intended to reduce the inventory cost of long-term support of victims in Sudan. Das and Hanaoka (2014b) extended the relief inventory model to consider stochastic demand and lead time for a large-scale disaster.

A salient concern of inventory management is deciding on when and how much to order to minimize the total cost. This paper aims to propose a dynamic programming model for a relief item using the following assumptions: a constant deterioration rate, declining demand, allowing backlogging with penalty, and RS cost. The solutions to the model determine an optimal replenishing schedule during a finite planning horizon to ensure a minimum total cost associated with the inventory system. The proposed model exhibits variations in both the replenishment cycle length and the ordered quantities.

The remainder of the paper is structured as follows. In Section 2, we present the definition of urgency. This section also presents the mathematical function of urgency. Section 3 presents mathematical notations and assumptions of the proposed model. Section 4 includes several sub-sections that explain the proposed model and a solution algorithm. Section 5 reports the results from numerical analyses using the proposed model. Finally, Section 6 presents the conclusions of this study and a summary of the study outcomes.

2. Definition of urgency and RS cost

Sheu (2007) coined the term "urgency" to represent the effectiveness of relief items in different areas. An application of urgency is presented by Das and Hanaoka (2014a). Urgency shows differences between DL and CL. For CL, a supplier receives revenue after delivering products. Each unit of the delivered product incurs the same revenue for the supplier. However, the DL supplier does not deliver relief items to receive monetary revenue. Instead, the supplier aims to deliver items that are more effective, whereas the CL supplier aims for higher revenue.

Urgency is characterized in the literature using several terms, including hierarchy (Özdamar and Demir, 2012) and priority (Holguín-Veras *et al.*, 2013). Sheu (2007) computed urgency in the spatial dimension. However, urgency may also be represented in the temporal dimension. The urgency of a particular relief item may change during the relief operation period. Therefore, urgency in the temporal dimension is necessary for designing inventory planning. Some relief items (e.g. medicine) follow increasing trend of urgency in an initial period which starts to decrease after reaching a peak. Again, products such as water are associated with constant urgency. In this study, we specifically examine the relief item that shows a continuous decrease of urgency.

A field survey conducted by Holguín-Veras *et al.* (2014) after the 2011 GEJE revealed that, "The trucking companies interviewed indicated that between 50% and 70% of the cargo that handled was not needed at all, and should not have been sent there." Holguín-Veras *et al.* (2014) continued by stating that, "The case of blankets (and water) is illustrative because during the first week after the tsunami disaster were high-priority supplies. However, as the weather warmed up, blankets ceased to be high priority and because a nuisance as hundreds of thousands blankets from dozens of prefectures in Japan and countries like Canada (that sent 25,000 blankets) descended on Tohoku." This example shows the high urgency of blankets in the aftermath of the earthquake and the lower urgency after a certain period of the earthquake.

If relief items cannot be delivered to victims on time, the system incurs a shortage cost, generated from victims' suffering. This cost is equivalent to a deprivation cost (Holguín-Veras *et al.*, 2013). Yushimito *et al.* (2010) used the idea of deprivation cost to locate a finite number of distribution centers to provide rapid response for disaster relief.

Generally, the previous DL models computed shortage costs without considering urgency. We introduce "RS cost" which changes with urgency.

Letting $f(\gamma,\mu)$ represent the urgency with parameters γ and μ, we assume that the functional form of urgency is the following:

$$f(\gamma,\mu) = 1+\gamma e^{-\mu t} \tag{1}$$

Equation (1) shows that urgency declines gradually. Here, we assume that relief urgency is higher in the initial stage and that it declines gradually. If γ is equal to zero, then the value of urgency becomes one.

3. Notations and assumptions

The objective of the proposed model is to ascertain the optimal sequence of replenishment point p along a finite planning horizon H that minimizes total system costs. To facilitate further discussion, the following notations are defined and are used throughout the paper.

Input

A_0	Fixed ordering cost for each order ($)
C_0	Operational cost of relief item ($/unit day)
H_0	Holding cost of relief item ($/unit day)
P_0	Shortage cost for a given urgent level because of relief item shortage ($/unit day)
$D(t)$	Demand is a function of two parameters a_0 and a_1 (unit/day)
γ,μ	Urgency parameter (no unit and positive value)
θ	Deterioration rate (day^{-1})

α,β Conversion factors of k and p, respectively
H Planning horizon (day)
t Independent variable (day)

Intermediate output
HC Holding cost for a cycle ($)
OC Operational cost for a cycle ($)
RS Realized shortage cost for a cycle ($)
$S(t)$ Shortage level at time t (unit)
SC Shortage cost for a cycle ($)
$W(j,p,k)$ Total cycle cost in replenishment cycle $[j, k]$ with replenishment scheduled at time p ($)

Final output
j Start time of each cycle (day)
p Arrival time of relief item in each cycle, $p = j + \beta$ (day)
k End time of each cycle, $k = j + \alpha$ (day)
$I(t)$ On-hand inventory at time t over the replenishment cycle $[0,T]$ (unit)
$N(t)$ Number of deteriorated item at time t (unit)
$Q(p)$ Ordered quantity at time p (unit)
TC_k Optimal cost of the cumulated total cost from time zero to time k ($)

The inventory model for the relief replenishing problem is based on the following assumptions:

(1) The demand rate decreases linearly during the planning horizon of length H:

$$D(t) = a_0 - a_1 t, \ 0 \leqslant t \leqslant H \tag{2}$$

therein a_0 and a_1 are constant, $a_0 \geqslant 0$, $a_1 \leqslant a_0/H$ to ensure that demand is nonnegative, and H is a positive integer. Here a single relief item is considered and is measured by "unit."

(2) Relief item shortages are backordered completely, incurring a shortage cost (or penalty cost).

(3) Inventory is reviewed continuously. Therefore, a decision maker has up-to-date knowledge related to the inventory level. Additionally, it is assumed that replenishment is instantaneous. Therefore a decision maker will receive relief items of an expected quantity. No time lag exists between relief ordering and relief arrival. It means that the lead time is zero. Therefore, the model emphasizes the situation after the arrival of relief.

(4) The total cost associated with the inventory system consists of holding, shortage, setup, and operational costs. The cost coefficients are constant.

(5) Relief items also deteriorate because of improper facilities. In practice, deterioration is applicable to several relief items such as fruits, medicine, and blankets. Some relief items (i.e. fruits) deteriorate because of their characteristics. Others deteriorate because of improper warehouse facilities. Generally, disaster-affected areas are not equipped with warehouse facilities and establish temporary structures in open parking lots and school playgrounds (Krishnamurthy *et al.*, 2013). Because relief

items (e.g. clothing, blankets) are kept outside (sometimes without proper cover), they lose their utility because of bad weather or for other reasons. A single item is considered with a constant deterioration rate (θ) over time. Because we consider product deterioration for poor warehouses, a constant deterioration rate can represent the post-disaster situation.

(6) Deterioration of units occurs only when the item is effectively in stock. No disposal cost is incurred. No replacement of deteriorated units occurs during planning horizon H.

(7) Figure 1 presents a pictorial representation of the replenishment policy for the deteriorating item with decreasing demand, where $[j,k]$ is the replenishment cycle and time p is the replenishment point to be scheduled within the inventory system cycle.

(8) Although the planning horizon is known a priory, the cycle length is not known.

4. Model

Generally, relief items available in the affected areas are either destroyed by the event or are quickly depleted, necessitating the rapid deployment of relief items to mitigate further human suffering. The planning horizon of the proposed system is H. The planning horizon is divided into several cycles. A cycle length is $[j,k]$ where j is the starting point of the cycle and where k is the cycle end point. In each cycle, a point p represents the relief arrival time. In the proposed system, no inventory is held at the beginning and at the end of replenishment cycle $[j,k]$ along the planning horizon. The cycle starts with accumulating shortages from time j until time p, at which point a replenishment is scheduled. The quantity of the relief item received at time p equals the sum of the demand backordered during period $[j,p]$, the demand requirement in period $[p,k]$ and waste during $[j,k]$.

4.1 RS cost

In this study, $[j,p]$ is called the out of stock duration (refer to Figure 1). Here, j stands for the starting point of the cycle; p denotes the replenishment point of the cycle. The shortage quantity is satisfied after arrival of relief at the next replenishment point. The shortage quantity can be regarded as delayed satisfied demand. It is equal to the demand quantity observed during $[j,p]$ duration. The shortage quantity is given at time t as the following:

$$\text{Shortage quantity, } S(t) = \int_j^t D(\tau)d\tau = \int_j^t (a_0 - a_1\tau)d\tau, \quad j \leqslant t \leqslant p \tag{3}$$

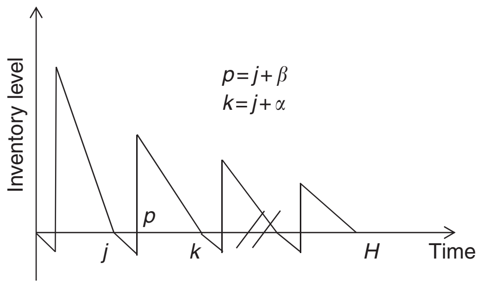


Figure 1.
A pictorial representation of the replenishment policy for a deteriorating item with declining demand

$$\text{Shortage cost, } SC = \int_j^t P_0 D(\tau)d\tau = \int_j^t P_0(a_0 - a_1\tau)d\tau, \quad j \leqslant t \leqslant p \tag{4}$$

Equation (3) computes the shortage quantity before arrival of relief items. Here, the range of independent variable t is from j to p. Actually, $D(t)$ is replaced by Equation (2). Here, P_0 denotes the shortage cost per unit attributable to the relief item shortage. Because P_0 and $D(t)$ are independent, Equation (4) can be solved easily. Equation (4) computes the shortage cost in a cycle and this shortage cost computation approach follows traditional CL. To represent DL, we have proposed a RS cost.

We assume that the urgency decreases exponentially in Equation (1). RS cost also changes accordingly. It is noteworthy that the RS cost does not represent a real monetary cost. It is a proxy for human suffering for shortage. It is natural to assume that human suffering is higher during high urgency. Therefore, the RS cost is also higher for high urgency. *RS* is computed by multiplying the urgency and the shortage cost. Then the value is integrated over [j,p] because a shortage is observed only during [j,p]:

$$\text{Realized shortage cost, } RS = \int_j^p \left(1+\gamma e^{-\mu t}\right) SC dt \tag{5}$$

$$= \int_j^p \left(1+\gamma e^{-\mu t}\right) \int_j^t P_0(a_0 - a_1\tau) d\tau dt \tag{6}$$

$$= P_0 \int_j^p \left(1+\gamma e^{-\mu t}\right)\left(a_0 t - 0.5 a_1 t^2 - a_0 j + 0.5 a_1 j^2\right) dt \tag{7}$$

In actuality, Equation (7) is easy to integrate.

4.2 Cycle holding cost (HC)

Let us consider the inventory level at time t, $I(t)$, during the cycle, that is, $j \leqslant t \leqslant k$. This inventory is depleted by the combined effects of demand and the deterioration rate (θ). It is noteworthy that no deterioration effect occurs if no inventory is on stock. Therefore, the variation of $I(t)$ with respect to time is shown below (Benkherouf, 1995):

$$\frac{dI(t)}{dt} = -I(t)\theta - D(t), \quad j \leqslant t \leqslant k \tag{8}$$

By multiplying $e^{\theta t}$ on both sides of Equation (8), and by replacing the demand function by Equation (2). One obtains the following (Benkherouf, 1995):

$$\text{Inventory quantity, } I(t) = e^{-\theta t} \int_t^k (a_0 - a_1\tau) e^{\theta t} d\tau \tag{9}$$

Now, H_0 represents the HC per unit time. The HC is incurred while there is effective inventory on hand. Therefore, the cycle HCs are computed by multiplying H_0 and $I(t)$ and then integrating over [p,k] by parts. Therefore, the HC for a cycle is written as shown below:

$$\text{Holding cost, } HC = \int_p^k H_0 I(t) dt = H_0 \int_p^k e^{-\theta t} \int_t^k e^{\theta t}(a_0 - a_1 t) dt\, dt \tag{10}$$

$$= \frac{H_0}{\theta}\int_p^k \left[a_0 e^{\theta k-\theta t} - a_1 k e^{\theta k-\theta t} + a_1\frac{e^{\theta k-\theta t}}{\theta} - \left(a_0 - a_1 \mathrm{t}\right) - a_1\frac{1}{\theta}\right)\right] dt \tag{11}$$

Now, the number of deteriorated items in a cycle is the difference between inventory at replenishment point in a cycle and demand in the cycle. It is calculated as:

$$N(t) = I(t) - \int_j^t D(t)dt \tag{12}$$

4.3 Operational cost

A cycle starts with accumulating shortages from time j to time p. After replenishment of relief items at time p, the stored relief item is expected to satisfy demand until time k. It is noteworthy that, the shortage quantity during the time period j to p is satisfied after the arrival of relief items. The ordered relief items for the cycle at time p equal the sum of the demand backordered during period $[j,p]$ and the demand requirement in period $[p,k]$:

$$\text{Ordered quantity,}\ \ Q(p) = S(p) + I(p) \tag{13}$$

here $S(p)$ is the backordered quantity (or shortage quantity) and $I(p)$ is the inventory quantity (or demand requirement in period $[p,k]$). The total quantity of relief items in a cycle is $Q(p)$.

Now, C_0 is the operational cost per unit:

$$\text{Operational cost,}\ \ OC = C_0 Q(p) = C_0(S(p) + I(p)) \tag{14}$$

After replacement with Equation (3) and Equation (9):

$$OC = C_0\left(\int_j^p (a_0 - a_1 t)dt + e^{-\theta p}\int_p^k e^{\theta t}(a_0 - a_1 t)dt\right) \tag{15}$$

4.4 Total cycle cost

The total cycle cost consists of RS cost, HC, operating cost (OC), and ordering fixed cost (A_0). It is noteworthy that the RS cost is incurred by affected residents, whereas other costs are incurred by the relief donating organization:

$$\text{Total cycle cost,}\ \ W(j,p,k) = A_0 + RS + HC + OC \tag{16}$$

Now, replacement with Equations (7), (11), and (15), the total cycle cost becomes the following:

$$\begin{aligned} W(j,p,k) = A_0 + P_0 &\int_j^p \left(1+\gamma e^{-\mu t}\right)\left(a_0 t - 0.5 a_1 t^2 - a_0 j + 0.5 a_1 j^2\right) dt \\ &+ \frac{H_0}{\theta}\int_p^k \left[a_0 e^{\theta k-\theta t} - a_1 k e^{\theta k-\theta t} + a_1\frac{e^{\theta k-\theta t}}{\theta} - \left(a_0 - a_1 \mathrm{t}\right) - a_1\frac{1}{\theta}\right)\right] dt \\ &+ C_0\left(e^{-\theta p}\int_p^k e^{\theta t}(a_0 - a_1 t)dt + \int_j^p (a_0 - a_1 t)dt\right) \end{aligned} \tag{17}$$

For given j and k, the optimal replenishment point p in the cycle is obtained by differentiating Equation (17). Because the value of p and k depends on the value of j, we replace $k = j+\alpha$ and $p = j+\beta$, where $\alpha > \beta$. After replacing the k- and p-values, we differentiate Equation (17) with respect to β and the outcome set equal to zero for locating β:

$$\frac{\partial W(j,p,k)}{\partial \beta} = 0 \tag{18}$$

After several algebra operations of Equation (18), the following were obtained:

$$\begin{aligned} W_\beta(j,p,k) = {} & \frac{H_0}{\theta^3}\left(-\theta e^{\theta\alpha-\theta\beta}(\theta a_0 - \theta a_1(j+\alpha) + a_1) + \theta^2 a_0\right) - (j+\beta)\theta^2 a_1 + \theta a_1) \\ & + P_0\left(a_0\beta - 0.5a_1(j+\beta)^2 + 0.5a_1 j^2 + \left(a_0\beta\gamma - a_1\gamma j\beta - 0.5a_1\gamma\beta^2\right)e^{-\mu j-\mu\beta}\right) \\ & + C_0\left(-a_0 e^{\theta\alpha-\theta\beta} + \frac{a_1 e^{\theta\alpha-\theta\beta}}{\theta}(\theta j + \theta\alpha - 1) + \frac{a_1}{\theta} + a_0 - a_1(j+\beta)\right) \end{aligned} \tag{19}$$

Because obtaining a closed-form solution of Equation (18) is difficult, the replenishment point p ($p = j+\beta$) is estimated using a numerical search technique. A bisection algorithm is used to ascertain the location of β^* (therefore, p^* for given j).

4.5 Sequences of replenishment points

The objective of the proposed model is to ascertain the optimal sequence of replenishment point p along the planning horizon H that minimizes total system costs. The optimal sequence of replenishment points may be determined by solving the following dynamic programming equations:

$$TC_k = Min\left\{TC_j + W\left(j, p^*, k\right)\right\} \tag{20}$$

$$TC_0 = 0 \tag{21}$$

Equation (21) shows that cost at the starting point is zero. The forward recursive procedure is used to ascertain the minimal total cost over planning horizon H. To solve the model, a dynamic programming algorithm is proposed and subsequently explained. The algorithm is a modified version of Chen (1998).

We assume that S_m, starting point of the replenishment cycle number m; E_m, end point of the replenishment cycle number m; $P^*_{j,k}$, optimal replenishment schedule in cycle $[j,k]$.

Step 0: Input parameters: A_0, C_0, H_0, P_0, θ, γ, μ, a_0, a_1
Step 1: Let $TC_0 = 0$, $j = 0$ and $m = 1$
    For $k = 1$ to H {
      Solve Equation (19) to obtain β^*
      $p^* \leftarrow j+\beta^*$
      Solve Equation (17) to obtain $W(j, p^*, k)$
      Let $TC_k = W(j,p^*,k)$, $P^*_{j,k} = p^*, S_m = 0,$ and $E_m = k$
      }
Step 2: Let $i \leftarrow 1$
For $k = 2$ to H {
    For $j = i$ to $k-1$ {
      Solve Equation (19) to obtain β^*

$p^* \leftarrow j+\beta^*$
Solve Equation (17) to obtain $W(j, p^*, k)$
IF $TC_k > TC_j+W(j, p^*, k)$ {
$TC_k = TC_j+W(j, p^*, k), P^*_{(j,k)} = j+\beta^*,\ \ m = m+1,\ \ S_m = j$ and $E_m = k$
$i = k+1$
For $t =\ i$ to H {
Solve Equation (19) to obtain β^*
$p^* \leftarrow k+\beta^*$
Solve Equation (17) to obtain $W(k, p^*, t)$
Let $TC_t = TC_k + W(k, p^*, t), P^*_{k,t} = k+\beta^*$
}
}
}
}
Step 3: Let $k = H$
While $(m \neq 1)$ {
Replenishment cycle $= [S_m, E_m]$
Replenishment schedule $= P^*_{S_m,E_m} = p^*$
Cumulated total cost $= TC_k$
$m \leftarrow m-1$
}

Thus, the algorithm finds optimal "cycle lengths" and "order splitting" for a given planning horizon, H.

5. Data and results

The model developed in Section 4 is applicable to the distribution of different long-term relief items. We present a numerical example to illustrate the solution concepts presented previously. Table I presents the key parameters.

The values used in the numerical example are useful for illustration purposes but they might not match values estimated by the ultimate users of the model. A user should gather parameter values before using the model. Historical relief operation data can be helpful for setting parameter values for the demand and deterioration rate. Cost parameters are local condition values. The values of a_0 and a_1 in the demand function are set, respectively, to 10 and 0.2. Some relief items deteriorate because of the lack of preserving facilities in a shelter. We assume a constant deterioration rate (θ) with value set to 0.08. The value of the shortage cost per unit (P_0) is higher than the value of the HC per unit (H_0). The shortage cost per unit in independent of urgency parameters. The operational cost per unit (C_0) is set between P_0 and H_0. The parameters for urgency (γ and μ) are set, respectively, to 2 and 0.08. Planning horizon (H) is assumed to be fixed and set to a value of 50.

Table II presents a scenario description and the computational results for different scenarios. Scenario "0" represents no order splitting, indicating that all relief items are

Table I.
Demand, cost, deterioration rate, and urgency parameter

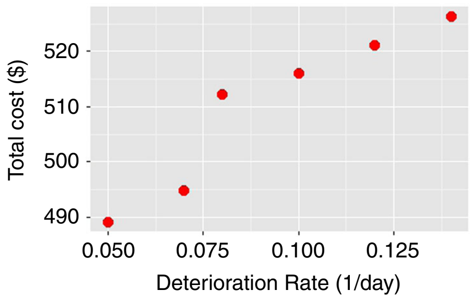
Demand (unit)		Cost per unit (\$/unit)			Fixed C (\$)	Deterioration rate	Urgency parameter	
a_0	a_1	C_0	H_0	P_0	A_0	θ	μ	γ
10	0.2	0.5	0.3	1	20	0.08	0.08	2

brought at once. Other scenarios have the order splitting property. The deterioration rates for scenarios 1-3 are varied and the urgency parameter (γ) value is kept fixed. In contrast, scenarios 2, 4, and 5 have characteristics of different urgency parameter (γ) values for a fixed deterioration rate (θ). The outcomes presented in Table II show that the total out of stock duration during the planning horizon is longer for higher deterioration rates.

Figures 2-4 are drawn using data from Table II. Figure 2 shows that the total cost increases nonlinearly as the deterioration rate increases. The total cost increases from

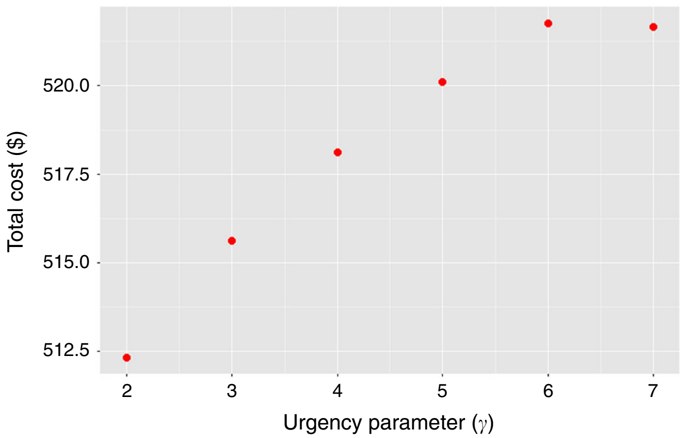
Scenario	γ	θ	Order splitting	Total cost ($)	Out of stock (day)	Total shortage cost ($)	Holding cost ($)	Deteriorated quantity (unit)
0	2	0.08	No	3,155.19	11.70	1,329.45	1,483.00	395.45
1	2	0.05	Yes	489.16	8.37	25.07	109.93	18.32
2	2	0.08	Yes	512.33	9.16	24.48	90.75	24.20
3	2	0.1	Yes	516.02	9.59	26.45	89.63	29.88
4	4	0.08	Yes	518.13	7.70	20.69	99.20	26.45
5	6	0.08	Yes	521.75	6.75	18.16	104.65	27.91

Table II. Scenario descriptions and computational results



Notes: Urgency parameter, $\mu = 0.08$; $\gamma = 2$

Figure 2. Relationship between deterioration rate and total cost



Note: While $\mu = 0.08$

Figure 3. Relationship between urgency parameter (γ) and total cost

the combined effect of incremental deterioration and the total duration of the out of stock items. Because the deterioration rate depends on the facilities, making investments to lower the deterioration rate is advised. Consequently, a tradeoff exists between investment cost and total cost. Figure 3 shows the relation between the total cost and the urgency parameter (the γ-values) and indicates that the curve becomes a flat line for higher γ values. Thus, the model shows sensitivity to urgency parameters.

A comparison among scenarios 2, 4, and 5 reveals that the total out of stock duration is shorter and the quantity deteriorated is larger for higher urgency values. Therefore, the model responds optimally to a changing environment.

Now, we define the service level (SL) as:

$$SL = 1 - \frac{\text{Out of stock duration}}{\text{Planning horizon}}. \tag{22}$$

Equation (22) shows that service level will be higher if the out of stock duration is shorter. The range of SL is from 0 to 1. Because DL has no distinctive index for measuring performance, we use Equation (22) for performance measurements.

Figure 5 presents the computed SL for each scenario. Scenario "0" represents no order splitting. All relief items are brought together in scenario "0." SL is observed to be the lowest for scenario "0." The SL changes with the combined effects of γ and θ.

Table III presents the outcomes of the model for scenarios 0 and 2. The first row represents the result of scenario 0 and shows that the total cost in this scenario is \$3,155.19. In contrast, the total cost after the planning horizon in scenario 2 is \$512.33, or 83.76 percent lower than the total cost for scenario 0. Scenario 2 has 13 cycles.

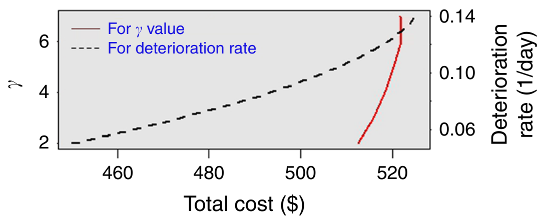


Figure 4.
Changes in total cost for urgency parameter (γ) and deterioration rate

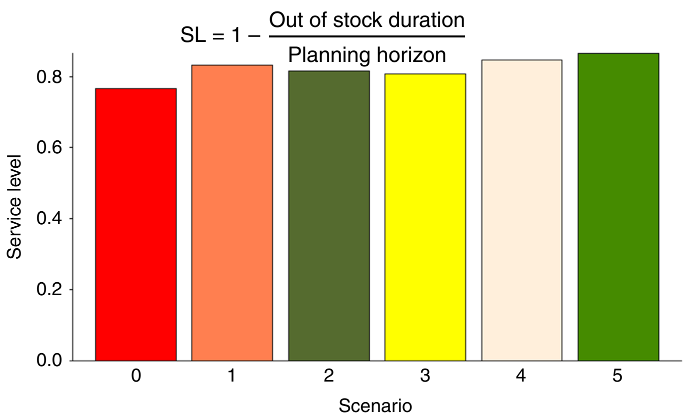


Figure 5.
Service levels in different scenarios

Cycle number	Cycle period [j,k]	Arrival time (day)	Cost end of cycle ($)[a]	Holding cost ($)	Shortage cost ($)	Ordered quantity (unit)	Deteriorated quantity (unit)
Scenario 0							
1	[0,50]	11.7	3,155.19	1,483.00	1,329.45	645.47	395.47
Scenario 2							
1	[0,2]	0.22	36.01	4.87	0.69	20.89	1.30
2	[2,5]	2.36	83.29	10.27	1.69	30.64	2.74
3	[5,8]	5.41	128.57	9.24	1.75	28.56	2.46
4	[8,11]	8.46	171.89	8.27	1.78	26.51	2.21
5	[11,14]	11.50	213.26	7.37	1.77	24.47	1.97
6	[14,17]	14.54	252.75	6.54	1.73	22.44	1.74
7	[17,20]	17.58	290.39	5.77	1.66	20.44	1.54
8	[20,24]	20.83	334.95	8.89	2.82	24.77	2.37
9	[24,28]	24.87	374.95	7.36	2.50	21.16	1.96
10	[28,32]	28.90	411.82	5.95	2.12	17.58	1.59
11	[32,36]	32.92	445.20	4.65	1.71	14.04	1.24
12	[36,43]	36.53	486.40	9.14	3.48	17.14	2.44
13	[43,50]	44.04	512.33	2.40	0.76	5.54	0.64
Total			512.33	90.72	24.46	274.18	24.200

Note: [a]"Cost end of cycle" is calculated from time, $t=0$

Table III. Computational result for scenarios 0 and 2

The start and end periods for each cycle are tabulated in the second column in Table II. Cycle lengths differ during the planning horizon. Table III shows the relief items' arrival time (p^*) after the start of each cycle. The replenishment point in each cycle is also tabulated in the third column. The fourth column represents the accumulated cost after the end of a cycle. Because of the different cycle lengths, the number of deteriorated items in each cycle changes. Generally, a longer cycle length engenders an item that deteriorates faster. Table III provides the HC, the shortage cost, the ordered quantity, and the deteriorated quantity for each cycle.

Figure 6 presents the inventory level in each cycle for scenario 2, and shows that the ordered quantity (arrived quantity equals ordered quantity) is not constant. Considering the decreasing demand trend, the ordered quantity also changes and it is the combined effect of demand and the deterioration rate.

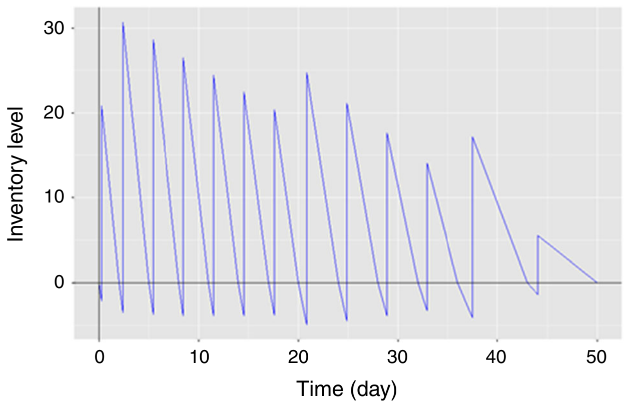


Figure 6. Inventory level for type A (declining demand and urgency incorporated)

To observe the effect of declining demand and urgency, we conduct a sensitivity analysis and compare the results obtained from four sets of cases. Table IV presents the properties of the four cases and Figure 6 through Figure 9 illustrate the results of the four cases. It is noteworthy that type A has the same properties as those of scenario 2.

An analysis of four cases shows that the first and the last cycles have irregular shapes. The irregularity of the first cycle occurred because the value of the initial cost (TC_0) is zero. The last cycle is irregular because the algorithm is based on a forward recursive procedure for a fixed time horizon. The model brings a zero inventory level after satisfying all the demand in the last cycle.

Among the four cases, the total number of replenishment points is highest in type B and lowest in type C. For types A and B, the quantity of delayed satisfied demand shows an increasing trend, which results from the exponential decrease in the urgency of relief items. However, type C shows a decreasing trend in the quantity of delayed satisfied demand, which occurs because of a gradual decrease in demand and unchanging urgency. Type D has a constant value for the quantity of the delayed satisfied demand (Figures 7 and 9).

Except for the first and last cycles, the inventory level after order arrival decreases for type A but has disturbances after certain cycles. The timing of the appearance of the disturbance depends on the slope of the demand function. In this example, the disturbance appears after seven cycles. In contrast, the inventory levels after order arrival for types B and D are constant. Finally, the inventory levels after order arrival decrease for type C. The findings of sensitivity analysis provide an understanding of the trend in order quantity changes and the quantity of delayed satisfaction.

6. Conclusion

The salient aim of this work was development of a mathematical model that can be applied for replenishment schedule planning. Particularly, this study was designed to

Table IV. Properties of four types of situations

Type	Properties	Mathematical explanation
A	Declining demand and declining urgency incorporated	$D(t) = 10-0.2t$, $\theta = 0.08$, and $\gamma = 2$
B	Constant demand and declining urgency incorporated	$D(t) = 10$, $\theta = 0.08$, and $\gamma = 2$
C	Declining demand and constant urgency	$D(t) = 10-0.2\text{t}$, $\theta = 0.08$, but $\gamma = 0$
D	Constant demand and constant urgency	$D(t) = 10$, $\theta = 0.08$, and $\gamma = 0$

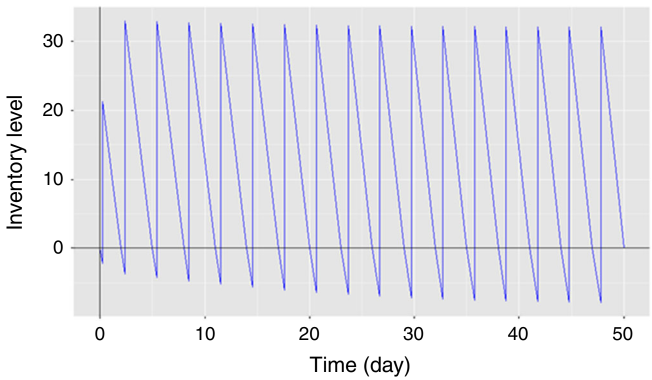


Figure 7. Inventory level for type B (constant demand and urgency incorporated)

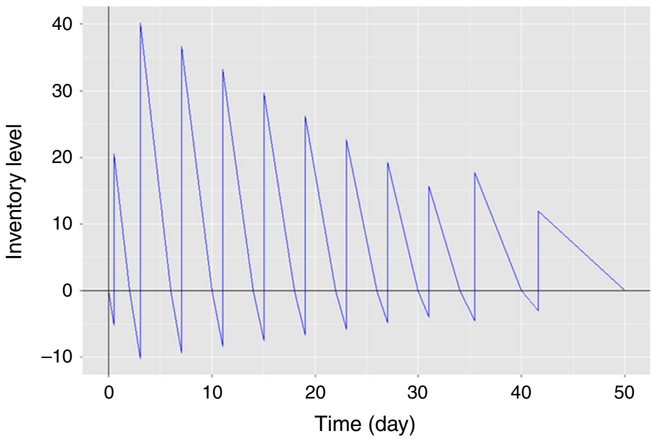


Figure 8. Inventory level for type C (declining demand and constant urgency)

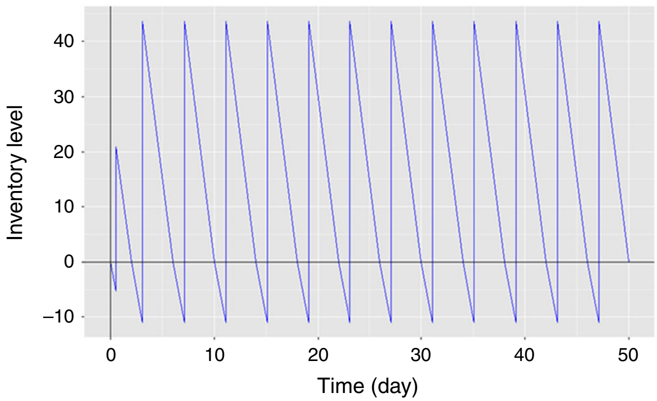


Figure 9. Inventory level for type D (constant demand and constant urgency)

examine whether the replenishing cycle changes with the inclusion of RS cost and declining demand. No report in the literature describes a study of this type of problem.

The proposed model permits variations in both replenishment intervals (and, therefore ordered quantity) and the service levels between replenishment cycles. Results show that the model generates a better solution than optimization models with fixed order intervals or a fixed service level. The optimization model with fixed order interval or fixed service level does not incorporate a decreasing trend of demand and urgency variation. Moreover, we have presented the model result obtained using six scenarios and four cases. Results shows a particular trend in each scenario, which proves that the model results are independent of particular values.

Aside from the academic implication stated above, this study has several practical implications. A logistics manager can use the model for resource planning including warehouse capacity and transportation capacity. Because cycle lengths are not constant in planning horizon, a logistics manager has opportunity to operate transport capacity effectively. In the case of declining relief demand case, the transport capacity can be reduced in a later stage of the planning horizon. In the case of constant relief demand, transportation capacity need not be altered during the planning horizon.

Moreover, urgency parameters can be adjusted to represent sensitivity to relief shortage.

The model is tested for several cases. Order splitting can be an effective mechanism to improve the response to natural disasters.

The results of the study engender the following conclusions:

- The urgency for the temporal dimension provides better inventory planning based on the effectiveness of relief items. While urgency is incorporated (for types A and B), the model reduces the shortage quantity in early stages and accommodates a larger shortage quantity in later stages. However, in the case of without urgency incorporation (for types C and D), there is no certain trend.
- The incorporation of RS cost and declining demand affect the service level of the system. The service levels vary among scenarios. The system has a higher service level (for scenarios 2, 4, and 5) for higher urgency. Again, the service level decreases for a higher deterioration rate (for scenarios 1, 2, and 3). In addition, the strategy of order splitting (presented in scenarios 1-5) is better than no order splitting (scenario 0).
- The introduction of deterioration into the model reveals that a higher deterioration rate causes a higher system cost. This issue is observed in scenarios 1, 2, and 3. As the deterioration rate depends on the facilities, a tradeoff exists between investment cost and total cost. A decision maker can invest in improved facilities to decrease the deterioration rate. However, if deterioration occurs because of product properties, then a logistics manager might search a substitutable relief item that deteriorates in lower rate.

Although the proposed model is flexible, the main restrictions to its practical implementation are twofold: continuous review of the stock is assumed and the shape of the urgency function is not known with certainty. Fortunately, organizations are undertaking efforts to collect relief operational data, thereby supporting a true urgency function to be generated in the future. In addition, the application of modern technology has been increasing recently. Finally, this model is applicable for those areas for which relief storage facilities are poor. Unfortunately, poor facilities are a commonly occurring phenomenon after a disaster.

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