Examining the impact of Zambia's curriculum in outcome-based education on cognitive development: insights from mental number line research

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Abstract

Purpose – This study explores how Zambia's outcomes-based education (OBE) curriculum supports the development of a mental number line, which is a fundamental concept for numerical understanding. It also examines the relationship between mental number line development and students' calculation abilities, identifying key issues that must be addressed to enhance numeracy skills.

Design/methodology/approach – A mixed-methods approach was adopted, combining the quantitative analysis of mathematical tests with qualitative observations of number line estimation tasks. Data were collected from 149 valid responses from Grade 6 and Grade 7 students in Southern Province, Zambia. Participants completed a test on four arithmetic operations with whole and decimal numbers and a 0–100 number line estimation task. Mental number line accuracy was assessed using the Percentage Absolute Error (PAE) and analysed using proportional judgement and estimation strategies.

Findings – The findings indicate that most students struggled with accurate mental number line representation. While estimation accuracy improved with grade level, many relied on counting or integer-based reasoning, demonstrating a limited conceptual understanding of numerical magnitude. No significant correlation emerged between mental number line representations and math performance, suggesting that computational skills depend more on procedural learning than on conceptual understanding. These findings suggest that the current curriculum does not sufficiently support cognitive development in numeracy.

Originality/value – This study underscores the importance of fostering students' cognitive development by using an outcome-based educational approach. It emphasises the significance of student-centred resources in boosting numerical understanding and the formation of mental number lines, thereby enhancing mathematics instruction across various learning settings.

Keywords Mathematics education, Numerical representation, Numerical estimation, Mathematical competency

Paper type Research article

1. Introduction

In line with global educational reforms, many African countries have adopted Outcome-Based Education (OBE) to equip learners with the competencies necessary for participation in 21st century society. For example, in 2013, Zambia shifted from a traditional content-based curriculum to an outcomes-oriented framework aimed at developing problem-solving and critical thinking skills (Kabombwe, Machila, & Sikayomya, 2021; Kabombwe & Machila, 2020). This shift reflects the widespread recognition of the need to prepare for real-world challenges and socioeconomic development.

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Underlying these reforms are serious challenges regarding the quality of education. It is estimated that about 250 million students worldwide have not mastered basic math skills despite years of schooling (UNESCO, 2015; World Bank, 2018). An education quality monitoring study by the Southern and Eastern Africa Consortium (SACMEQ) found that approximately 80% of Zambia's sixth-grade primary students are below Level 3, the equivalent of basic numeracy skills (Awich, 2021).

Basic numeracy in the SACMEQ refers to the ability to interpret and calculate information using sentences, simple graphs, and arithmetic operations, including basic numerical operations used in everyday life. Students at Level 4 and above, on the other hand, are able to analyse more complex information and solve problems using multiple operations with whole numbers, fractions, and decimals in the proper sequence (SACMEQ, 2017).

The OBE approach emphasises quantifiable learning outcomes, such as mathematical problem-solving skills, and is designed to improve students' ability to analyse and solve complex problems. However, the acquisition of these applied mathematical skills requires the underlying number-cognitive skills to be appropriately developed (Laski & Siegler, 2007; Zhu, Cai, & Leung, 2017).

In particular, the development of number sense and mental number lines is strongly associated with mathematical attainment (Siegler & Opfer, 2003).The mental number line is the ability to visually and conceptually place the magnitude of numbers on a straight line, and has a significant impact on numerical reasoning and computational flexibility (Siegler & Opfer, 2003). As shown in Figure 1, early developmental number lines are logarithmic (unequal spacing between small and large numbers), but as learning progresses, they move to linear number lines with proportional spacing (Siegler & Booth, 2004). The formation of the mental number line is strongly associated with broader mathematical competencies, including magnitude comparison and set-size estimation (Laski & Siegler, 2007; Ruiz, Kohnen, & Bull, 2024). These findings highlight the foundational role of number line understanding in supporting more advanced numerical reasoning skills.

Among the various applications of number sense, understanding decimal numbers poses a particular challenge due to their continuous nature and position on the number line. Understanding decimal numbers is a mathematical concept that many students have difficulties with (Stacey & Steinle, 1999; Siegler & Lortie-Forgues, 2017), requiring a continuous conceptual understanding that is distinct from the quantity sense of whole numbers (Hiebert, 1992). Therefore, it is essential to establish basic number cognitive skills and strengthen decimal understanding in Zambian mathematics education to enable achievement

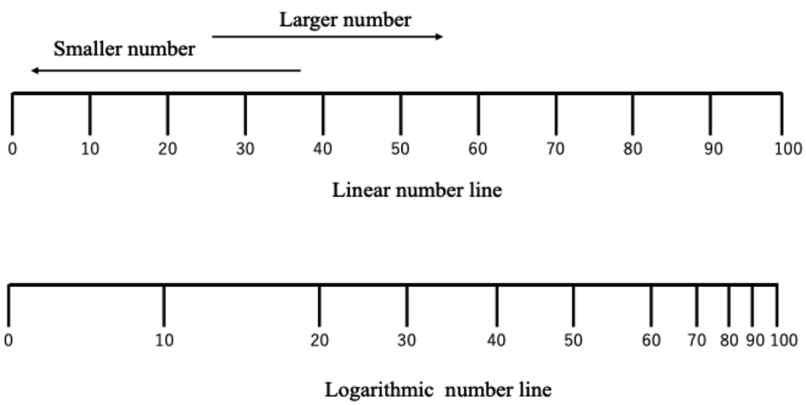

Figure 1. The image represents linear and logarithmic number lines. Note: Adapted from Fritz, Ellis, Demsky, and Guros (2013) and Longo and Lourenco (2007)

at levels 4 and above. This study analysed the relationship between mental number line development and numeracy in Zambian primary school children and examined its function as a cognitive foundation for the curriculum. To address these challenges, this study explored the development of number sense and its implications for numeracy education in Zambia. Specifically, it examined the relationship between mental number line ability and curriculum outcomes through the following research questions:

(1) What are the characteristics of the development of mental number lines among Zambian primary school children?

(2) How does mental number line ability correlate with computational ability in Zambian primary school children?

(3) What effects does the Zambian OBE mathematics curriculum have on the development of mental number lines and numeracy skills?

2. Literature review

2.1 Cognitive development of number concepts and characteristics of Zambian children

Research on the cognitive development of numbers indicates that humans are born with innate cognitive abilities (Wynn, 1992; Xu, Spelke, & Goddard, 2005). This early cognitive ability develops through experiences in daily life, leading to the formation of more advanced concepts. The quantitative recognition of integers is based on understanding them as discrete quantities, whereas the recognition of decimal numbers requires understanding them as continuous quantities (Newcombe, Levine, & Mix, 2015). Although it has been noted that early number cognition development is a predictor of future rainfall academic achievement, there is limited research on the developmental process of number cognition in Zambian children.

Subitizing skills and the development of mental number lines have received particular attention in studies of number cognition skills in Zambian children. Subitizing is the ability to recognise a small number of sets instantly, and can be classified into two types: innate and conceptual subitizing. The former refers to the ability to instantly recognise numbers 1–3, which forms the basis of numerical understanding. The latter, on the other hand, refers to the ability to recognise numbers as a unified whole and influences the formation of more advanced number concepts (Nakawa, 2016). Existing research confirms that Zambian children have innate subitizing skills, but suggests that they are underdeveloped concerning conceptual subitizing skills (Nakawa, 2016). Such cognitive traits may pose a barrier to learning as they make an integrated understanding of integers and decimal numbers difficult.

Another important component of cognitive skills is the development of mental number lines. The mental number line refers to the ability to visually and conceptually place the size of numbers on a straight line, and has been reported to correlate strongly with the development of numerical reasoning and calculation skills (Siegler & Opfer, 2003).

Studies on the development of mental number lines in Zambian children include Abe (2022), Sudo, Kume, Bagenda, and Fujino (2022), and Kume, Sudo, Bagenda, and Fujino (2022). These studies have indicated that the development of mental number lines in Zambian Grade 6 students lags behind that of Grade 2 students in Western country (Abe, 2022), which may be due to a lack of numeracy skills (Kume *et al.*, 2022).

However, the existing research has several limitations. In addition to sample size limitations, many studies have focused on mental models of integers and have not adequately examined the link to understanding decimal numbers. Additionally, much of the research on the relationship between numeracy and mental number lines remains hypothetical and lacks empirical analysis to clarify causal relationships. This gap in cognitive development research is particularly significant in Zambia, where the Outcome-Based Education (OBE) curriculum emphasises applied problem-solving. Without a strong foundation in numerical cognition, students may struggle to fully engage with the curriculum's objectives, leading to a misalignment between the intended learning outcomes and actual mathematical comprehension.

2.2 Background and challenges of outcome-based education in Zambia

The introduction of the current curriculum in Zambia is in line with global trends promoting outcome-based education to bridge the gap between labour market demands and skills imparted in the classroom (Mungalu & Ndhlove, 2021; MoGE, 2013).

According to Tyler's rationale for curriculum development, three major sources should inform curriculum construction: studies of learners, studies of contemporary life, and suggestions from subject matter experts (Tyler, 1949; Ishii, 2016). Zambia's recent curriculum reform appears to align with this framework; however, it has primarily drawn on the latter two sources—namely, the demands of modern society and recommendations from experts—emphasizing the cultivation of competencies needed in the global context. In contrast, the first source, which concerns learners' developmental readiness, has received insufficient attention. As Baba and Nakai (2011) point out, curriculum development grounded in the actual cognitive and developmental characteristics of Zambian learners remains limited due to the scarcity of relevant empirical research.

This imbalance poses critical challenges for curriculum implementation in the classroom. Recent cognitive research suggests that many Zambian learners may lack the numerical understanding required to engage effectively with the curriculum. In particular, studies indicate that the development of the mental number line—a key cognitive structure for understanding magnitude, quantity estimation, and operations—is significantly delayed compared to learners in high-income countries (Abe, 2022). Such cognitive limitations create a mismatch between the expected curriculum outcomes and the actual learning conditions of students (Kabombwe & Mulenga, 2019; Mulenga & Kabombwe, 2019).

Thus, it is essential to consider an integrated perspective that combines curriculum theory with insights from cognitive development. Educational design that disregards learners' readiness risks undermining the very goals that OBE seeks to achieve. While the Zambian mathematics curriculum emphasizes critical thinking and conceptual application, classroom instruction remains focused on rote memorization and procedural practice. This disconnect stems from a gap between curriculum goals and actual teaching practices (Kabombwe & Mulenga, 2019).In particular, acquiring the concept of decimal numbers requires an understanding of the continuity of numbers, which cannot be fully acquired through simple calculation procedures (Siegler & Lortie-Forgues, 2017).

Furthermore, the assessment methods employed are not always consistent with the curriculum objectives. Rote learning tends to be prioritised in the administration of examinations, and the development of critical thinking and problem-solving skills is not fully achieved (Kabombwe *et al.*, 2021). The impact of this assessment approach often leads learners to be biased towards memorising calculation procedures rather than developing their understanding of numerical concepts, which may, in turn, inhibit the development of abstract mathematical understanding, such as the concept of decimal numbers (Stacey & Steinle, 1999). The OBE curriculum has been shown to be problem-based. encouraged; however, these problems have the potential to rely on procedural solutions. If teachers only require correct answers, the development of basic number cognitive skills may remain insufficiently deep, and understanding of mathematical content and more complex problem-solving may be limited.

Given these challenges, assessing learners' readiness becomes crucial for informing both instruction and curriculum design. However, little empirical research has been conducted on the cognitive foundations of Zambian learners—especially with regard to the transition from natural numbers to decimals. Existing studies have primarily focused on the understanding of whole numbers, while little is known about how learners develop the concept of decimal numbers.

This study seeks to address this gap by investigating the relationship between mental number line development and calculation ability among primary school learners in Zambia. Whereas natural number understanding is rooted in discrete quantity recognition, decimal understanding requires the ability to conceptualize continuous quantities. In the Zambian context, this conceptual shift has not been sufficiently supported by either instructional strategies or assessment tools. By analysing how learners move from whole number to decimal concepts, this study aims to provide insights into their underlying cognitive structures and to inform the development of teaching strategies and curriculum development better aligned with learners' actual developmental levels.

3. Methodology

3.1 The mental number line and its theoretical background

Since the 1990s, the relationship between numerical cognition and quantity has been extensively studied, resulting in the introduction of the mental number line concept (Case & Okamoto, 1996). This framework represents a cognitive structure in which numerical magnitudes are spatially organised along a continuum. The mental number line represents a critical cognitive structure that integrates numerical magnitude and spatial relationships, forming the basis for mathematical reasoning and problem solving (Siegler & Opfer, 2003).

The development of the mental number line entails a transition from logarithmic representations characterised by unequal spacing between numbers to linear representations in which spacing reflects proportional numerical relationships (Siegler & Opfer, 2003; White & Szűcs, 2012). This progression indicates a more profound comprehension of numerical magnitude and has been demonstrated to be associated with enhancements in a range of mathematical abilities, including estimation, computation, and problem solving (Siegler & Booth, 2004; Laski & Siegler, 2007).

Two principal analytical frameworks are typically employed to examine the evolution of the mental number line. The first is the shift representation. This study examined qualitative changes in numerical representation over time, focusing on the transition from logarithmic to linear representations. It is common for young students to overestimate small numbers and underestimate large numbers, which reflects their understanding of numerical magnitudes logarithmically. However, as children develop, their estimates become more accurate and consistent with linear representations (Barth & Paladino, 2011; Siegler & Opfer, 2003). An alternative is proportional judgement. This method classifies estimation strategies based on the reference points used during numerical tasks. These strategies comprise three distinct models. The first model is the Unbounded Power Function (UPF), which relies minimally on reference points and typically focuses only on the lower endpoint of the number line. The second model, the 1-Cycle Model, incorporates both endpoints and uses them as reference points for estimation. Finally, the 2-Cycle Model employs three reference points: the lower endpoint, upper endpoint, and midpoint, allowing for more precise numerical magnitude estimations (Slusser, Santiago, & Barth, 2012; Barth & Paladino, 2011).

The accuracy of the numerical estimates was quantified using the Percentage Absolute Error (PAE), which was calculated as follows:

$$\text{PAE} = \frac{|\text{ estimated number} - \text{presented number}|}{\text{range of the number line}} \times 100$$

Lower PAE scores indicate enhanced estimation accuracy and alignment with linear representations.

Despite extensive research on the mental number line, many studies have focused on Western populations, with a paucity of data from African contexts. Abe (2022) investigated the cognitive development of sixth- and seventh-grade students in Zambia and identified distinct growth patterns in their test scores. While some students exhibited proficiency in representing quantities in the 0–10 range using linear representations, the majority demonstrated difficulty extending this understanding to the 0–100 range, where both linear and logarithmic representations were required. These findings emphasise the distinctive challenges that Zambian students encounter in developing mental number lines and underscore the importance of contextually adapted research.

This study builds on existing frameworks by focusing on the 0–100 range, in which Zambian students have demonstrated difficulties. This study aimed to examine the development of mental number lines and their correlation with calculations in this population.

3.2 Research design

This study was conducted in the Southern Province of Zambia and involved 154 students in grades 6 and 7. After data screening, 149 valid responses were retained for statistical analysis.

To ensure methodological rigour and cultural relevance, the research design integrated established methodologies with adaptations to the local context of the study. Public primary schools in the Monze area were selected to capture the socioeconomic diversity of the participants. Informed consent was obtained from the school principals and teachers before the study. Participants were selected based on their grade, where decimal numbers had already been studied.

A mixed-methods approach was adopted, incorporating quantitative data from tests of calculation problems and number line tasks and qualitative observations of students' behaviour. Each participant was provided with an A4 sheet containing a 24 cm long number line labelled “0” at the left endpoint and “100” at the right endpoint. Target numbers (13, 30, 49, 75, 80, 10.2, and 85.6) were presented verbally in a randomised order. The students were organised into small groups of approximately 10 participants to facilitate a manageable testing environment. The researchers provided clear instructions to all participants and conducted practice trials to ensure task comprehension. The students were then instructed to mark the locations of target numbers on the number line. For ambiguous responses, such as multiple markings on the line, the researchers conducted follow-up interviews to clarify students' estimation strategies. Observational notes were taken throughout task administration to document the students' behavioural approaches, such as their reliance on visual anchors or verbal self-guidance.

The data analysis process was structured into two key dimensions: estimation accuracy and strategies. Estimation accuracy was assessed by calculating the Percentage Absolute Error (PAE) values for each target number, which allowed the researchers to evaluate the degree to which students' responses were consistent with linear representations. The estimation strategies were classified using proportional Judgement, which identifies patterns of reference point usage, such as reliance on endpoints or the inclusion of midpoints. Statistical analysis was performed using R software, with descriptive statistics summarising the mean PAE values for each grade and numerical range. Inferential statistics, including ANOVA and regression analysis, were used to examine differences in estimation accuracy between grades and the relationship between mental number line development and computational ability.

4. Findings

4.1 What are the current developmental representations of mental number lines among primary school students in Zambia?

Analyses were conducted using both function and percentage judgement representations to measure the formation of the Zambian students' mental number line. To determine the accuracy of the mental number line in each analysis, it was measured using PAE, which checks the discrepancy between the actual number position presented and the number position estimated by the student. Figures 2–4 show the plot diagrams of the representation of the mental number line based on the difference between the actual position of the number presented and the point estimated by the student(plotted as circles) and the theoretical value (plotted as crosses).

The results of the analysis of the mental number line by function representation are shown in Table 1, with 44% of the linear and logarithmic representations falling within the function representation and the remainder being nonconforming. The number of students with nonconforming representations is presented in Table 2. Consequently, it cannot be said that half of the upper secondary school students in Zambia have sufficiently developed mental number lines.

The results of the mental number line analysis based on proportional judgements are presented in Table 3. Regarding the method of estimating the number, when 49 was presented, one student used the midpoint, but 88% of the students either counted one by one, marked with a stick from 0, used arbitrary numbers such as 3, 5, 10, or estimated the number without using any clues.

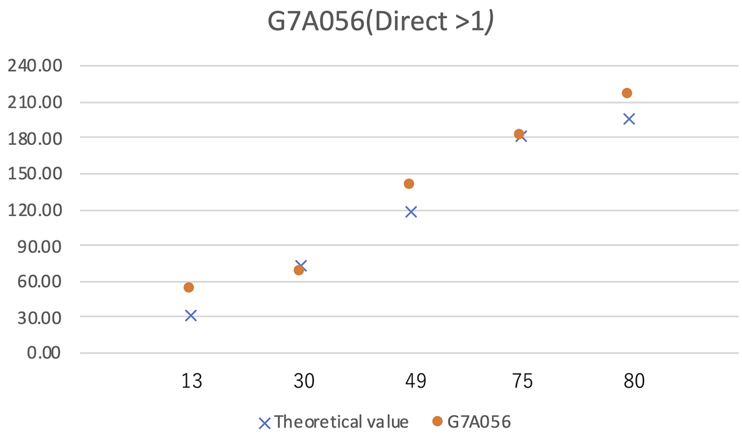

Figure 2. Example direct >1 (linear function representation) in this survey

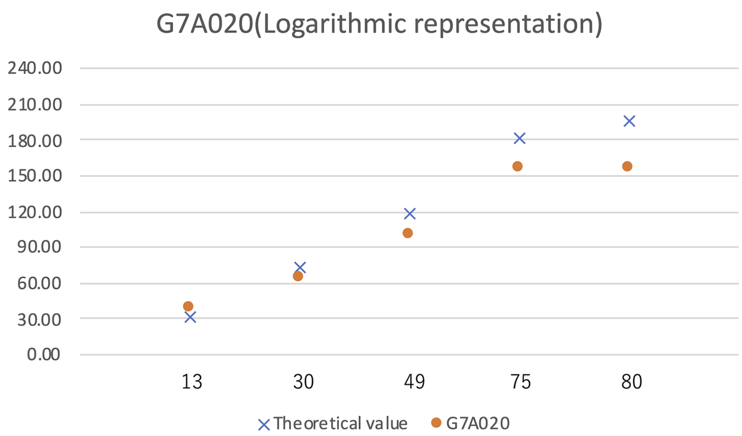

Figure 3. Example logarithmic representation in this survey

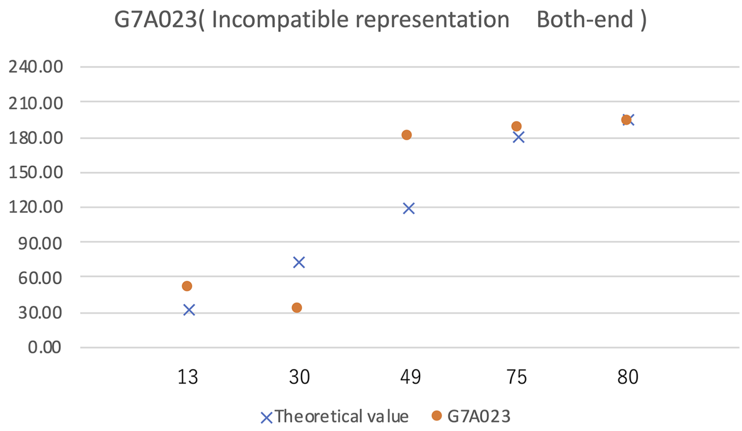

Figure 4. Example incompatible representation in this survey

Table 1. Number of people by function representation and PAE mean

Representation	*N*	PAE mean
Direct = 1	0	
Direct<1	0	
Direct >1	35 (23%)	67.8
Logarithmic	16 (11%)	35.0
Incompatible	98 (66%)	119.8
Total	149	

Table 2. Number of persons of incompatible representations

	Representation	*N*
Incompatible	Left end	8
	Right end	2
	Both-end	21
	Mid	33
	Large and small	1
	Irregular	35
	Other	1

The students' answers are listed in the order of Student A, B, and C from top to bottom in Figure 5. Student A counted up by 1 from 0 to 30 but counted up by 5 after 30, estimating 49. In addition, the interval between the bars was somewhat constant; however, the sense of the interval remained constant even when the number changed. Additionally, the interval of a tick is different from its actual interval. Student B's answer counts by 10 but estimates 49 to be behind 80. There are two ways of looking at this: one is that he has not developed a relative sense of number 49. Second, he sees 49 as 94 and estimates it to be greater than 80. Student C marked a point on the diagram next to 49, approximately in the middle, and estimated based on this point.

These results indicate that the majority of Zambian upper elementary school students have not yet acquired linear mental number-line representations. The high percentage of non-linear

Table 3. Number of student by proportional judgement and PAE mean

Model	*N*	PAE mean
2-cycle	1 (1%)	31.8
1-cycle	17 (11%)	170.6
UPF	131 (88%)	105.6
Incompatible		
Total	149	

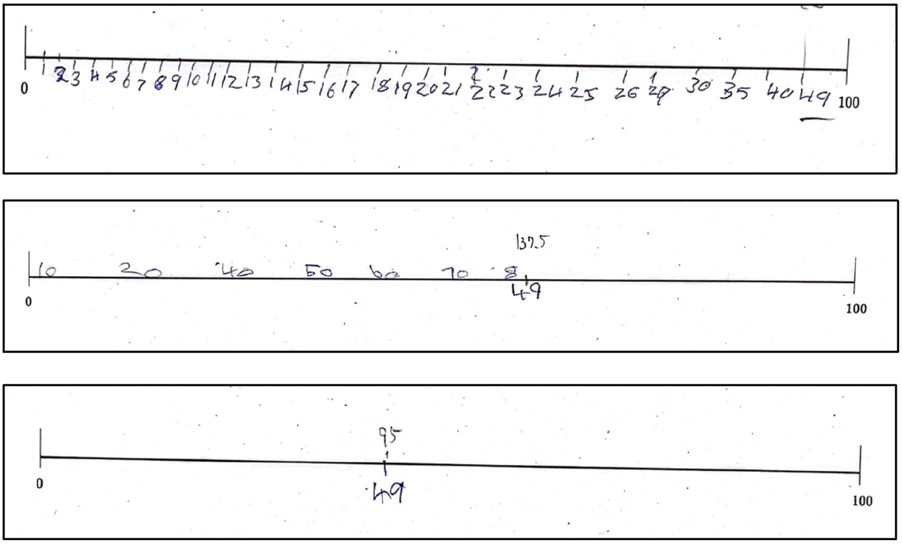

Figure 5. Examples of student's responses (A, B, C)

representations indicates that students' ability to estimate numbers, even numbers they normally use, such as up to 100, is difficult to estimate.

4.2 What is the relationship between mental number line proficiency and the computational performance of Zambian students?

The mental number line is positively correlated with the later acquisition of mathematical skills and various mathematical abilities (Siegler & Opfer, 2003). Kume *et al.* (2022) hypothesized that the development of mental number lines is a factor contributing to the low computational ability of students in Zambia. Therefore, we calculated the correlation coefficient between the mean PAE scores and the computational scores for each student using the statistical software R. The results of the calculation scores by function representation and percentage judgement are summarised in Tables 4 and 5, respectively.

The correlation with the function representation PAE showed that the calculated score included natural numbers and decimal numbers ($r = -0.32$) and the calculated score for natural numbers only ($r = -0.11$).

The correlation of proportional judgements with PAE calculation ability was $r = 0.17$ for calculation scores including natural numbers and decimal numbers, and $r = 0.17$ for calculation scores for natural numbers only.

Table 4. Calculated scores by function representation

N	98	16	35
Representation	Incompatible	Logarithmic	Linear
Mean score	10.1	13.8	12.7
Lowest score	0.0	6.0	1.0
Highest score	19.0	19.0	20.0

Table 5. Calculation scores by proportional judgement

N	131	17	1
Model	UPF	1 cycle	2 cycle
Mean score	10.8	13.6	12.0
Lowest score	0.0	0.0	12.0
Highest score	20.0	20.0	12.0

The lower the PAE, the more linear the formation of the mental number line; the more negative the correlation between the PAE and calculation scores, the higher the correlation with the calculation scores. Only the PAE for functional PAE and the calculation scores for natural and decimal numbers combined showed a weak correlation; the rest of the results were close to being uncorrelated.

The weak but negative correlation between mental number line accuracy (PAE) and calculation scores supports the hypothesis that students with more linear representations tend to perform better in computational tasks. However, the overall weak correlation also implies that other factors may mediate this relationship. The higher math score averages for children in 1 cycle and 2 cycle models than for those in UPF model support the hypothesis that the development of number line estimation strategies is associated with appropriate number concept understanding.

Two decimal numbers (10.2 and 85.6) were presented in the number line task. As these numbers were not indicated within the appropriate range on the number line, they were analysed independently, and their trends are shown in Table 6.

The results in Table 6 show that 72% of the students applied their knowledge of natural numbers to capture decimal numbers. For example, Student D's response indicated that 16% of students estimated the answer by treating both the integer part and the decimal part as separate whole numbers, as they had done during the introduction of decimal numbers (see Figure 6).

The findings show a large proportion of students relied on their understanding of whole numbers to interpret decimal values on the number line. This suggests that the conceptual shift from discrete to continuous quantities remains a significant challenge, which may hinder students' overall decimal understanding.

Table 6. Trends in estimates of decimal numbers

Position of the estimated number	*N*	
Other	32	21%
Estimated in 2 numbers (10.2 divided into 10 and 2)	24	16%
Estimate 10.2 or 85.6 on the side of 0	9	6%
Estimate 10.2 or 85.6 as 100 side or more than 100	84	56%
	149	

Figure 6. Example of decimal number on number line

5. Discussions

Based on these findings, this section examines the extent to which the Zambian OBC in mathematics contributes to the development of students' mental number lines and cognition. The discussion is structured from two key perspectives:

5.1 Relationship between numerical representation and computational skills among primary school children in Zambia

In response to RQs 1 and 2, empirical investigations revealed that Zambian sixth- and seventh-graders remained at the basic stage of making proportional and strategic-number judgements. In contrast, second graders in the USA generally exhibit a linear number line, with fourth graders extending this linearity to four-digit numbers (Siegler & Booth, 2004).

Numerical learning generally begins with natural numbers because discrete quantities are more intuitive. However, as mathematical cognition progresses, the concept of continuous numbers (e.g. decimal numbers, fractions, and real numbers) becomes essential for higher-order abstraction. The findings show that many Zambian students struggle to understand continuous numbers, which hinders their development of abstract operations. In particular, 66% of the students exhibited incommensurate numerical representations, suggesting a preference for discrete over continuous numerical estimation.

In contrast to previous studies in other countries (Siegler & Opfer, 2003), no strong significant correlation was found between the formation of mental number lines and computational abilities. The lack of a significant correlation between memorisation and computational skills suggests that Zambian students may rely more on procedural learning than on conceptual numeracy. This is consistent with the observation that classroom instruction often emphasises memorisation and procedural fluency over conceptual number sense (Kabombwe & Mulenga, 2019). This instructional bias may result in students being able to perform arithmetic operations well but lacking the deep numerical understanding required for flexible numerical estimation. Therefore, teaching methods and materials must be improved to integrate the development of numerous concepts and computational procedures.

Observations of Zambian students' explanations revealed teaching methods that relied heavily on counting aids, such as fingers and tally sticks, as well as verbal explanations. This reliance on concrete and discrete methods may hinder students' transition to more sophisticated numerical representations, such as linear rote counting lines.

The results of the textbook analysis also support these limitations of the study. Nakawa (2012) noted that Zambian mathematics textbooks lack visual and concrete representations in favour of symbolic and verbal explanations. A similar pattern was observed in the analysed textbooks (Oxford, 2015), where decimals introduced in Grade 5 were represented with minimal visual aids and relied heavily on verbal explanations. This bias towards symbolic methods constrains students' ability to develop the robust mental models needed to move from concrete to abstract mathematical reasoning.

5.2 Structural challenges in the cognitive foundations of the OBE mathematics curriculum

This study highlights a critical gap in the cognitive foundations of mathematics learning in Zambia. Although students have mastered basic arithmetic skills, their mental number lines remain underdeveloped, hindering the formation of a strong conceptual number base.

Baba (2010) and Nakawa (2012) highlight that the Zambian curriculum systematically introduced basic number concepts such as counting, arithmetic operations and fractions in a spiral fashion, reinforcing learning across multiple grades. However, the current curriculum, although it has these characteristics, tends to prioritise applied knowledge and may not adequately consider the basic development of number cognition. The results revealed that the current curriculum emphasises applied problem-solving skills while neglecting to build a solid foundation for numeracy cognition. This is particularly evident in the limited use of visual and experiential learning materials, such as number lines, in decimal teaching (Nakawa, 2012). The strong correlation between number line proficiency and numeracy estimation skills suggests that incorporating structured number line activities into early mathematics education may deepen students' conceptual understanding of numerical magnitude.

5.3 Implications for curriculum design and teaching materials

Recent studies have emphasized the effectiveness of visual and manipulative tools in supporting decimal understanding. Cramer *et al.* (2015) proposed the grid model as a teaching material, which uses a 10×10 array where one large square represents 1.0 and each small square corresponds to 0.01. This representation enables learners to grasp decimal quantities while maintaining a base-ten structure, and is particularly effective for visualizing operations like addition and subtraction.

Furthermore, Cramer *et al.* (2015) suggest linking the grid model with the number line, thereby helping students bridge the conceptual gap between discrete and continuous quantities (see Figure 7). Such instruction would help Zambian students who rely on discrete representations to understand continuous quantitative representations. Introducing such manipulatives in early grade mathematics may foster structural understanding of number relationships and promote flexible thinking across different representations.

In terms of curriculum design, it is recommended that activities involving both discrete (e.g., unit blocks) and continuous (e.g., number lines) representations be integrated from the early stages of mathematics education. For example, in Japan, a unit called Length, which is studied in the second grade of elementary school, connects discrete and continuous expressions. Although the Zambian curriculum includes number line activities, these are typically introduced in Grade 5 or later. Based on the present findings, we argue for earlier and more structured use of number line-based activities to support learners' developmental readiness and strengthen their conceptual foundation in quantity and operations.

For teacher training, professional development programs should focus on (1) the pedagogical rationale for using visual models such as the grid and number line, (2) how to interpret learner difficulties in representing continuous quantities, and (3) how to scaffold student learning through targeted use of such materials. These elements could form the basis for sustainable in-service training modules or collaborative lesson planning practices (e.g., lesson study).

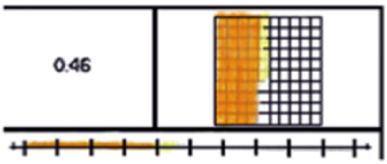

Figure 7. Material of linking the grid model with the number line (Cramer *et al.*, 2015)

6. Conclusion

This study examined in detail how the understanding of numerical concepts develops in Zambian mathematics education through the relationship between mental number lines and computational skills. In particular, this study focused on the development of numerical concepts, identifying learning issues related to the transition from whole numbers to decimals, and the impact of the Outcome-Based Education curriculum on the development of numerical skills. The analysis showed that although Zambian primary school children have basic numeracy skills, much of this relies on procedural understanding, and their conceptual understanding of numbers is not well formed. This trend is also reflected in the lack of a significant correlation between mental number line development and computational skills, suggesting that the emphasis on procedural learning in Zambian mathematics education may inhibit the construction of numerical concepts in children.

The novelty of this study is that it examined how the development of mental number lines is related to computational skills in a developing country's educational setting. Existing studies have discussed the relationship between mental number line development and computational skills based mainly on data from developed countries and have not adequately considered the impact of the learning environment in developing countries. This study is unique in that it analyses how the curriculum influences the understanding of number concepts in the Zambian education system, particularly in that it points out that the OBE curriculum emphasises applied problem-solving skills but may not adequately develop the foundations of number concepts. While previous research has suggested that computational skills are generally associated with an understanding of several concepts, this study showed that this association can be significantly transformed by the educational environment. In other words, the study suggests that in an educational environment where procedural learning is the predominant method, improved computational skills may not be accompanied by a good understanding of number concepts.

The findings of this study have implications for teaching and curriculum design in mathematics. The current OBE curriculum emphasises the development of applied problem-solving skills but may not adequately support the development of basic numeracy. They also suggest that educational policies require improved teaching methods to promote conceptual connections between whole numbers and decimal numbers. In particular, the introduction of visual teaching methods that utilise number lines and teaching materials that promote conceptual understanding may support the development of numeracy cognition in children. For instance, based on the teaching materials of Cramer *et al.* (2015) and examples from the Japanese curriculum, I presented a methodology for developing the cognitive foundation of Zambian learners in this study. Incorporating a more visual and concrete approach to teaching number concepts is likely to contribute to the improvement of Zambian students' numerical understanding. On the other hand, it is necessary to have appropriate discussions on whether it is suitable for society and culture.

In addition, the development of diagnostic assessments and a more detailed assessment of children's number concept development will contribute to future improvements in mathematics education.

This study had several limitations. First, the sample size was limited and may not fully reflect the state of mathematics education in Zambia. Therefore, future research should be conducted on a larger scale, covering a more diverse range of regions and grades in the same study. Although this study focused on the relationship between the mental number line and computational skills, the relationship between the development of reasoning skills and conceptual understanding has not been fully explored. Future research should analyse in more detail how the mental number line contributes to the development of mathematical reasoning and identify effective teaching strategies to support the development of numerical concepts.

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