Students' Paper No. 946.

"A Survey of the Present Position in Road Transition-Curve Theory." †

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Dr. Alexander Thom thought that considerable time could be saved by the use of his Standard Tables *, which had apparently been overlooked by the Author. Those Tables were based on the clothoid, so that identical results should be obtained by their use as by the use of equations (5) (p. 334 §) and (10) (p. 337 §) and other expansions given by the Author. The latter were, in Dr. Thom's opinion, quite unsuitable for routine field work. It was that consideration which engendered the idea of preparing standard Tables in the first place. Facilities for finding X and Y were included, thereby obviating the use of the expansions given on p. 339 §. For the straightforward work, however (such as the examples given in the Appendixes to the Paper), it was not necessary to obtain X and Y at all, since Dr. Thom had given direct expansions for the shift and subtangent, and had prepared a Table for application to all ordinary cases. To illustrate the saving effected by those Tables Dr. Thom applied them to the example in Appendix I (p. 351 §).

The rate of change of acceleration was 1·15 feet per second per second per second. In the notation of the Standard Tables

$$\text{acceleration} = \frac{V^2}{R} = \frac{V^2S}{k^2} = \frac{V^3t}{k^2},$$

and rate of change of acceleration

$$= \frac{V^3}{k^2}.$$

† Journal Inst. C.E., vol. 11 (1938–39), p. 327 (March 1939).

* "Standard Tables and Formulæ for Setting out Road Spirals." Pitman, London, 1935.

§ Page numbers so marked refer to the Paper (Footnote (†) above).—Sec. Inst. C.E.

Therefore $$k = \sqrt{\frac{V^3}{1{\cdot}15}} = 419{\cdot}023.$$

The minimum value of R was $\frac{V^2}{0{\cdot}3g} = 356{\cdot}292$ feet,

and, since $$LR = k^2,$$

then $$L = 492{\cdot}804 \text{ feet}.$$

Using small letters for the basic curve,

$$l = \frac{L}{k} = 1{\cdot}17607,$$

and the angle turned through in the transition was

$$\frac{l^2}{2} = 0{\cdot}691567 \text{ radians} = 39 \text{ degrees } 37 \text{ minutes } 25 \text{ seconds}.$$

Up to that stage the calculation followed that of the Author, the difference being merely in the presentation. For the deflexion angle (Table A, p. 353 §) corresponding to $S = 300$ feet, using again the notation of the Standard Tables,

$$s = \frac{300}{k} = 0{\cdot}71595.$$

With only a simple interpolation, Table I (p. 22 §§) gave directly 4 degrees 53 minutes 32 seconds, which agreed to within 1 second with the Author's value. When the instrument was brought forward to station 300, the back angle was $\frac{s^2}{2} - \phi$, and the forward angle was approximately $\frac{1}{2}sc + \phi_c$, as given by the Author. It should be noted, however, that the forward angle was really $\frac{1}{2}sc + \phi_c - \delta$, where δ was a correction which might, in some circumstances, be quite appreciable. The correction arose from the slight inaccuracy of the osculating-circle method. That was fully discussed in the Standard Tables. In the example the deflexion angle between stations 300 and 450 was

$$\begin{aligned} \tfrac{1}{2}sc + \phi_c - \delta &= (7^\circ\ 20'\ 32'') + (1^\circ\ 13'\ 25'') - 4'' \\ &= 8^\circ\ 33'\ 57''. \end{aligned}$$

§ *Ibid.*

§§ Page numbers so marked refer to Dr. Thom's book (Footnote (*), p. 457.)—SEC. INST. C.E.

The shift and subtangent were easily obtained:

$$\text{shift} \qquad = N = \frac{L^2}{24R} - k \triangle n;$$

$$\text{subtangent} \qquad = (R + N) \tan \tfrac{1}{2}\alpha + \frac{L}{2} - k \triangle m.$$

Table III (p. 27 §§) then gave

$$\triangle n = 0{\cdot}00117 \text{ and } \triangle m = 0{\cdot}00936,$$

the application of which gave

shift = 27·91 feet and subtangent = 731·19 feet,

agreeing with the Author's results. It could be seen that the Standard Tables gave the same results as the Author obtained, in a fraction of the time. Furthermore, they enabled more complicated cases, such as the transition between compound curves, to be dealt with.

Dr. Thom pointed out that the examples were not worked so as to retain the running chainage, and he thought that that did not receive sufficient consideration in Great Britain.

Mr. H. A. Warren referred to the Author's deprecation (p. 328 §) of certain empirical methods used in the design of railway transition-curves. He pointed out that the shift, which might be evaluated from $\frac{1}{24C^2}\left(\frac{V^2}{R}\right)^3$‡, allowing a maximum value of 8 feet per second per second for $\frac{V^2}{R}$, and using values of C recently experimentally obtained *, became negligible. On that basis alone the transition could be dispensed with.

The fact remained, however, that the superelevation had to be gained gradually, and so the length of the transition became, once more, a function of the rate of change of superelevation. The empirical rules regarding rate of gain of superelevation mentioned by Mr. Orchard on p. 328 § were not therefore to be entirely despised. Where, however, the track was for a highway and not a railway there were many other variables to be considered. Upon the entry of a road vehicle into a transition-curve there would be forces arising from vertical accelerations due to the application of superelevation, and also due to rotation about a longitudinal axis consequent upon the increasing angle of banking. In evaluating those,

§§ Footnote (§§), on p. 458.

§ *Ibid.*

‡ H. A. Warren, "Highway Transition Curves." Journal Inst. M. & Cy. E., vol. lxv (1938–39), p. 311 (August 16, 1938).

* H. A. Warren and E. R. Hazeldine, "Highway Transition Curves." Journal Inst. M. & Cy. E., vol. lxv (1938–39), p. 1021 (March 28, 1939).

the width of the road, the method of tilting the surface about one edge or the centre line, the wheel-base, and the track-width of the vehicle would be involved. The accelerations would be present only in the small interval of time between the passage of the front and rear wheels across the line at which the rate of gain of superelevation commenced, and the provision, or otherwise, of a vertical curve there would affect the values. Those were the considerations which set a limit upon the value of C to be adopted, except that at low speeds and with steering gear ratios of about 6 to 1, the rate of turning the steering column might be the limiting factor. Very rarely indeed, if ever, would the rate of gain of radial acceleration itself, *manifested as an increasing centrifugal force*, produce either discomfort or danger so long as the ultimate value of $\frac{V^2}{R}$ was restricted to about 8 feet per second per second.

On p. 329 § the Author stated that Professor Royal-Dawson had provided a graduated empirical formula providing for an increase in the rate of gain of centripetal acceleration for speeds below 30 miles per hour. Recent experimental work had shown the reverse to be the case, borne out by theory on the assumption of a natural rate of turning the steering column being maintained constant at all velocities †. It would be of interest if the Author could state his reasons for having expressed the opinion (p. 330 §) "that Professor Royal-Dawson's values are at present the best compromise." It was unfortunate that such misnomers as "increased manœuvrability" and "standard rate of turning" should be reproduced in the Paper as affecting, or meaning C, the rate of gain of radial acceleration; the former had no scientific meaning, and the latter was a function only of radius of curvature and velocity, and had no necessary connexion with transition-curves at all. Such expressions led only to confusion.

On p. 341 § the Author used the "unit-chord" advocated by Professor Royal-Dawson, and defined as "the length of polar ray whose polar deflexion is 16 minutes." That peculiar and arbitrary unit had its main justification in a slight simplification in the calculations for deflexion angles not greater than 4 degrees; that was to say, only within the limit where the curve would be as accurately and more simply defined by the cubic parabola equation. The natural parameter for use with the lemniscate was the maximum polar ray, at 45 degrees, and the introduction of unnecessary units such as "unit-chords" and "speed-chords" only made a perfectly straightforward subject appear unnecessarily confused.

Mr. Robert Young thought that the Paper was a useful survey of what had been done regarding the design of road curves, and would be useful to engineers who wished to acquaint themselves with the salient

§ *Ibid.* † Warren and Hazeldine, p. 1025. (Footnote (*), p 459, *ante*.)

features of the subject with a minimum of investigation. The calculations set forth in the Appendixes to the Paper would hardly be undertaken by engineers wishing to set out transitions in the field.

It would never be possible to cater for road traffic with the same degree of certainty as for rail traffic. Some attempt, however, should be made to anticipate the conditions for which the curves were to be designed, with a view to approaching, as nearly as possible, that degree of certainty. He advocated the addition to signposts of figures showing the maximum speeds at which the bends should be negotiated. Those speeds would be compatible with the speeds for which the curves were designed and with the nature of the road surface. A distinction should be made between the maximum safe speed for a dry road and that for a wet road, both speeds being displayed on signs at the roadside. He considered that the white line, generally used, should be supplemented by a 6-inch by 4-inch curb to prevent vehicles, whilst negotiating the bend, from encroaching on the wrong traffic-lane. That precaution was especially desirable at sharp bends.

Mr. Young thought that greater superelevations could be employed on sharp curves on main roads in country districts. It would be seen from the expression governing superelevation, that the centrifugal force was directly proportional to the square of the speed and inversely proportional to the radius of the curve. Owing to greater visibility on the outside of the curved carriageway, the speed of vehicles might be appreciably in excess of that of vehicles on the inside half. The superelevation might, therefore, be increased on the outside half of the carriageway. In the case of a curved carriageway with parallel kerbs the superelevational change could be defined by the 6-inch by 4-inch kerb referred to, or, if the outside and inside radii were the same, the two superelevations could be separated by the central reserve given by the consequent widening for carriageways of similar width. Horse-drawn traffic should be discouraged and cyclists segregated completely. Mr. Young pointed out that on the *Autobahnen* in Germany, all traffic other than motor vehicles was prohibited.

In view of the fact that careful calculations had been evolved with a view to defining the road surface in line and level, and methods used to put the results of those calculations into practice, it was Mr. Young's opinion that, in most cases, the conditions, as designed, were only temporary, and other conditions (often dangerous) resulted from settlement of the sub-grade. In order that the surface, as designed, could be maintained as permanent as possible, as much attention should, in the future, be given to the soil mechanics of road formations, as in Germany and the United States.

The Author, in reply, indicated that he was aware of the existence of Dr. Thom's Standard Tables, and agreed with Dr. Thom that the use of Tables did save time and that they were essential for routine field work. He did not consider, however, that those Tables were sufficiently complete for that purpose. He pointed out that the title of the Paper indicated that its sole purpose was to present the pure theory of transition-curves, in the hope that the gaps left by existing Tables would thereby be filled.

The extreme accuracy which might in certain cases be required, such as in the setting out of long tunnels, could hardly be obtained from existing Tables, but was readily available by the method set forth in Appendix II of the Paper. Dr. Thom's method of calculating the first example was very interesting but he had omitted the more laborious parts of the process which recourse to his Tables did not entirely and satisfactorily remove. Dr. Thom's Table I §§ gave the x and y co-ordinates and the polar deflexion angles for certain intervals of length round the spiral, all the linear dimensions being given in terms of a unit and thus requiring transposition to practical dimensions. Unless, therefore, the engineer made, when using those Tables, the same assumption that the Author made in the working of the first example in Appendix I (but not in the working of the example in Appendix II), namely that the chord length and arc length were equal, the inconvenient offset method of setting out was the only one available. Dr. Thom appeared to object to such an assumption on the ground that it did not retain the running chainage. Furthermore, Dr. Thom's book did not give any tabular information of the deviation for the different polar deflexion angles, of the length of the transition for a particular design speed, or of the minimum radius and centrifugal ratio, where those did not reach their limiting values. In those circumstances, the Author could not concede Dr. Thom's claim that his Tables gave the same information as the Author's calculations.

The Author had deprecated, as unnecessary, existing empirical methods for the setting out of transition-curves and, in particular, for assessing their length or the necessary distance for the introduction of superelevation. He would point out that Mr. Warren's value of 8 feet per second per second for $\frac{V^2}{R}$, although suitable for road curves, was quite unsuitable for railway work, and although a reduction of that to the more usual value of 4 feet per second per second would result in a still smaller shift, there was no reason whatever for assuming that the values of C as determined in Mr. Warren's experiments were applicable to railway practice, even they were considered seriously for road work. For that and other reasons it did not appear that his contention that transition-curves could be dispensed with was basically correct. Furthermore, the rate of change of superelevation was a variable which could most conveniently be kept subservient to the theoretical length of transition, and not the reverse as Mr. Warren contended. The entire basis of the modern theory was

§§ Footnote (§§), p. 458.

the necessity for some method of calibrating the sharpness of a transition curve. That was done by using the value of C in conjunction with the design speed of the curve, enabling the scale of the transition-curve to be determined, whether that was measured by the American Degree system or by Professor Royal-Dawson's Unit Chord system. If that method were adopted, the distance over which the superelevation was to be applied could be calculated quite simply from the theoretical principles and without recourse to empirical rules.

For instance, a rate of gain of centripetal acceleration of C feet per second per second per second was equivalent to a rate of gain of centrifugal ratio $\left(\frac{W}{P} = B\right)$ of $\frac{C}{g}$ per second.

If W denoted the width of the road,
v ,, the speed of the vehicle in feet per second,
and k ,, the proportion of the centrifugal ratio which was at any instant to be taken up by superelevation (Professor Royal-Dawson suggested a value of 0·4),

then the rate of gain of crossfall or superelevation was, at any time, 1 in $\frac{g}{kC}$ per second. The final crossfall, however, was to be 1 in $\frac{1}{kB}$, and therefore the time (t seconds) taken to reach that superelevation was

$$\frac{g}{kC} \div \frac{1}{kB} = \frac{gB}{C},$$

and the distance over which the superelevation should be applied was

$$\frac{vgB}{C} \text{ feet.}$$

Furthermore, it could easily be shown that if the superelevation were applied about the centre line of the road, then the gradient of the outside and inside edges of the road would be 1 in $\frac{2vg}{WCk}$, and that might be added, with due regard to sign, to any existing gradient of the road.

The precise effect of the large number of variables mentioned by Mr. Warren as limiting the value of C did not admit of analytical treatment, and recourse had to be made to experiment to determine the value of C which might be adopted. With that in view, and in order to obtain information for a research on skidding, the Author had conducted, before the publication of Mr. Warren's experiments but after the writing of the Paper, a series of tests employing exactly the same method of tracing out the curves as did Mr. Warren. The experiments were conducted mainly with a small car, but a pedal-cycle was also used. It was thought that a pedal-cycle would act as a valuable standard since it was the lightest and the most easily monœuvred of all road vehicles, and possessed the capability of banking and thereby obtaining the effect of perfect superelevation.

The preliminary conclusions arising from those experiments were, to a certain extent, the same as Mr. Warren's, but in the absence of similar experiments conducted with a heavy road vehicle such as a double-decked bus, the Author was of the opinion that it was necessary to exercise rather more restraint in drawing conclusions than Mr. Warren appeared to have done. It was agreed that the possible rate of turning of the steering column might be a limiting factor in fixing the value of C. Against that, however, it was found that the value of C obtained with a pedal-cycle was not generally greater than that obtained with the car in spite of the speed with which the steering wheel of the pedal-cycle, for mechanical reasons only, could be turned. The value of C obtained in comfort did show a general tendency to increase as the vehicle speed increased, but the increase was not merely proportional to the square of the speed, as would be the case if it were assumed that the rate of turning of the steering column remained the same in all cases. The Author agreed, however, unreservedly with Mr. Warren that the value of the centrifugal ratio affected the comfort of turning to a far greater extent than the value of C, but he thought that the figure of 0·25 for the limiting centrifugal ratio could, subject to the possibilities of skidding, be increased materially without causing discomfort. For the value of C the Author had obtained, generally, high values, and could state that a value of 6 feet per second per second per second was quite practicable, although not to be recommended for general use. He had dismissed as unreliable any values much in excess of that figure, but he thought that Mr. Warren had not been quite so reserved. Contrary to Mr. Warren's expressed opinion, he did not find the method of determining experimentally the value of C adopted by both, to be reliable at speeds in excess of 25 or, at the most, 30 miles per hour.

The reason for stating (p. 330 §) "that Professor Royal-Dawson's values are at present the best compromise" was that at the time that that observation was made no experimental data existed beyond Mr. Shortt's work, and in the absence of such it was better to err on the safe side.

The "manœuvrability" of a vehicle depended, amongst other factors, on its weight and its tendency to body-sway, and therefore it directly affected the possible "rate of turning"; the Author could not, therefore, understand Mr. Warren's contention that they were entirely independent of C, the rate of gain of radial acceleration, nor would he accept the statement that the rate of gain of radial acceleration had no connexion at all with transition-curves.

There was no objection to the use of the maximum polar ray of the lemniscate as a unit of measurement, except that it was rather large;

§ *Ibid.*

it was just as arbitrary as Professor Royal-Dawson's unit, which the Author had adopted as he saw neither wisdom in, nor necessity for, adding a further unit to those already in use.

The Author agreed with Mr. Young's suggestion that the safe speed of a curve should be indicated on a signpost at the beginning of the curve; there was, however, a limit to the number of road directions which a driver could be expected to read—a limit which would prohibit the addition of a separate safe speed for wet conditions. The effect of rain in lowering the coefficient of friction was a very variable factor, and the ideal to be attained was a road surface which was equally safe when wet as when dry.