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"Soil Mechanics: The Factor of Safety of Clay Banks."†

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Mr. A. H. A. Hogg observed that the construction given by the Author seemed to lead to the result:

$$\text{Turning moment} \doteq \left\{\frac{1}{\eta} KSr \sin\beta + \frac{S}{C} Wr \frac{\cos\alpha \sin\epsilon}{\sin(\beta - \epsilon)}\right\}$$

where α denoted the inclination of AB to the horizontal, β denoted the angle between OO′ and AB, and

$$\sin\epsilon = \frac{C}{S}\sin\left(\tan^{-1}\frac{\tan\phi}{\eta}\right);$$

$\frac{\tan\phi}{9}$, $\frac{\tan\phi}{8}$, etc. appeared to be misprints for $\frac{2\tan\phi}{10}$, $\frac{3\tan\phi}{10}$, etc.

If, instead of OO′, the perpendicular distance of O′J from O were made equal to $\frac{rS}{C}$, the equation became:

$$\text{Turning moment} = \frac{1}{\eta}\left\{KSr + \frac{S}{C} Wr \cos\alpha \frac{\sin\epsilon}{\sin\beta\sin(\beta-\epsilon)}\right\}$$

and $$\sin\epsilon = \frac{C}{S}\sin\beta\sin\left(\tan^{-1}\frac{\tan\phi}{\eta}\right).$$

Usually in practice, ϕ was small and β did not differ greatly from 90 degrees, so that both equations gave an approximation to

$$\eta = \frac{r(KS + W\cos\alpha\tan\phi)}{\text{Turning moment}},$$

which was the result obtained from the ordinary "curved slip-surface" construction if it were assumed that $W\cos\alpha$ represented the total normal pressure on the sliding surface. It might perhaps legitimately be argued that the assumptions made in estimating the stability of an embankment differed so much from the truth that a few extra inaccuracies did not matter, but it was desirable that the assumptions made in any solution should be clearly recognized.

† Journal Inst. C.E., vol. 18 (1941–42), p. 293 (June 1942).

Apart from its mathematical ingenuity, the new method compared unfavourably with that which involved division of the "curved wedge" into vertical strips [1]. The latter was almost certainly more accurate, and took very little longer than the new method, whilst it could be applied without difficulty to soil which was not homogeneous. Moreover, students found it simple to understand, remember, and apply, whereas the mathematical basis of the new construction was not immediately obvious. That was important, because the proper application of any method depended largely upon whether or not it was really understood by students to whom it was demonstrated. An increase in complexity could be justified only by a gain in accuracy or an appreciable saving in time.

Mr. C. R. J. Wood observed that the matter dealt with in the Paper had already been the subject of very extensive literature, principally on the Continent (Terzaghi, Hultin, Krey, Caquot, etc.). So far, however, no direct solution had been discovered for finding the position and radius of the surface of rupture that offered the least resistance to a movement of the mass, and one had to proceed by the method of successive approximations.

The method of determining the factor of safety set out by the Author was of easy application; it was also more correct than the older method, since it gave the factor of safety on the cohesion and frictional resistance combined, instead of assuming that the whole frictional resistance was taken up first and calculating the factor of safety on the cohesion only.

It would be interesting to know how the Author would modify his method to suit cases where another material overlay the clay—that was, where the surface of rupture was not entirely in homogeneous clay.

The Author, in reply, observed that the expression given by Mr. Hogg for the "turning moment" was evidently intended to give an expression for the "resisting moment," and if such was the case it gave the correct idea for obtaining its value. The perpendicular distance of the line O′J from O was $\frac{rS}{C}$, and by equating the moments about O the value for η could be found from the equation:

$$\frac{KSr}{\eta} + \frac{S}{C}\left\{\frac{\cos\alpha\sin\epsilon}{\sin(\beta+\epsilon)} - \cos(\beta-\alpha)\right\} Wr\operatorname{cosec}\beta = 0,$$

where $\sin\epsilon = \frac{C}{S}\sin\left(\tan^{-1}\frac{\tan\phi}{\eta}\right)\sin\beta$; α denoted the inclination of AB to the horizontal; and β denoted the angle between OO′ and AB.

Substituting the values applicable to the example shown in *Fig. 1* (p. 294, *ante*) it would be found that $\eta = 1{\cdot}63$, which agreed with the rapid graphical construction given.

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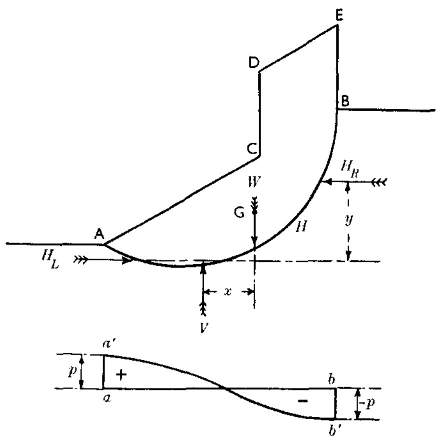
[1] L. F. Cooling and D. B. Smith, "The Shearing Resistance of Soils", Journal Inst. C.E. vol. 3 (1935–36), Engineering Research, p. 342 (June 1936).

The radii of the friction circles given by the graphical construction were given by $r \sin\left(\tan^{-1}\dfrac{\tan\phi}{\eta}\right)$ where π had the values, $\dfrac{1}{0{\cdot}2}, \dfrac{1}{0{\cdot}4}, \dfrac{1}{0{\cdot}6}$, etc.

If the method advocated by Mr. Hogg were applied to an example where W was near the point B in *Fig. 1*, it would readily be seen that that method assumed that a vertical force could be balanced by a force inclined to the right, which was, of course, impossible. That proved that the method given by the Author was more correct than the older method. The Author's method had the additional advantage that it gave the true value for the factor of safety of the bank.

The fact that the study of earthworks was very complex was all the

Figs. 2.



more reason why it should be studied in greater detail, so that progress could be made.

The Author wished to thank Mr. Wood for the interest he had evinced in the Paper, and for the valuable support given to the more accurate method. The Author was in complete agreement with the views expressed by Mr. Wood. For a bank made up of a number of layers of different classes of earth the Author would propose the following modification to his method. On examining the two methods which had been advanced for investigating the stability of such banks, it would be seen that in the first method, due to Petterson and Fellenius, the bank was assumed to be in equilibrium if there was no tendency for the mass of earth above the surface of rupture to rotate, and that in the second method, which was introduced by Hultin, the bank was assumed to be in equilibrium if the vertical components of the forces at the surface of rupture balanced and likewise the horizontal components of the forces balanced. Neither of those methods was quite correct, since the conditions for equilibrium were not completely fulfilled. The additional condition to be satisfied in the first method was that the forces at the surface of rupture should balance, and in the second that there should be no tendency for rotation. The problem could be solved by a combination of the two methods given above, and the introduction of a method for correcting the intensity of pressure at the surface of rupture. In *Figs. 2*, ACDEB represented the bank and superimposed load, AHB the surface of rupture, and G the position of the centre of gravity of the total load W, produced by the mass of earth above the surface of rupture and the superimposed load.

Let H_R denote the sum of all horizontal components of the forces acting from the right on the mass of earth above the surface of rupture;

H_L ,, the sum of all the horizontal components of the forces acting from the left on the mass of earth above the surface of rupture;

V ,, the sum of the vertical components of the forces resisting W;

y ,, the height of the point of application of H_R above the point of application of H_L;

x ,, the horizontal distance between the line of action of V and the line of action of W;

M_C ,, the sum of the moments of the forces tending to rotate the mass of earth above the surface of rupture in a clockwise direction; and

M_A ,, the sum of the moments produced by the frictional and cohesive forces acting along the surface of rupture.

Since the point of application of H_R was usually at a higher level than the point of application of H_L, if the bank sloped upwards towards the right (as in *Figs. 2*), the horizontal forces exerted a couple $H_R y$ which necessitated that the line of action of V should be to the left of the centre of gravity of W, such that $H_R y = Vx = Wx$. That was basically the same principle which was automatically adopted in the ϕ-circle method, which meant that there was a modification to the distribution of the load on the ground to prevent rotation. That resulted in intensities of pressure such as those represented by the ordinates of the curve $a'b'$ in *Figs. 2*, being added algebraically to the pressure produced at the surface of rupture by the weight of earth and superimposed load above the point considered. Adopting the "straight-line" theory of pressure-distribution, as a first approximation, the added intensity of pressure might be considered to vary uniformly from p lb. per square foot at a to $-p$ lb. per square foot at b (*Figs. 2*).

The necessary conditions for equilibrium were that :

$$H_R - H_L = 0 \qquad \ldots \ldots \qquad (1)$$

$$V - W = 0 \qquad \ldots \ldots \qquad (2)$$

$$M_C - M_A = 0 \qquad \ldots \ldots \qquad (3)$$

or

$$H_R y - Wx = 0.$$

and the problem was reduced to finding the values of p, the maximum added pressure on the ground, and η, the factor of safety which enabled those three conditions to be satisfied. That was done by first finding values of $\frac{1}{\eta}$ for which $M_C - M_A = 0$ for a number of different definite values of p, say p_1, p_2, and p_3. Then finding values of $\frac{1}{\eta}$ for which $H_R - H_L = 0$ for the same values of p as indicated in *Fig. 4*. With values of p as abscissas and values of $\frac{1}{\eta}$ as ordinates, the curves $M_C - M_A = 0$ and $H_R - H_L = 0$ were plotted as shown in *Fig. 5*, and the intersection x gave the true values for p and $\frac{1}{\eta}$. The required factor of safety of the bank was therefore given by $\frac{1}{0A}$ in *Fig. 5*.

It had been pointed out to the Author—and he agreed—that it would have been better to use a printed "r" in *Fig. 1* instead of the script "r."

Fig. 3.

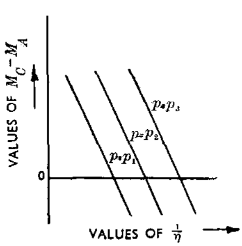


Fig. 4.

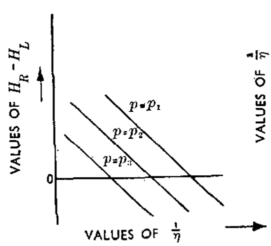


Fig. 5.