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# The influence of amplitude and frequency in the compaction of concrete by table vibration*

by A. R. Cusens, Ph.D., A.M.I.C.E.

**Contribution** by A. G. A. Saul, B.Sc.(Eng.)
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I have read Dr Cusens's paper with interest. I should like to suggest, however, that there is another manner in which the results may be interpreted. In his Figure 4, for instance, he shows a straight-line relation between the amplitude of vibration of the table and the reciprocal of the time taken to compact various mixes. Since in this test only one frequency was used, the Figure to a different scale shows a straight-line relation between the velocity of vibration of the table and the power taken up by the table or, in other words, the effectiveness of the vibration.

In a classical vibratory system the power can be expressed by

$$P = \frac{FV}{2} \cos \phi$$

wherein $F$ and $V$ are the vibratory force and velocity which are related by a constant and $\phi$ is the angle of phase which becomes zero at resonance. If the unknown variable $\phi$ be ignored, the power developed is thus proportional to the square either of the force or of the velocity. There is no suggestion that frequency, as such, or acceleration has any influence.

The above relation is complicated, however, by the fact that concrete will not vibrate at the same amplitude as the vibrator, except in certain circumstances, and for a table, because of the force of gravity, the weight of the concrete must be added to its vibratory force and the velocity due to gravity must be added to its vibratory velocity, the latter addition tending in practice to be proportionally greater. This being the case, it is interesting to see this straight-line relation between the velocity of the table and the power in the concrete, a relation which implies that the velocity in the concrete varies with the square root of that in the table or other source of vibration.(1) In an unpublished report(1) I noted indeed that this was true of internal vibrators but I also pointed out that it is only an approximation over the lower ranges of a family of curves of which these apparent straight lines form parts. The curves are in reality loci of functions of cos $\phi$.

Then there is the conception of a minimum effective amplitude of the table, which may here be regarded as a minimum effective velocity or a minimum effective force in the concrete. This latter makes sense because such a force can be visualized as one which just fails to break the tough skin of a minute air bubble. Such a force, however, can be produced at any amplitude by varying the frequency. Applying theory to various experimental results(2), I arrived at a figure of 0·248 kg.cm/sec.cm$^2$ as the minimum effective power; at the relevant frequency this is developed at an amplitude of 0·0012 in. as compared with 0·0015 in. as shown by Dr Cusens, the former amplitude being in the concrete and the latter in the table, the linkage at such low velocity being very close.

In the other tests the velocity of the table may be calculated by dividing the acceleration by the angular velocity. If these velocities be plotted against the reciprocal of the time, which is really equivalent to plotting them against the power, the straight-line relation at each frequency is not upset, the lines on the Figure are displaced and their relative position and slope altered. They thus become more closely grouped together. Our interpretation of the results has thus to be revised in favour of higher frequencies.

From Series II, mix A, Figure 3, the following results are obtained.

| Acceleration | Velocity of table (cm/sec) | Reciprocal of time of compaction (min$^{-1}$) | Ratio |
|---|---|---|---|
| FREQUENCY : 12,000 vib/min | | | |
| 2g | 1·5 | 0·3 | 5 |
| 6g | 4·8 | 0·9 | 5·3 |
| FREQUENCY : 3,000 vib/min | | | |
| 2g | 6·4 | 0·7 | 9·3 |
| 8g | 25·5 | 2·7 | 9·5 |

It can be seen at once that the linkage or ratio between the vibratory velocity of the table and that induced in the concrete (or related power, or reciprocal of time) is better at low velocities than at high and that it is better with the high frequency than with the low. In the competition the high frequency was not given a chance because the highest velocity of the table at 12,000 vib/min was 4·8 cm/sec, but even at this figure it was able to beat 3,000 vib/min at 6·4 cm/sec.

When it is considered that the powers related to these table velocities are proportional to their squares, it becomes evident that had more power been provided

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at 12,000 vib/min, this frequency would have clearly showed its superiority over the lower frequency. Within these limits, it is actually better than any of the other frequencies except 6,000 vib/min, which is the best probably because it happens to be the nearest to the average resonant frequency throughout the test. This fact is of no great significance since in the test with mix B 12,000 and 8,480 vib/min tie for first place, while 6,000 vib/min is far behind; 3,000 vib/min is a consistent loser and only shows up reasonably in the tests with mixes C and E of which all results tend to be equally poor.

The suggested conclusions to be drawn from these tests are that whereas, other factors being equal, power transmitted to concrete in the form of vibration varies approximately with the vibratory velocity of the source of power regardless of frequency, for vibrating tables certain reservations should be made since a certain frequency may most nearly approach the average resonant frequency throughout the test. In view of the shallow section of concrete involved, such a frequency is likely to be a high one. During the early period of vibration before the concrete has been shaken down, a low frequency and large amplitude is more effective, and a high frequency thereafter. It follows that with drier mixes, which are more difficult to shake down, a low frequency may over a given period be the more effective; a compromise would be a medium frequency.

The most efficient transmission of power takes place at low velocity and this efficiency rapidly decreases as velocity and power increase. Under practical conditions, however, where several kilowatts may be consumed by a table which transmits only a couple of hundred watts to the concrete, efficiency is of small account and the most powerful vibrator, regardless of frequency, will give the quickest results. A very powerful vibrator may, however, under certain conditions, disrupt rather than compact the concrete.

### *REFERENCES*

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2. ibid. p. 21.

## **Contribution** by J. M. Plowman, B.Sc.(Eng.), Ph.D., A.C.G.I., D.I.C., A.M.I.C.E.

Dr Cusens raises several interesting points in his paper. He reports rotational instability for gradings Y, Z (Table 1) and C, E (Table 2); grading Y having a very high proportion of fine material, one would expect instability. Grading Z is really a mortar, and gradings C and E lie in the unstable zone which I reported(1,2) although some deviation may be expected because the maximum size of aggregate for these gradings was ⅜ and not ¾ in.

On page 84 it is stated that rotational instability will cease if the acceleration is reduced and then increased. This occurs in only a few marginal cases; the majority of unstable mixes never settle down. Mixes liable to rotate do not all have the appearance described by Dr Cusens, some looking dry and harsh and others like normal concrete.

Tests made some ten years ago(3,4) show a reduction in the time required for compaction on increasing the acceleration but do not support very closely the contention that an increase in acceleration from 3 to 4½ *g* halves the time required for compaction.

Since Dr Cusens submitted his paper, further work by Farrar and myself at King's College, London, has been published.(5,6)

These publications report a series of tests in which moulds of different size were used on a vibrating table and the accelerations were measured by pick-ups inside the concrete. As a result of these tests a general expression was evolved for the acceleration $f$ at any point distant $x$ from a source of vibration of acceleration $f_o$

$$f = f_o \cosh Nx$$

where $N$ is a constant dependent upon the mix. Such a relation was shown to agree closely with experimental curves. The constant $N$ includes terms such as the viscosity of the concrete. Work is far advanced on (a) determination of a similar law for shutter-vibrated and immersion-vibrated concrete, and (b) the viscosity of concrete and cement pastes. (It should be noted that the word viscosity is used in a loose sense since concrete does not behave as a perfect liquid.)

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5. FARRAR, N. S. Vibration waves in concrete. *The Engineer*. Vol. 206, No. 5354. 5th September 1958. pp. 378–380.
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## **Reply** by the author

I am very glad to have Mr Saul's contribution in view of his extensive work on the theoretical aspects of the vibration of concrete. It is unfortunate that it has not been possible to repeat the tests of series III at other frequencies. In order to obtain the curves of

Figure 4, some 240 cubes had to be cast and tested, so that any extension of the series would be a large undertaking. Without such an extension, I cannot accept Mr Saul's suggestion that Figure 4 shows anything more than an amplitude/time relationship.

Mr Saul's remarks to the effect that the square of velocity is a better criterion of the effectiveness of vibration than the acceleration are very logical. The intensity of vibration at any point within the concrete is proportional to the square of the velocity of vibration at that point and it is reasonable to suppose that the effectiveness of vibration should depend on the intensity, i.e. the power available. However, my own feeling is that the breakdown of the cement paste (the decrease of structural viscosity) under vibration, which is the basic necessity for compaction, must be directly proportional to vibratory force rather than to intensity or power. Within the range of frequencies commonly used in table vibration (3,000—6,000 vib/min), it makes little difference which of the two criteria is adopted. I must agree that if the range is extended to 12,000 vib/min, the results of series II fit more neatly into the pattern if the square of velocity is taken as the criterion of the effectiveness of vibration. More results are required to clarify this point. In the tests of series II, mechanical and electrical difficulties inherent in the apparatus prevented the use of higher amplitudes at 12,000 vib/min. In general, mechanical troubles always arise in table vibration at high frequencies. If it is necessary to maintain constant power at higher frequencies, increasing values of vibratory force are necessary. It may not be clear whether the effectiveness of vibration in concrete depends on acceleration or the square of velocity, but it is certain that fatigue failures in metal parts of the vibrator and table depend upon force and the number of reversals of stress. Moreover, damping increases with frequency—certainly in metal and wood and presumably in concrete.

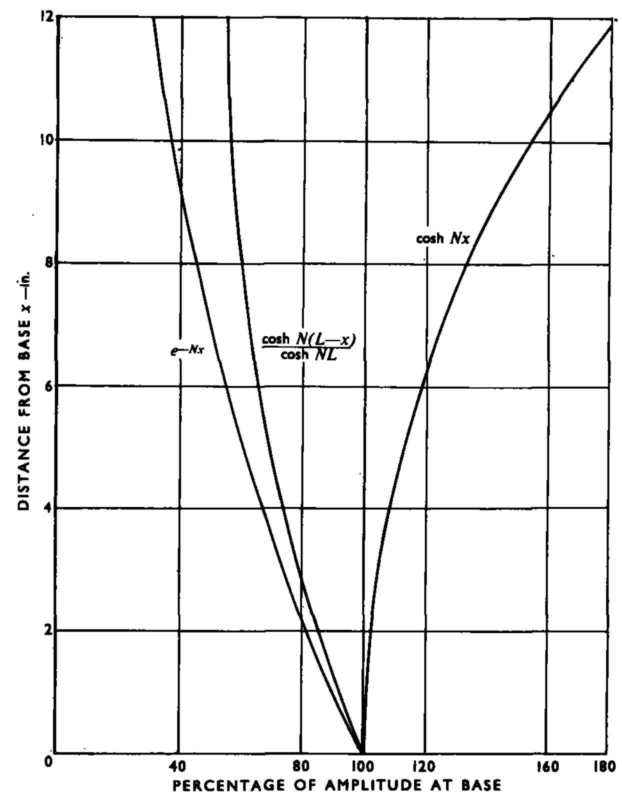

*Figure 1: Relation between amplitude and distance from base of specimen.*

Dr Plowman mentions his recent work with Mr Farrar on the passage of vibration waves through concrete. I have already commented on this elsewhere(1) and can only repeat that the expression

$$f = f_o \cosh Nx$$

quoted by Dr Plowman cannot represent a decay curve if $x$ is measured from the source $f_o$. This is clearly illustrated in Figure I. An analysis of the absorption of vibration waves in a specimen of height $L$ yields the solution

$$f = f_o \frac{\cosh N(L - x)}{\cosh NL}$$

and this function is shown in Figure I, taking $L = 12$ in. If the specimen is assumed to be of infinite height, $f = f_o e^{-Nx}$, an expression which is accurate enough for normal purposes. For simplicity, I have used amplitudes rather than accelerations in Figure I. Since Dr Plowman's tests were performed at a constant frequency, this does not affect the issue.

## REFERENCE

1. CUSENS, A. R. Vibration waves in concrete. *The Engineer*. Vol. 206, No. 5361. 24th October 1958. p. 646.