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Yield criterion for initially isotropic reinforced slab*

by M. W. Kwiecinski, M.Sc.(Tech.), D.Sc.(Tech.) (Warsaw)

Contribution by R. M. Dinnat, M.Sc. *Georgia Institute of Technology*

Dr Kwiecinski proposes a theory postulating the absence of a twisting moment in the yield line. Possibly contrary experimental evidence exists in tests, conducted at the University of Illinois[1], on simple-span slabs loaded by equal line-loads applied at the third points. In such a test set-up, kinematics requires that the yield line remain parallel to the supports and therefore to the applied moment vector. If there were no twisting moment in the yield line, one would expect that the direction of one principal compressive concrete strain would always be perpendicular to the yield line; yet four tests were reported in which it deviated from this direction, thus implying the existence of concrete shearing stresses in the yield line.

I am currently engaged in a testing programme at the Georgia Institute of Technology designed to provide information concerning the necessary conditions for the existence of a twisting moment in a yield line and, if it exists, its order of magnitude. The first phase is concerned with the magnitude and direction of the force induced by an opening slab crack into a reinforcement bar crossing it at a known angle. Preliminary results indicate that the existence of a twisting moment in a yield line is possible, and that the arrangement of bars crossing the yield line and the boundary conditions of the slab are important controlling factors.

I would greatly appreciate further discussion by the author on his postulation of the absence of a twisting moment in a yield line.

REFERENCE

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Yield criterion for initially isotropic reinforced slab*
Some tests on the yield criterion for a reinforced concrete slab†

by M. W. Kwiecinski, M.Sc.(Tech.), D.Sc.(Tech.) (Warsaw)

Contribution by G. M. Mills, M.Sc., A.M.I.C.E., A.M.I.Struct.E. *Bradford Institute of Technology*

I have recently carried out some preliminary tests before embarking on a more comprehensive investigation of reinforced concrete slabs. The object of this preliminary series was to determine the effect of yield lines crossing the reinforcement at varying angles on the yield moment of a reinforced concrete slab section. Like Dr Kwiecinski, I suspected that the simple Johansen criteria were not accurate for this case and that the kinking of the reinforcement could increase the yield moment to a value of μ times the yield moment predicted from the Johansen criteria when the angle was 45°.

In devising an experimental procedure to determine the value μ, the following points were considered.

1. The test should be designed to eliminate secondar effects present in two-way spanning slabs, namely, arching, membrane action, torsional forces, etc. This

*Pages 97 to 100 of *Magazine* No. 51.

†Pages 135 to 138 of *Magazine* No. 52.

suggested a simply supported slab in a rig designed to eliminate torsion completely.

2. As the experimental difference was likely to be greatest when the angle between the yield line and the bars was 45°, the tests were arranged to achieve such an angle.

3. The effects of steel reinforcement both with and without a clearly defined yield stress should be considered.

4. The results of the test with " angled " bars should be compared with the results of the tests on similar beams with straight bars. A number of investigators have found[1] that typical values of M_{test}/M_{theory} (where the theory is a rational one based on the equilibrium of internal forces) range between 1·0 and 1·2. Where the number of variables is small, the variance may be much smaller and a coefficient of variation of less than 5% can usually be expected. To obtain an approximation for the coefficient of variation, four beams (1–4) were made as similar as possible.

The shape of section adopted for the tests and the arrangement of reinforcement in the slabs with angled bars are shown in Figure I. The slabs were tested over a span of 48 in. with two-point loading applied 3 in. to each side of the centre. The load was applied by a 50-ton Denison testing machine in the 5-ton loading range (reading to nearest 0·005 ton). Initially, the load was increased in steps of 0·1 ton but, as the ultimate load was approached, the increments were reduced to 0·01 ton at intervals of not less than 4 min. The average time taken for each test was 1¼ h. Enough readings of deflexion were taken to enable the total rotation over the central 6 in. to be deduced. For the steel with a clearly defined yield, the rotation at failure was approximately 0·015 rad; for the steel with no defined yield, the rotation at failure averaged 0·03 rad.

Detailed results for the beams tested are given in Table I. The theoretical moment was calculated by using a simplified formula suggested by Thomas[2].

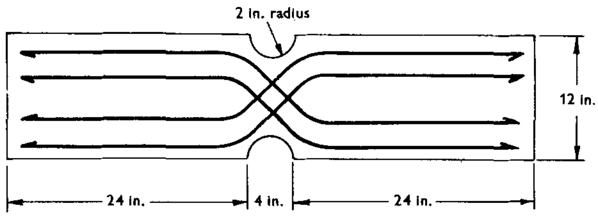

Figure 1

For the steel with no clearly defined yield, the equivalent yield for calculations was taken as the 0·2% proof stress.

If it is assumed that μ is constant for these test beams, a statistical analysis appears to indicate that the probability of μ exceeding 1·1 is greater than 50 to 1 against.

Although the results of the tests reported by Dr Kwiecinski in his second paper appear to show that μ has a value between 1·15 and 1·2, his calculated values are based on the assumption that at least one of the bars crossing a yield line is ineffective and contributes nothing to the strength of the slab. However, I have shown[3] that this assumption is incorrect and that the resistance of a bar which is curtailed close to a yield line is mainly a function of the " ultimate bond stress " of the bar. From the information given by Dr Kwiecinski, the embedment required to achieve a bond resistance equal to the yield strength of the bar would probably not exceed 3 in.

A study of Figure 2 of the second paper indicates that the yield line crossed a total of eight bars. I therefore suggest that the observed pulling-out of one of the bars was a secondary consequence of failure and that the effective width of the slab used in the calculations be amended to allow for the full yield resistance of this bar. If this is done μ would decrease to 1·04 for $\alpha = 45°$, and presumably a similar decrease would obtain for the other angles. In assessing whether these

TABLE I

Beam	Type of steel	Bar pattern	Average d_t (in.)	u (lb/in²)	T_y (lb)	M_{test} (lb. in. × 10³)	M_{theory} (lb in. × 10³)	$\frac{M_{test}}{M_{theory}}$	Remarks
1	cold-worked	crossed	1·71	5,600	8,500	11·5	9·65	1·19	
2			1·67	7,000	8,600	12·2	9·85	1·235	angle 43½°
3			1·86	5,950	8,600	12·7	10·7	1·195	angle 46°
4			1·97	5,150	8,400	12·2	11·05	1·12	angle 46°, slab damaged before test
5		straight	1·95	5,700	8,900	18·4	15·9	1·16	
6	annealed	crossed	1·67	5,800	13,250	14·4	14·0	1·03	angle 45½°
7		straight	1·95	5,950	11,100	20·5	19·5	1·05	
8			1·92	6,100	12,100	22·6	20·9	1·085	

revised values are (statistically) significantly greater than 1·0, details of the variance of the values for each individual test would be required and it is to be regretted that such information has not been included in the published results. Owing to the uncertainty about the contribution of the reinforcement in the type of slab tested by Dr Kwiecinski in which yield lines pass very near to the ends of curtailed bars, it seems to me that this type of test is unsatisfactory for the particular problem investigated. Such uncertainty is virtually eliminated in the method of test which I adopted (Figure I).

Attention may be drawn to the difficulty of proving the nullity of Dr Kwiecinski's hypothesis particularly for suggested low values of μ. To show that μ lies within a range of $\pm 0{\cdot}05$ of a given value with a probability of 100 to 1 would require several times the number of tests that I have carried out. The range of values used in these tests for T_y/ubd_1 was 0·07 to 0·15 and, although it seems unlikely that higher values of T_y/ubd_1 would show different values for μ, this possibility remains to be investigated.

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3. MILLS, F. G. *The effect of bar cut-off on the ultimate load of reinforced concrete beams.* Thesis submitted to the University of Manchester for the degree of M.Sc. 1965. pp. 148.

Reply by the author

Mr Dinnat and Mr Mills are thanked for their interesting contributions. As I have not read Houbolt's thesis it is difficult to comment on it. From what Mr Dinnat says, it would seem that the middle third of a slab was subjected to pure uniaxial bending (homogeneous distribution of moments). Thus the direction of yield lines in this area might be to some extent accidental, e.g. because of certain defects of the models. In other words this domain could have been in a sense " unstable ".

My postulate of the vanishing of twisting moment on a yield line was an attempt to strike a balance between the demands for simplification and the requirements of strict mathematical limit analysis of slabs, and to produce a yield criterion which would not be at variance with present-day limit analysis (Johansen's is) and which would at the same time be simple: hence the decision to introduce a concept of partial kinking and to make the extent of it dependent upon an assumption that there should ideally be no twist on yield lines. This means that a yield line is generated in a direction of maximum principal moment. In particular, when one proposes to construct consistently a yield condition (yield surface) in the form $F(m_x, m_y, m_{xy}) = K$, it appears reasonable and neat to assume that at the generic point in a slab the direction of the principal moment coincides with the yield line itself. This avoids the known contradiction that, on the one hand, a yield line is formed because of the occurrence of a maximum (principal) moment of some critical magnitude and that, on the other, the state of ultimate moments on a yield line is not " twistless ".

Mr Mills suggests that my values of ultimate moments were based on the *assumption* that some bars near the edges of the slab had been ineffective. Actually, it was not an assumption but a *fact* ascertained from the inspection of their behaviour during the tests; it was thus not a secondary effect of failure but rather an essential feature. It therefore appeared reasonable to exclude the contribution of these curtailed bars to the total value of ultimate moment on the yield line. However, I agree that this type of reinforcement, although perfectly realistic as far as the actual structure is concerned, did necessitate amending the ultimate moments obtained from the tests in the manner carried out in the paper. Mr Mills's arrangement of bars appears to overcome this difficulty and enables one to discuss the test values directly.

The statistical approach was not applied in the preliminary programme: it was felt that the number of tests was too small to be dealt with statistically.

That my yield criterion is justified (amongst other respects, in the sense of the absence of a twisting moment on a yield line) and that, at the same time, there is a defect in the Johansen yield criterion, can be demonstrated by a simple test which need not be actually carried out. Consider a rectangular, orthotropically reinforced slab (Figure II) subjected to simple bending by means of a central line load. According to the kinematical boundary conditions as well as all the requirements of equilibrium, a yield line should be generated parallel to both supports, and the value of twisting moment clearly ought to be zero. But, if the directions of reinforcing bars are not parallel to the sides (i.e. inclined at α and $90° - \alpha$ to the given edge), the non-vanishing value of twisting moment $m_{nt} = m\,(1 - \varkappa) \sin\alpha \cos\alpha$ is, according to the Johansen " rectangular " yield criterion, automatically involved ($\varkappa$ is the coefficient of orthotropy). This shows that, in this simple case, the Johansen criterion breaks down, apparently violating the equilibrium conditions of the problem. The partial-kinking yield criterion is able to dispose of this built-in shortcoming of the " rectangular " yield criterion. It would be perhaps difficult to maintain that an attempt to establish an improved yield criterion was not worth making.

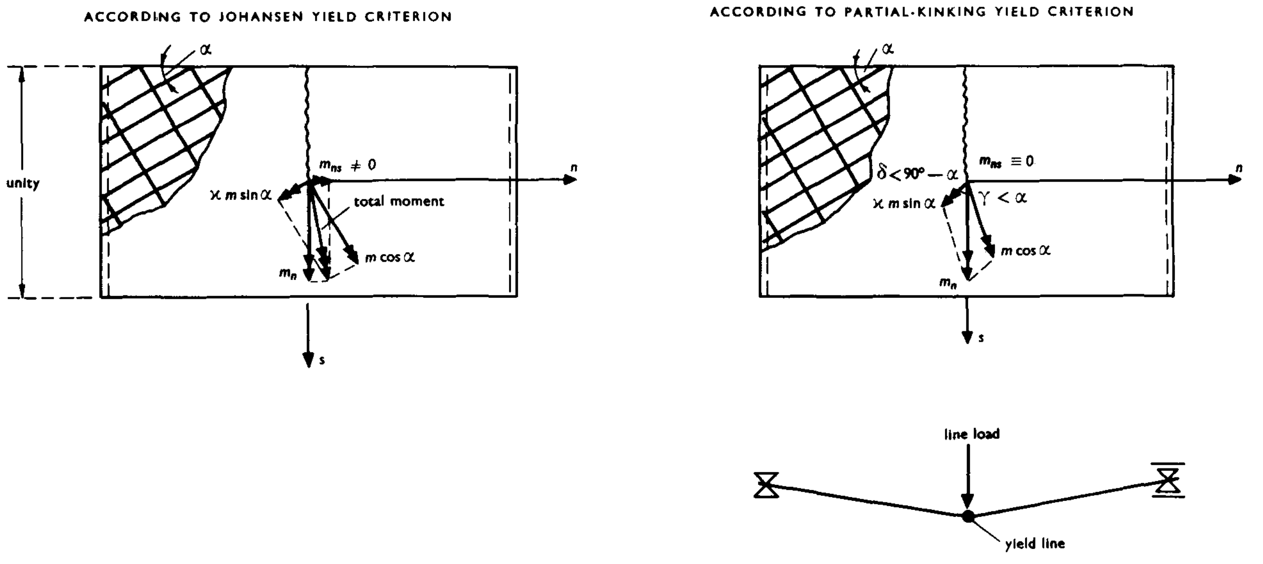
ACCORDING TO JOHANSEN YIELD CRITERION
ACCORDING TO PARTIAL-KINKING YIELD CRITERION
α
unity
$m_{ns} \neq 0$
$\varkappa\, m \sin\alpha$
total moment
$m \cos\alpha$
m_n
n
s
α
$m_{ns} \equiv 0$
$\delta < 90° - \alpha$
$\gamma < \alpha$
$\varkappa\, m \sin\alpha$
$m \cos\alpha$
m_n
n
s
line load
yield line

Figure II