The influence of cement paste content on the creep of lightweight aggregate concrete*

by S. E. Rutledge, BSc, MSc, MEIC
and Professor A. M. Neville, MC, TD, DSc(Eng), PhD, MSc, MICE, MIStructE, MEIC, MASCE

Contribution by H. G. Geymayer, Dr techn, Dipl Ing
U.S. Army Engineer Waterways Experiment Station

The authors are congratulated upon their very interesting extension to lightweight aggregate concrete of Neville's rational creep concept[1] and upon the agreement they achieved between the test results and empirically derived expressions for time-dependent creep as a function of cement paste content.

To stimulate discussion, a question will be raised here regarding the theoretical validity of their formulae 1 and 2. In their original version both formulae were derived by Pickett[2] for the case of shrinkage in a theoretical study of the effect on shrinkage of one small spherical particle of aggregate in a large body of concrete, the surrounding concrete being considered to be a homogeneous material. On the assumption that both the particle and the rest of the body are linearly elastic, Pickett derives an expression for the reduction in total shrinkage of the body due to the one small embedded non-shrinking particle. Neville, in his 1964 paper[1], tentatively applied the same solution to the more complicated case of creep. However, when Pickett's approach is being used, I feel it should be remembered that creep is very stress-dependent, whilst shrinkage is not. Not only will the enclosed non-creeping particle restrain the total contraction of the body, thereby reducing creep, but it will also have an effect on the load-induced stress distribution and the magnitude of stresses in the surrounding concrete and thus doubly influence creep.

To evaluate tentatively this double influence upon creep in the simple case of hydrostatic loads, we will follow Pickett's route and consider the small spherical particle embedded in the centre of a larger concrete sphere, which is exposed to a uniform external pressure p_o representing load-induced stresses that cause creep.

The restraint of the small sphere as the large sphere tends to contract elastically under the hydrostatic pressure will cause the following stresses in the large sphere[3].

$$\sigma_r = \frac{p_o b^3(r^3 - a^3)}{r^3(a^3 - b^3)} + \frac{p_i a^3(b^3 - r^3)}{r^3(a^3 - b^3)} \quad \ldots\ldots\ldots(1)$$

$$\sigma_t = \frac{p_o b^3(2r^3 + a^3)}{2r^3(a^3 - b^3)} - \frac{p_i a^3(2r^3 + b^3)}{2r^3(a^3 - b^3)} \quad \ldots\ldots(2)$$

where σ_r = normal stress in the radial direction
σ_t = either of two normal stresses perpendicular to the radius
r = radial co-ordinate
a = radius of inner sphere
b = radius of outer sphere
p_o = unit pressure on outer face of the large sphere
p_i = unit pressure between inner and outer spheres

Radial displacement δ of any point in the outer sphere is:

$$\delta = \frac{r}{E}\left[(1 - \mu)\sigma_t - \mu\sigma_r\right] \quad \ldots\ldots\ldots\ldots(3)$$

where E and μ are Young's modulus and Poisson's ratio, respectively, of the outer sphere.

If the radial displacements of the inner and the outer spheres are equated for $r = a$, it can be shown that:

$$p_i = \frac{3p_o(1 - \mu)}{\left[2\frac{a^3}{b^3}(1-2\mu) + 1 + \mu\right] - \frac{2E}{E_a}(1-2\mu_a)\left(\frac{a^3}{b^3} - 1\right)}$$

or if $a >> b$ so that $a^3/b^3 \doteq 0$

$$p_i = \frac{3\,p_o\,(1 - \mu)}{1 + \mu + 2\frac{E}{E_a}(1 - 2\mu_a)} \text{ or } p_i = \alpha p_o \ldots\ldots(4)$$

where

$$\alpha = \frac{3(1 - \mu)}{1 + \mu + 2\frac{E}{E_a}(1 - 2\mu_a)}$$

*Pages 69 to 74 of *Magazine* No. 55

The unit pressure around the enclosure is, therefore, α times the applied uniform external pressure on the outer sphere. Returning now to Pickett's and Neville's train of thought, we can compute the reduction of the volume creep ΔCVp_o due to the restraining effect.

$$-3\Delta CVp_o = \frac{3\bar{p}\Delta V}{E}\left(\frac{1-\mu}{2}\right)\frac{3b^3}{b^3-a^3}\ldots\ldots(5)$$

where $\Delta V = 4a^3\pi/3$ and $\bar{p}$ is the unit pressure between the inner and the outer spheres caused by the restraint to creep.

Assuming that creep is a linear function of the stresses, we may then compute the reduction of the available space for the inner sphere if an unrestrained creep deformation had been possible. The radial displacement δ_c due to creep $\delta_c = rC[(1-\mu)\sigma_t - \mu\sigma_r]$, where C is a creep coefficient and for $r = a$, $\sigma_r = -\alpha p_o$ and $\sigma_t = -p_o(3-\alpha)/2$

$$\delta_c = -\frac{acp_o}{2}[3(1-\mu) - \alpha(1+\mu)]\ldots\ldots(6)$$

and, if we further assume that $\mu = 0$ for creep,

$$\delta_c = -\frac{acp_o}{2}(3-\alpha) = -p_o ca\beta\ldots\ldots\ldots(7)$$

where

$$\beta = \frac{3-\alpha}{2}$$

Equating the reduction in volume of the particle caused by pressure $\bar{p}$ on it to the total reduction of space available to it within the larger body

$$\frac{3(1-2\mu_a)p\Delta V}{E_a} = 4\pi a^2(\delta_c - \delta_{\bar{p}})1\ldots\ldots\ldots\ldots\ldots(8)$$

where E_a amd μ_a are the elastic constants of the particle and $r = a$, we get from equations 7 and 8

$$\Delta CV = \alpha\beta Cp_o\Delta V\ldots\ldots\ldots\ldots\ldots(9)$$

Inserting this into Pickett's or Neville's further derivation, we finally arrive at

$$\log\frac{C_p}{C} = \alpha\beta\log\frac{1}{1-g-u} = n\log\frac{1}{1-g-u}$$

with

$$n = \frac{3\alpha-\alpha^2}{2}$$

or

$$C = C_p(1-g-u)^{\alpha\beta} = C_p(1-g-u)^n\ldots(10)$$

representing the creep term for a hydrostatically loaded body of concrete, which is, of course, not the normal case at all.

For the regular case of a uniaxially loaded concrete specimen, the effect of the enclosure upon the (elastic) stress field of the surrounding concrete and its restraining effect on creep deformations are not so readily obtained. No attempt at a solution will be made here. However, it seems obvious at this point that the exponent n in Neville's creep equation $C = C_p(1-g-u)^n$ can hardly correspond to Pickett's term,

$$\alpha = \frac{3(1-\mu)}{1+\mu+2\frac{E}{E_a}(1-2\mu_a)}$$

but has to assume a far more complex form, depending upon the loading conditions for the specimen. This is felt to explain some surprising results for the modular ratios that the authors calculated on the basis of $n = \alpha$, e.g. the dramatic decrease of E/E_a with time under load in the wet-cured specimen.

In view of the restricted knowledge that we presently have on μ, μ_a and E_a, it is indeed of little practical significance whether the exponent n can be satisfactorily approximated by a simple or a complex function of the elastic properties of the concrete and the aggregate and/or what the function looks like. In practice the exponent in Neville's creep equation will be experimentally evaluated, and at this point its relation to elastic coefficients is of academic interest only. Knowledge will hopefully increase, and the question may become more important in the future.

Reply by the authors

Dr Geymayer's contribution is an interesting development in the field of stress and deformation within concrete, and we hope that he will extend it to cases of more practical loading than a hydrostatic load. One or two points may, however, be made. First, the difference between the solution for creep and for shrinkage is not as large as suggested by Dr Geymayer. Although, as he says, shrinkage is not stress-dependent, restrained shrinkage (as is the case with a non-shrinking aggregate particle) induces stress and thus affects the behaviour of the system. Admittedly, the non-elastic behaviour of the surrounding material (concrete) has to be taken into account. In our case, this was done by reducing the instantaneous value of E; the method was explained in an earlier paper[1]. We feel that this is no less justified than some of the assumptions made by Dr Geymayer, as for instance that $\mu = 0$ for creep. Tests on creep under triaxial stress, conducted at Calgary, show that this is far from true.

In any case, there is little difference between the numerical values of α and of n. Considering wet-stored concrete after 28 days under load, $\alpha = 1{\cdot}03$ and $n =$

1·02. For long periods under load, taking $\alpha = 1{\cdot}25$, we find $n = 1{\cdot}10$. As Dr Geymayer says, α is determined experimentally, and we found our approach to work well for concrete made with normal weight aggregate[1], lightweight aggregate (the paper under discussion), and, more recently, semi-lightweight aggregate[4].

We claim no more than that the method used is of help in determining the influence of composition of the mix upon creep.

REFERENCES

1. NEVILLE, A. M. Creep of concrete as a function of its cement paste content. *Magazine of Concrete Research*. Vol. 16, No. 46. March 1964. pp. 21–30.
2. PICKETT, G. Effect of aggregate on shrinkage of concrete and a hypothesis concerning shrinkage. *Journal of the American Concrete Institute. Proceedings* Vol. 52. 1956. pp. 581–590.
3. TIMOSHENKO, S. and GOODIER, J. N. *Theory of elasticity*. Second edition. New York, McGraw-Hill Inc., 1951. pp. 359.
4. WARD, M. A., JESSOP, E. L. and NEVILLE, A. M. *Some factors in creep of lightweight aggregate concrete*. RILEM Symposium on Lightweight Aggregate Concretes, Budapest, March 1967.