Discussion on a paper published in the *Magazine of Concrete Research* Volume 20, Number 62 : March 1968

Fatigue of concrete and its effect upon prestressed concrete beams*

by F. Sawko, MSc and G. P. Saha, MSc, PhD

Contribution by E. W. Bennett, MSc, PhD, MICE, MIStructE
University of Leeds

Information on the fatigue of prestressed concrete beams is still so limited that it may seem ungrateful to criticise any new work, particularly that of two former colleagues. One cannot, however, refrain from questioning the authors' conclusion that the ultimate load improves after repeated loading up to 50–60% of the ultimate strength, and the way in which it has been reached. It is based upon their own five fatigue tests together with previous work. Among the previous work quoted, however, is that of Rimmer (reference 6 of the paper), who tested beams of a type similar to the authors' beam No. 1 in the same laboratory and observed no significant difference in strength in 12 beams tested at various levels of repeated loading. It is essential to allow for the considerable scatter of fatigue results, particularly in concrete, and the ratios in the last column of Table 3, with the possible exception of 2(iii)/2(i), lie well within the range for the usual order of standard deviation in static tests, not to mention fatigue tests.

The authors explain their rather incautious conclusion by hardening of the compressive zone and softening of the tensile zone, resulting in an increased lever arm at failure. The writer suggests that the comparatively low level of repeated compressive stress could not have caused more than a slight modification in the shape of the stress distribution of the much more highly stressed compressive zone at failure. The form of this modification may be appreciated from the typical compressive stress-strain curve for concrete illustrated in Figure 1. The modulus of elasticity increases rapidly with the first repetitions of stress and was found in tests by Muir and Bennett[1] to become stable after about 300,000 cycles. On a single loading to higher stresses, however, the stress-strain relationship reverts to the original curve, sometimes termed the maiden line. Thus, in a beam at failure, the increase of the stress gradient is restricted to the lower part of the compressive zone. Comparing this with the stress distribution for a single loading to failure (Figure 1), it will be seen that a greater length of the falling branch of the stress-strain curve is required to maintain the area of the stress diagram to balance a given tensile force in the bottom steel. Whether the lever arm, and hence the ultimate moment, increases or decreases depends therefore upon the shape of the curve, which is very variable, but the effect either way is likely to be small.

In Figure 5 the authors show the flexural strains due to the cyclic loading; it should be made clear that, on account of the prestress, the neutral axis of stress is much lower than the neutral axis of strain in this diagram, and probably no more than one inch from the bottom. In most of the lower half of each beam, the concrete would have been alternating between two levels of compressive stress and the influence of any 'softening', as opposed to actual cracking of the tensile zone, would be very small.

It must be pointed out that the load factor specified in the British Code of Practice CP 115:1959 (Clause 309) has a *maximum* and not a minimum value of 2 as stated by the authors, and may lie between 1·5 and 2 according to the ratio of live load to total load.

*Pages 21 to 30 of *Magazine* No. 62.

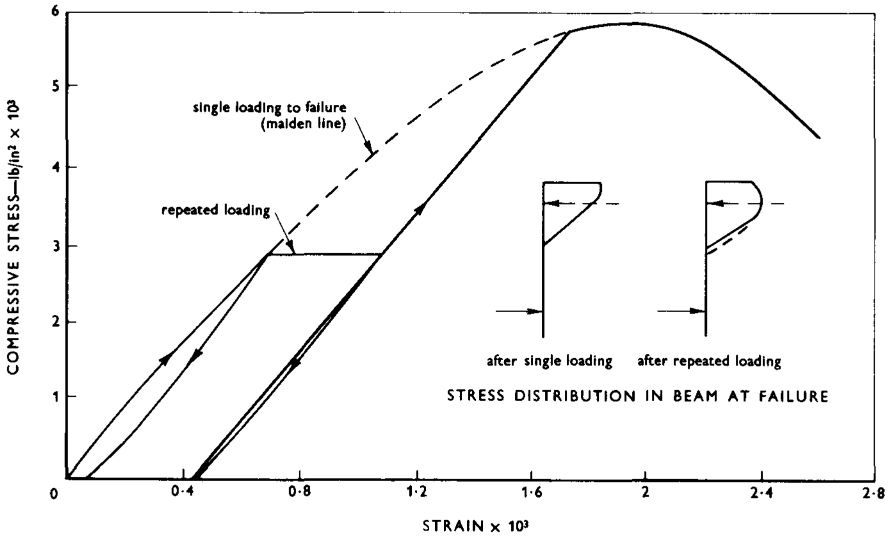

Figure 1: Typical stress-strain curve for concrete.

Contribution by N. K. Raju

University of Leeds

The simultaneous loading of two beams by a single dynamic jack does not necessarily ensure equal load distribution between the beams, because the stiffness and deformations are not likely to be exactly the same for both beams during the course of the fatigue test, and there is probably some restraint due to friction in the loading arrangement. Were any measures adopted to verify the actual load applied to each beam?

The equivalent diagram shown in Figure 6c implies that the flexural rigidity (*EI*) of the concrete section has been increased as a result of the cyclic loading. This contradicts the load-deflexion diagrams presented in Figures 4 and 8, where the fatigued beams have a lower stiffness than they did before the start of the fatigue tests.

In dealing with the mechanism of fatigue in concrete, the authors contend, without any direct experimental evidence, that repeated loading, resulting in permanent set of concrete, causes a kind of solid-body compaction leading to an increase in strength and stiffness. Recent experiments conducted by the writer on 122 plain concrete prisms, subjected to repeated axial loads of up to 50% of the static strength(2), have revealed that the modulus of elasticity was not significantly different after a million repetitions, although the material became more elastic in the sense of exhibiting a much narrower hysteresis loop. A marginal beneficial effect upon the static strength of 4·3% was observed which is believed to have been due to the more rapid drying of the fatigued specimens.

In the absence of any significant increase in the stiffness of fatigued concrete, the prevailing mechanism underlying the more elastic behaviour of concrete after repeated loading appears to be the redistribution of some of the internal load from the mortar to the harder aggregate. This does not imply an increase in over-all stiffness or strength in contrast to the hypothesis of solid-body compaction proposed by the authors.

Contribution by C. S. Chandrasekhar

University of Leeds

The conclusion that "Since a load factor of at least 2 is specified (in the Code of Practice for prestressed concrete), fatigue is never likely to be a design criterion from the strength point of view" is not fully justified, and it is desirable to qualify any conclusions on fatigue strength by stating the degree of effective prestress. If a prestressed beam has a low effective prestress, as may happen with a partially prestressed (FIP-CEB Class III) beam, there may be extensive cracking, though of limited width, of the beam even at a working load of half the ultimate load. Under these conditions the steel is stressed to a very great extent, and there is the possibility of bars fracturing under fatigue. In one recent test by the writer, a 6 × 12 in. beam with a low effective prestress of 260 lb/in² at the tension edge was subjected to a repeated loading cycle varying from $\frac{1}{4}\,u_w$ to $\frac{1}{2}\,u_w$ at the rate of 250 c/min. It was observed that, during the first 2 million cycles of loading, the elastic deflexions increased by nearly 10% for each million cycles of loading, evidently owing to extensive cracking which became progressively deeper and reduced the stiffness of the section. The beam failed by fracture of the untensioned bars after sustaining nearly $4{\cdot}6 \times 10^6$ repetitions, at a load of between 2800 and 5000 lb, whereas the ultimate load of a corresponding beam was 11,500 lb.

The authors' conclusion that excessive deflexions could be eliminated by re-stressing the beams during the maintenance period is not borne out by their tests. A comparison of the deflexions of beams 2(iii) and 2(iv) before fatigue and after re-stressing (Figure 8) reveals a general increase of the deflexions by as much as 23% [beam 2(iii)] to 30% [beam 2(iv)]. Re-stressing would have little effect upon the residual deflexion.

The term 'creep softening' used by the authors is not well defined but might be accounted for by micro-cracking, which would result in reduced stiffness of the tension zone. In this connexion the writer would like to know the method of crack detection and measurement employed by the authors in the tests. Although it is probable that visible cracking had not occurred at the stated loadings, it cannot be said that there was no micro-cracking (which occurs at a tensile stress much lower than the modulus of rupture). This extensive micro-cracking could explain the rise in the neutral axis of the beam (i) after fatigue (Figure 5).

The strain values in Figure 5 appear incredibly large; should the multiplier be 10^4?*

Reply by the authors

The authors are grateful to Dr Bennett and Mr Raju and Mr Chandrasekhar for their comments. They would, however, like to emphasize that their explanation of the fatigue mechanism of concrete was not based solely upon their own work. They used the results of previous work on fatigue which was carried out by different research workers in this field. On the basis of the extensive literature available, the authors formulated their hypothesis for the mechanism of fatigue of concrete, and designed a series of tests which enabled a direct check on this hypothesis to be made. The authors' contribution does not therefore lie in the experimental field of study but rather in suggesting a hypothesis which seems to fit in with all the observed phenomena of plain concrete and reinforced and prestressed beams subjected to fatigue loading.

The authors accept Dr Bennett's comments that concrete is a variable material, but point out that the specimens were cast and all the tests carried out under strict laboratory conditions where the variability is not expected to be very much at random. According to the authors' hypothesis, only beam 2(iii) was supposed to demonstrate an appreciable increase in strength due to fatigue whereas the other beams were not expected to demonstrate such a large increase. The authors would therefore hesitate before putting down this increase to the effect of variability in concrete.

Dr Bennett does not give the source from which his Figure I is reproduced. Dr Bennett's stress-strain curve suggests that fatigue does not play any part at all in increasing the ultimate stress of plain concrete and the authors therefore cannot agree with this stress-strain diagram presented. The authors would refer Dr Bennett to his own paper[1] dealing with the fatigue tests of plain concrete prisms and the following paragraph is quoted from his paper:

"The 'run out' specimens which did not fail in fatigue were tested statically and showed a consistent increase in strength over control specimens which had not undergone repeated loading. In eleven tests, the percentage increase of strength varied between 1 and 23 with an average increase of 11%. This beneficial effect of understressing has been observed by a number of experimenters, but no satisfactory explanation appears to have been offered."

Dr Bennett himself has discovered the increase in strength of plain concrete specimens subjected to fatigue and his findings therefore invalidate the stress-strain diagram reproduced here.

Furthermore, the increase in the ultimate load of both reinforced and prestressed concrete beams observed by various experimenters was reviewed in the original paper. An increase in strength of up to 43% in reinforced concrete beams failing by yielding of steel was reported in one case[3].

The authors clearly stated that creep softening takes place only when tension develops in the specimen. It is now established that a smaller load is required to produce the same width of crack in a fatigued beam than that required for its non-fatigued companion. This phenomenon of lower resistance to cracking due to fatigue creep has been called by the authors 'creep softening'. 'Micro-cracking' is a term used to express the actual width of crack in a concrete specimen, whereas 'creep softening' is used to express the cracking resistance of the concrete in relation to its non-fatigued state. Thus, in the authors' opinion, in creep-softened concrete, micro-cracking will develop at a lower load than in virgin concrete. They hope that this explanation answers Mr Chandrasekhar's query.

The British Code of Practice CP 115:1959 surely recommends a minimum load factor of 2 for live loading, and it is the live load which causes fatigue loading in a structure.

The authors feel that Mr Chandrasekhar has failed to note the contents of clause 303b(ii) of the British Code of Practice for Prestressed Concrete (CP 115: 1959) dealing with the permissible tensile stresses in bending. The Code specifies the permissible tensile stresses under maximum working load, the values of which are such that a no-cracking condition under normal working load is always ensured. Thus, for a beam designed under CP 115, the development of 'extensive cracking' is irrelevant. Partial prestressing is not within the scope of CP 115:1959, but the possibility of fatigue fracture on steel at a badly cracked section due to fatigue loading has already been discussed in the authors' paper in the section 'Summary of previous findings'.

All the beams of group 2, with the exception of beam 2(i), were subjected to fatigue after cracks had

*Yes. Ed. See Corrigenda on page 175.

developed and re-stressing was carried out after fatigue. Owing to propagation of cracks deep into the section, and the development of further cracks during fatigue, the stiffness of these beams after fatigue was reduced. This explains why an increase in deflexion was noted even after beams 2(iii) and 2(iv) had been re-stressed. It would be wrong, however, to state that "Re-stressing would have little effect upon the residual deflexion". In a simply supported beam, re-stressing to the original force would produce a deflexion which would depend upon the eccentricity of the prestressing tendon and the percentage of prestressing force lost in the cable due to shrinkage, creep, etc. According to CP 115:1959 the loss of this prestress due to shrinkage, creep and relaxation of steel is approximately 22% of the initial prestress.

Crack detection was carried out by using a magnifying lens, a hand lamp and an illuminated microscope. The microscope read to 0·001 in., although it was possible to detect cracks as small as 0·0001 in. A proportional discontinuity in the reading of the electrical-resistance strain gauge at the tensile face also gave an indication of the development of cracks.

In reply to Mr Raju, the spreaders and cross-spreaders were mounted on rollers to ensure point loads. The horizontality of the surface was ensured by using a spirit level. The distance from the loading point to supports of each member was checked carefully. Moreover, the load dispersal was checked on two identical steel beams and the result was found to be very satisfactory. In the final test to destruction, each beam was loaded individually—only during the fatigue test were the two beams loaded simultaneously.

The equivalent *EI* diagram of Figure 6c was not drawn quantitatively and hence is not a direct measure of the stiffness of the beams tested. The equivalent *EI* of any section is the resultant of the creep hardening of the compressive zone and the creep softening of the tensile zone, the net effect of which influences the stiffness of the member.

It was stated before that the authors' hypothesis was based largely upon the results of the vast number of fatigue tests of concrete carried out by many investigators. They have no knowledge of the details of Mr Raju's tests and hence cannot comment on them. It should be noted, however, that Dr Bennett[1] obtained an increase in strength of up to 23% in the same laboratory when using the same fatigue testing machine as Mr Raju on similar test specimens. Mr Raju suggests that the increase in strength is due to more rapid drying out of the fatigue specimens. It is conceivable that there would be a marginal increase in strength resulting from more rapid hydration due to temperature increase until the specimens were 28 days old. The authors' test specimens were a minimum of 41 days old at testing, and the effects of hydration or drying would be negligible.[4]

The authors cannot agree with Mr Raju's suggestion that "any significant increase in stiffness of fatigued concrete" is absent. The increase in stiffness at higher loads can be seen in tests of Bate (reference 21 of the paper), of Verna et al.[2] and of many others reviewed in the paper. The increase in stiffness was also noted in several of the authors' tests on reinforced and prestressed concrete model slab bridges, a typical example of which is shown in Figure II[5]. The increase in stiffness of the compressive zone is markedly demonstrated at higher loads when, because there are deep cracks, the uncracked part of the section plays a significant role in deflexions.

The authors thank Mr Chandrasekhar for pointing out a typographical error.

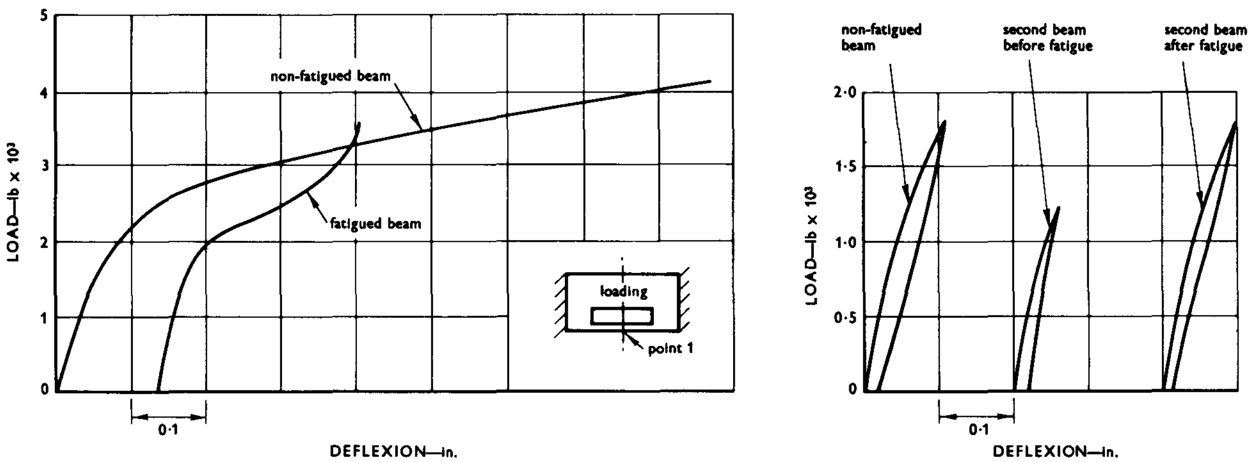

Figure II: Typical example of results of authors' tests on model bridge decks[4].

REFERENCES

1. BENNETT, E. W. and MUIR, S. E. ST. Some fatigue tests of high-strength concrete in axial compression. *Magazine of Concrete Research.* Vol. 19, No. 59. June 1967. pp. 113–117.
2. BENNETT, E. W. and RAJU, N. K. *Cumulative fatigue damage of plain concrete in compression*. Paper presented at International Conference on Structure, Solid Mechanics and Engineering Design. University of Southampton, April 1969. pp.94/1-94/14. (Proceedings to be published by John Wiley & Sons Inc.)
3. VERNA, J. R. and STELSON, T. E. Repeated loading effect on ultimate static strength of concrete beams. *Journal of the American Concrete Institute. Proceedings* Vol. 60, No. 6. June 1963. pp. 743–750.
4. TROXELL, G. E. and DAVIS, H. E. *Composition and properties of concrete.* First edition. New York, McGraw-Hill Book Company, Inc., 1956. p. 184.
5. SAWKO, F. and SAHA, G. P. *The effect of fatigue on the ultimate load behaviour of concrete bridge decks.* Paper presented at the Second International Symposium on Concrete Bridge Design, Chicago, April 1969.

CORRIGENDA

Page 25, Figure 5:
for $\times 10^3$ *read* $\times 10^4$ (*twice*)

Page 26, Table 3, Column headed 'Repeated loading test':
for 2,250† *read* 2,500†
(final) (final)

Page 28, column 2, first paragraph, last line but one:
for 0·0004 in. *read* 0·004 in.