

Allowing for membrane action in the plastic analysis of rectangular reinforced concrete slabs*

by B. Hayes, BSc, PhD

CORRIGENDA

Page 208: In equation 5 the first n should not be squared.

In the equation before equation 6, at the end of each expression in parentheses, $\sin \varphi$ should be in the denominator of the fraction.

Page 209: equation 21:

$$\text{for } \dots \frac{2K}{3 + g_0} \dots \text{ read } \dots \frac{2K}{3(3 - g_0)} \dots$$

equation 22:

$$\text{for } \dots + \frac{B'}{3} \dots \text{ read } \dots - \frac{B'}{3} \dots$$

Contribution by C. T. Morley, BA, PhD

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Dr Hayes has rightly pointed out that the membrane forces which can be developed in reinforced concrete slabs are limited in magnitude by the strength of portions of the slab acting as beams in their own plane. In the analysis based upon Figure 5, there are some puzzling assumptions, which are presumably introduced to simplify the algebra. Firstly, element 1 in plan is effectively a symmetrically loaded beam simply supported at points somewhere on the inclined yield lines; the bending moment in this beam is surely a maximum at mid-span, rather than at section EC? Secondly, at point B the normal membrane forces per unit length on the central and inclined yield lines are apparently taken to be equal although, as is easily seen in the case when the slab is cracked right through, these forces should differ, depending upon the ratio K between short-span and long-span steel.

The membrane forces given by equations 5 and 6 (or 18 and 19) are independent of deflexion. The predicted rigid-plastic load-deflexion curve is not then likely to be correct at zero deflexion, when membrane forces, if they do not vanish, will be constant along yield lines because neutral axes will be parallel to the slab surface. As Sawczuk (reference 3 of the paper)

makes clear, beyond some critical deflexion, W_c , behaviour of the type analysed by Dr Hayes will lead to lower loads, and so supersede the behaviour considered by Kemp (reference 2 of the paper) and others^(1,2) in which the membrane forces increase gradually with deflexion.

This explains why "the transverse in-plane bending hinges are observed to form after the main pattern of yield lines is established". Once the slab is cracked right through in the centre, these other methods predict load-deflexion curves which tend to a constant finite slope as deflexions become large. The question arises whether the critical deflexion, W_c , always exists: it is conceivable that, for some combinations of slab properties, the two plastic load-deflexion curves never intersect, and the in-plane bending hinges do not form.

Another interesting question is whether or not it is an experimental fact that elastic deflexions are such that measured load-deflexion curves first reach the rigid-plastic curve beyond the critical deflexion, W_c . The test results indicate that this may well be so, in which case the paper will prove to be of considerable practical importance.

Reply by the author

Dr Morley has raised a number of interesting points in connexion with the paper. Firstly, it must be stressed that the analysis was carried out to investigate the correlation between Sawczuk's energy approach

and an alternative equilibrium method. Therefore, a mechanism corresponding to that found to be critical by Sawczuk has been considered. Alternative mechanisms involving in-plane hinges at the middle of element

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It could also be investigated by the proposed method. Secondly, the simplified in-plane stress distribution employed in the analysis leads to close correlation with experimental results. It is conceded that, for orthotropic reinforcement, the assumed stress distribution violates the yield criterion when the slab is cracked right through. However, from the experimental results, it appears that this anomaly has only a marginal effect upon the solution. It would be of interest to investigate the sensitivity of the solution to changes in the assumed in-plane stress distribution.

The estimation of the point at which measured load-deflexion curves first reach the rigid-plastic curve is a problem pertinent to all methods of rigid-plastic analysis, whether the possibility of the formation of

in-plane bending hinges is considered or not. The results indicate that the concept of the formation of in-plane hinges is useful in predicting the eventual behaviour of the slab, although for some slab properties considerable deflexions are required to reach even the simple yield-line theory collapsed load.

REFERENCES

1. CALLADINE, C. R. Simple ideas in the large-deflexion plastic theory of plates and slabs. *Engineering Plasticity*—papers for the Conference held in Cambridge, March 1968. Cambridge University Press, 1968. pp. 93-127.
2. MORLEY, C. T. Yield-line theory for reinforced concrete slabs at moderately large deflexions. *Magazine of Concrete Research*. December 1967. Vol. 19, No. 61. pp. 211-222.